[nex52] Bistable chemical system

Consider the master equation of the birth-death process specified by transition rates of the form $W(m|n) = T_+(n)\delta_{m,n+1} + T_-(n)\delta_{m,n-1}$ with

$$T_+(n) = k_1 An(n-1) + k_3 A, \qquad T_-(n) = k_2 n(n-1)(n-2) + k_4 n.$$

This process describes two simultaneous chemical reactions $A + 2X \leftrightarrow 3X, A \leftrightarrow X$ that exhibit bistable states in a certain parameter range. The concentration of A is taken to be constant.
(a) Construct a product expression for $P_s(n)$ from the detailed-balance condition as explained in [nln17]. Use the three parameters $B = k_1 A/k_2, R = k_4/k_2, Q = k_3/k_1$.
(b) Show that that for $R/Q = 1$ we thus obtain the Poisson distribution.
(c) Plot the solution of the extremum condition $T_+(n-1) = T_-(n)$ as a graph B versus n over the range $0 \leq n \leq 100$ for the two cases (i) $Q = 100$, $R = 500$ and (ii) $Q = 100$, $R = 1200$. Identify the extremum positions n_{extr} for $B = 70$ in both cases.
(d) Plot $P_s(n)$ versus n over the range $0 \leq n \leq 100$ for $B = 70$ and cases (i), (ii) for Q, R.

Solution: