[nex55] Velocity correlation function of Brownian particle I

Consider a Brownian particle of mass m constrained to move along a straight line. The particle experiences two forces: a drag force $-\gamma\dot{x}$ and an uncorrelated (white-noise type) random force $f(t)$. Calculate the velocity autocorrelation function $\langle v(t_1)v(t_2)\rangle_0$ of a Brownian particle for $t_1 > t_2$ as a conditional average from the formal solution (see [nex53])

$$v(t) = v_0 e^{-\gamma t/m} + \frac{1}{m}\int_0^t dt'\, e^{-(\gamma/m)(t-t')} f(t')$$

of the Langevin equation with a random force of intensity I_f. Show that for $t_1 > t_2 \gg \gamma/m$ the result only depends on the time difference $t_1 - t_2$. Use equipartition, $\frac{1}{2}m\langle v^2\rangle = \frac{1}{2}k_\mathrm{B}T$, to determine the temperature dependence of the random-force intensity I_f.

Comment: By conditional average we mean that the initial velocity has the value v_0. For $t_1 > t_2 \gg \gamma/m$ the memory of that initial condition fades away.

Solution: