## [nex50] Pascal distribution and Planck radiation law

Consider a quantum harmonic oscillator in thermal equilibrium at temperature $T$. We know from [nex22] that the population of energy levels $E_n = n\hbar\omega_0, n = 0, 1, 2, \ldots$ is described by the Pascal distribution,

$$P(n) = (1-\gamma)\gamma^n, \quad \gamma = \exp(-\hbar\omega_0/k_BT).$$

Suppose that the heat bath is provided by a blackbody radiation field of energy density $u(\omega)$ and that the oscillator interacts with that field exclusively via emission and absorption of photons with energy $\hbar\omega_0$. If we describe the aproach to thermal equilibrium of the oscillator by a master equation, the transition rates must, therefore, be of the type, $W(m|n) = T_+(n)\delta_{m,n+1} + T_-(n)\delta_{m,n-1}$, familiar from birth-death processes (see [nln17]). Here $T_+(n)$ reflects the absorption of a photon and $T_-(n)$ the emission of a photon if the oscillator is in energy level $n$.

Now we assume (as Einstein did) that the transition rates are of the form $T_+(n) = Bu(\omega_0)$ and $T_-(n) = A + Bu(\omega_0)$, where the term $A$ reflects spontaneous emission and the terms $Bu(\omega_0)$ induced emission or induced absorption. Infer from the compatibility condition of these transition rates with the Pascal distribution at equilibrium the $T$-dependence of $u(\omega)$ at fixed frequency $\omega_0$.

**Solution:**