[nex54] Autocorrelation function of Wiener process.

The conditional probability distribution,

$$P(x+\Delta x, t+dt|x,t) = \frac{1}{\sqrt{4\pi D\, dt}} \exp\left(-\frac{(\Delta x)^2}{4D\, dt}\right),$$

which characterizes the realization of a Wiener process, depends on ly on dt but not on t. Use the regression theorem,

$$\langle x(t)x(t+dt)|[0,0]\rangle = \int dx_1 \int dx_2\, x_1 x_2 P(x_2, t+dt|x_1, t) P(x_1, t|0, 0),$$

to show that the autocorrelation function only depends on t but not on dt. Find that dependence.

Solution: