[nex29] Drift equation.

The Fokker-Planck equation with no diffusive term reads

$$\frac{\partial}{\partial t} P(x,t|x_0,0) = -\frac{\partial}{\partial x}\,[A(x,t)P(x,t|x_0,0)],$$

Show that this drift equation has has a solution of the form

$$P(x,t|x_0,0) = \delta\big(x - x_S(t)\big),$$

where $x_S(t)$ is the solution of the (deterministic) equation of motion $dx/dt = A(x,t)$ with initial condition $x_S(0) = x_0$.

Comment: In the $6N$-dimensional phase space of a classical system of N interacting particles, the drift equation for a phase point $\mathbf{x}$ is the Liouville equation. There exists a solution of the form $\delta\big(\mathbf{x} - \mathbf{x}_S(t)\big)$, representing the motion of a phase point through phase space. The function $\mathbf{x}_S(t)$ is the solution of the canonical equations, which have the form $d\mathbf{x}/dt = \mathbf{A}(\mathbf{x},t)$. The function $\mathbf{A}(\mathbf{x},t)$ is constructed from the Poisson bracket of $\mathbf{x}$ with the system Hamiltonian.

Solution: