[nex34] Random walk in one dimension: unit steps at unit times

Consider the conditional probability distribution $P(n, t_N|0, 0)$ describing a biased random walk in one dimension as determined by the (discrete) Chapman-Kolmogorov equation,

$$P(n, t_{N+1}|0, 0) = \sum_m P(n, t_{N+1}|m, t_N)P(m, t_N|0, 0),$$

where $t_N = N\tau$ and

$$P(n, t_{N+1}|m, t_N) = p\delta_{m,n-1} + q\delta_{m,n+1}$$

expresses the instruction that the walker takes a step of unit size forward (with probability p) or backward (with probability $q = 1-p$) after one time unit τ. Convert this equation into an equation for the characteristic function $\Phi(k, t_N) = \sum_n e^{ikn}P(n, t_N|0, 0)$, then solve solve that equation, and determine $P(n, t_N|0, 0)$ from it, all by elementary means.

Solution: