

[nex23] Statistically independent or merely uncorrelated?

Consider a classical spin, described by a 3-component unit vector

$$\mathbf{S} = (S_x, S_y, S_z) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta).$$

Let us assume that the spin has a completely random orientation, meaning a uniform distribution on the unit sphere. Show that the stochastic variables $\cos \theta, \phi$ are uncorrelated and statistically independent, whereas the stochastic variables S_x, S_z are uncorrelated but not statistically independent. This difference is testimony to the special role of canonical coordinates (here $\cos \theta, \phi$) in statistical mechanics.

Solution: