[nex53] Formal solution of Langevin equation

Consider a Brownian particle of mass m constrained to move along a straight line. The particle experiences two forces: a drag force $-\gamma\dot{x}$ and a white-noise random force $f(t)$. The Langevin equation, which governs its motion, is expressed as follows:

$$\frac{dx}{dt} = v, \qquad \frac{dv}{dt} = -\frac{\gamma}{m}\,v + \frac{1}{m}\,f(t).$$

Calculate, via formal integration, the functional dependence of (a) the velocity $v(t)$ and (b) the position $x(t)$ on the random force $f(t)$ for initial conditions $x(0) = 0$ and $v(0) = v_0$. For part (a) use the standard solution for the initial-value problem:

$$\frac{dy}{dt} = -ay + b(t) \quad \Rightarrow \; y(t) = y_0 e^{-at} + \int_0^t dt' e^{-a(t-t')} b(t').$$

For part (b) integrate by parts to arrive at the result

$$x(t) = v_0 \frac{m}{\gamma}\left(1 - e^{-\gamma t/m}\right) + \frac{1}{\gamma}\int_0^t dt' \left(1 - e^{-\gamma(t-t')/m}\right) f(t').$$

Solution: