Jump moments of discrete variables

Consider the master equation

$$\frac{d}{dt} P(n, t) = \sum_m [W(n|m)P(m, t) - W(m|n)P(n, t)]$$

of an integer random variable $n$ for two stochastic processes:
(a) Random walk: $W(n|m) = \sigma \delta_{n+1,m} + \sigma \delta_{n-1,m}$.
(b) Poisson process: $W(n|m) = \lambda \delta_{n-1,m}$.

Calculate the jump moments $\alpha_l(m) = \sum_n (n - m)^l W(n|m)$ for $l = 1, 2$.

Then calculate the time evolution of the mean value $\langle n \rangle$ and the variance $\langle n^2 \rangle$, consistent with the initial condition $P(n, 0) = \delta_{n,0}$. Rather than first calculating $P(n, t)$, solve the equations of motion for the expectation values: $d\langle n \rangle/dt = \langle \alpha_1(n) \rangle$, $d\langle n^2 \rangle/dt = \langle \alpha_2(n) \rangle + 2\langle n \rangle \langle \alpha_1(n) \rangle$.

Solution: