Consider a Brownian particle of mass $m$ constrained to move along a straight line. The particle experiences two forces: a drag force $-\gamma v$ and a white-noise random force $f(t)$. In [nex118] we inferred from the Langevin equation an ODE for the mean-square displacement and solved it to obtain

$$\langle x^2(t) \rangle = 2D \left[ t - \frac{m}{\gamma} \left( 1 - e^{-\gamma t/m} \right) \right].$$

Here the task is to calculate $\langle x^2(t) \rangle$ from the (steady-state) velocity autocorrelation function,

$$\langle v(t_1)v(t_2) \rangle = \frac{k_B T}{m} e^{-(\gamma/m)|t_1-t_2|}$$

determined in [nex55], via integration with initial condition $x(0) = 0$.

Solution: