[nex32] Jump moments of discrete variables

Consider the master equation

$$\frac{d}{dt}P(n,t) = \sum_m [W(n|m)P(m,t) - W(m|n)P(n,t)]$$

of an integer random variable n for two stochastic processes:
(a) Random walk: $W(n|m) = \sigma\delta_{n+1,m} + \sigma\delta_{n-1,m}$.
(b) Poisson process: $W(n|m) = \lambda\delta_{n-1,m}$.
Calculate the jump moments $\alpha_l(m) = \sum_n (n-m)^l W(n|m)$ for $l = 1, 2$.
Then calculate the time evolution of the mean value $\langle n\rangle$ and the variance $\langle\langle n^2\rangle\rangle$, consistent with the initial condition $P(n,0) = \delta_{n,0}$. Rather than first calculating $P(n,t)$, solve the equations of motion for the expectation values: $d\langle n\rangle/dt = \langle\alpha_1(n)\rangle$, $d\langle n^2\rangle/dt = \langle\alpha_2(n)\rangle + 2\langle n\alpha_1(n)\rangle$.

Solution: