

[nex36] Free particle with uncertain position and velocity

Consider a physical ensemble of free particles with unit mass moving along the x -axis. The initial positions and velocities, x_0, v_0 , are specified by a Gaussian joint probability distribution: $P_0(x_0, v_0) = (2\pi)^{-1} \exp(-x_0^2/2 - v_0^2/2)$.

(a) Find the joint probability distribution $P(x, v; t)$ for the position and velocity at time t . Infer from this result the probability distributions $P(x; t)$, $P(v; t)$ for the position and the velocity separately. Calculate the average position $\langle x(t) \rangle$ and the variance $\langle\langle x^2(t) \rangle\rangle$ thereof.

(b) Find the conditional probability distribution $P(x|v; t)$ for the positions x at time t of particles that have velocity v . Calculate the conditional averages $\langle x^n(t)|v \rangle \equiv \int dx x^n P(x|v; t)$, $n = 1, 2$ for the positions of particles that have velocity v , and infer from these results the conditional variance $\langle\langle x^2(t)|v \rangle\rangle$.

Solution: