

[nex24] Transformation of statistical uncertainty.

From a given stochastic variable X with probability distribution $P_X(x)$ we can calculate the probability distribution of the stochastic variable $Y = f(X)$ via the relation

$$P_Y(y) = \int dx P_X(x) \delta(y - f(x)).$$

Show by systematic expansion that if $P_X(x)$ is sufficiently narrow and $f(x)$ sufficiently smooth, then the mean values and the standard deviations of the two stochastic variables are related to each other as follows:

$$\langle Y \rangle = f(\langle X \rangle), \quad \sigma_Y = |f'(\langle X \rangle)| \sigma_X.$$

Solution: