Consider the master equation
\[
\frac{d}{dt} P(n, t) = \sum_m [W(n|m)P(m, t) - W(m|n)P(n, t)]
\]
for the probability distribution \( P(n, t) \) of the linear birth-death process. It is specified by the transition rates

\[
W(m|n) = n\lambda \delta_{m,n+1} + n\mu \delta_{m,n-1},
\]
where \( \lambda \) and \( \mu \) represent the birth and death rates, respectively, of individuals in some population.

(a) Determine the jump moments \( \alpha_l(m) = \sum_n (n - m)^l W(n|m) \) for \( l = 1, 2 \).

(b) Calculate the time evolution of the mean value \( \langle n \rangle \) and the variance \( \langle \langle n^2 \rangle \rangle \) for the initial condition \( P(n, 0) = \delta_{n,n_0} \) by solving the equations of motion for the expectation values,

\[
\frac{d}{dt} \langle n \rangle = \langle \alpha_1(n) \rangle, \quad \frac{d}{dt} \langle n^2 \rangle = \langle \alpha_2(n) \rangle + 2\langle n\alpha_1(n) \rangle,
\]
introduced in [nlh59].

(c) Plot \( \langle \langle n^2 \rangle \rangle \) versus \( t \) for three cases with (i) \( \lambda > \mu \), (ii) \( \lambda = \mu \), and (iii) \( \lambda < \mu \). Interpret the shape of each curve.

**Solution:**