[nex33] Random walk in one dimension: unit steps at random times

Consider the conditional probability distribution $P(n,t|0,0)$ describing an unbiased random walk in one dimension as determined by the master equation,

$$\frac{d}{dt}P(n,t|0,0) = \sum_m \Big[W(n|m)P(m,t|0,0) - W(m|n)P(n,t|0,0)\Big],$$

with transition rates

$$W(n|m) = \sigma\delta_{n+1,m} + \sigma\delta_{n-1,m}.$$

Here 2σ is the time rate at which the walker takes steps of unit size. The mean time interval between steps is then $\tau = 1/2\sigma$.

(a) Convert the master equation into an ordinary differential equation for the characteristic function, $\Phi(k,t) = \sum_n e^{ikn}P(n,t|0,0)$, solve it, and determine the probability distribution $P(n,t|0,0)$ from it via inverse Fourier transform.

(b) Set $n\ell = x$ for the position of the walker, where ℓ is the step size, and consider the limit $\ell \to 0, \sigma \to \infty$ such that $\ell^2\sigma = D$. Then determine $P(x,t|0,0)$ in this limit.

(c) Plot $P(n,t|0,0)$ versus n for various fixed t in comparison with the asymptotic $P(x,t|0,0)$.

Solution: