Random walk in one dimension: unit steps at unit times

Consider the conditional probability distribution \( P(n, t_{N+1}|0,0) \) describing a biased random walk in one dimension as determined by the (discrete) Chapman-Kolmogorov equation,

\[
P(n, t_{N+1}|0,0) = \sum_m P(n, t_{N+1}|m, t_N)P(m, t_N|0,0),
\]

where \( t_N = N\tau \) and

\[
P(n, t_{N+1}|m, t_N) = p\delta_{m,n-1} + q\delta_{m,n+1}
\]

expresses the instruction that the walker takes a step of unit size forward (with probability \( p \)) or backward (with probability \( q = 1-p \)) after one time unit \( \tau \). Convert this equation into an equation for the characteristic function \( \Phi(k, t_N) = \sum_n e^{ikn} P(n, t_N|0,0) \), then solve that equation, and determine \( P(n, t_N|0,0) \) from it, all by elementary means.

Solution: