

**[nex45] Random light switch.**

The position of the light switch is described by the stochastic variable  $X$ , which can assume the two values  $x = 0$  (lights off) and  $x = 1$  (lights on). Some agent switches the lights on/off randomly at the rate  $\gamma$ . This means that the average interval of continuous brightness/darkness is  $\tau = 1/\gamma$ .

- (a) Set up the master equation for  $P(x, t|x_0)$  and solve it.
- (b) Find the asymptotic distribution  $P_s(x) = \lim_{t \rightarrow \infty} P(x, t|x_0)$ .
- (c) Find the conditional average  $\langle X(t)|x_0 \rangle \doteq \sum_x x P(x, t|x_0)$  and then  $\langle X(t) \rangle_s = \lim_{t \rightarrow \infty} \langle X(t)|x_0 \rangle$ .
- (d) Use the regression theorem  $\langle X(t)X(t') \rangle_s \doteq \sum_{xx'} P(x, t|x', t') P_s(x')$  to determine the (stationary) autocorrelation function  $\langle \langle X(t)X(t') \rangle \rangle_s \doteq \langle X(t)X(t') \rangle_s - \langle X(t) \rangle_s \langle X(t') \rangle_s$ .

**Solution:**