

### [nex38] Life expectancy of the ever young

The probability distribution of lifetimes in some population is  $f(t)$  with an average lifetime

$$T = \int_0^{\infty} dt t f(t)$$

for individuals.

(a) Show that the conditional probability distribution for the remaining lifetime of individuals of age  $\tau$  is

$$P_c(t|\tau) = \frac{f(t)}{C(\tau)} \theta(t - \tau), \quad C(\tau) \doteq \int_{\tau}^{\infty} dt f(t),$$

where  $\theta(t)$  is the Heaviside step function.

(b) If we define the *life expectancy*  $T_{\tau}$  as the average remaining lifetime for an individual of age  $\tau$  express  $T_{\tau}$  in terms of  $P_c(t|\tau)$ .

(c) Find the function  $f(t)$  for a population (e.g. free neutrons) whose life expectancy is independent of the age of the individual, i.e. for the case where  $T_{\tau} = T$  holds. Then infer an explicit expression for the conditional probability distribution  $P_c(t|\tau)$ .

**Solution:**