

**[nex49] Air in leaky tank III: detailed balance**

A tank of volume  $V$  has a small leak and exchanges molecules of air with the environment. The environment has a constant density  $\rho$  of molecules. The master equation for the probability distribution  $P(n, t)$  of air molecules in the container is specified by transition rates of the form

$$W(m|n) = T_+(n)\delta_{m,n+1} + T_-(n)\delta_{m,n-1}$$

with  $T_+(n) = \rho$  and  $T_-(n) = n/V$ .

(a) Determine the stationary distribution  $P_s(n) = P(n, t \rightarrow \infty)$  from the detailed balance condition,  $T_-(n)P_s(n) = T_+(n-1)P_s(n-1)$ , via the recurrence relation derived in [nlh17].

(b) Compare the peak position  $n_p$  of the stationary distribution with the mean value  $\langle n \rangle$ .

**Solution:**