Markovian nature of diffusion process and Cauchy process.

Demonstrate that the diffusion process and the Cauchy process are Markov processes by showing that the respective conditional probability densities

\[ P(x|x_0; \Delta t) = \frac{1}{\sqrt{4\pi D\Delta t}} \exp \left( -\frac{(x-x_0)^2}{4D\Delta t} \right), \quad P(x|x_0; \Delta t) = \frac{1}{\pi} \frac{\Delta t}{(x-x_0)^2 + (\Delta t)^2} \]

satisfy the (integral) Chapman-Kolmogorov equation

\[ P(x_1|x_3; \Delta t_{13}) = \int dx_2 P(x_1|x_2; \Delta t_{12}) P(x_2|x_3; \Delta t_{23}) \quad \text{with} \quad \Delta t_{13} = \Delta t_{12} + \Delta t_{23}. \]

What property of the characteristic function \( \Phi(k, \Delta t) = \int d(x-x_0) e^{ik(x-x_0)} P(x|x_0; \Delta t) \) is instrumental in this context?

Solution: