[nex41] Ornstein−Uhlenbeck process: general solution.

(a) Show that the Fokker-Planck equation of the Ornstein-Uhlenbeck process can be solved by separation of variables and that the general solution can be expressed in terms of Hermite polynomials:

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial x}(\kappa x P) + \frac{1}{2}\gamma\frac{\partial^2 P}{\partial x^2}; \qquad P(x,t) = \sum_{n=0}^{\infty} a_n H_n\left(\sqrt{\frac{\kappa}{\gamma}}\,x\right) e^{-n\kappa t} e^{-\kappa x^2/\gamma}.$$

(b) Show that a unique stationary solution $P_S(x)$ is approached in the limit $t \to \infty$ for arbitrary of initial conditions.
(c) Determine the expansion coefficients a_n for the particular initial distribution $P(x,0) = \delta(x - x_0)$.

Solution: