Air in leaky tank III: detailed balance

A tank of volume $V$ has a small leak and exchanges molecules of air with the environment. The environment has a constant density $\rho$ of molecules. The master equation for the probability distribution $P(n,t)$ of air molecules in the container is specified by transition rates of the form

$$W(m|n) = T_+(n)\delta_{m,n+1} + T_-(n)\delta_{m,n-1}$$

with $T_+(n) = \rho$ and $T_-(n) = n/V$.

(a) Determine the stationary distribution $P_s(n) = P(n,t \to \infty)$ from the detailed balance condition, $T_-(n)P_s(n) = T_+(n-1)P_s(n-1)$, via the recurrence relation derived in [nln17].

(b) Compare the peak position $n_p$ of the stationary distribution with the mean value $\langle n \rangle$.

Solution: