[nex26] Markovian nature of diffusion process and Cauchy process.

Demonstrate that the *diffusion* process and the *Cauchy* process are Markov processes by showing that the respective conditional probability densities

$$P(x|x_0;\Delta t) = \frac{1}{\sqrt{4\pi D\Delta t}}\,\exp\left(-\frac{(x-x_0)^2}{4D\Delta t}\right), \qquad P(x|x_0;\Delta t) = \frac{1}{\pi}\frac{\Delta t}{(x-x_0)^2+(\Delta t)^2}$$

satisfy the (integral) Chapman-Kolmogorov equation

$$P(x_1|x_3;\Delta t_{13}) = \int dx_2\, P(x_1|x_2;\Delta t_{12})P(x_2|x_3;\Delta t_{23}) \quad \text{with} \quad \Delta t_{13} = \Delta t_{12} + \Delta t_{23}.$$

What property of the characteristic function $\Phi(k,\Delta t) = \int d(x-x_0)e^{ik(x-x_0)}P(x|x_0;\Delta t)$ is instrumental in this context?

Solution: