## [nex25] Poisson process.

Consider the discrete Poisson process specified by the master equation,

$$\frac{\partial}{\partial t} P(n,t) = \sum_m \Big[ W(n|m)P(m,t|0,0) - W(m|n)P(n,t|0,0) \Big], \qquad W(n|m) = \lambda \delta_{n-1,m},$$

for the discrete stochastic variable $n = 0, 1, 2, \ldots$ and with the initial condition $P(n,0|0,0) = \delta_{n,0}$. Convert the master equation into a differential equation for the generating function $G(z,t)$, then solve that equation, and determine $P(n,t|0,0)$ via power expansion.

Applications of the Poisson process include the following: (i) *Radioactive decay.* Macroscopic sample of radioactive nuclei observed over a time interval that is short compared to the mean decay time of individual nuclei. The average decay rate is $\lambda$. $P(n,t|0,0)$ is the probability that exactly $n$ nuclei have decayed until time $t$. (ii) *Shot noise.* Electrical current in a vacuum tube. Electrons arrive at the anode randomly. The average rate of arrivals is $\lambda$. $P(n,t|0,0)$ is the probability that exactly $n$ electrons have arrived at the anode until time $t$.

**Solution:**