[nex57] Mean-square displacement of Brownian particle II

Consider a Brownian particle of mass m constrained to move along a straight line. The particle experiences two forces: a drag force $-\gamma v$ and a white-noise random force $f(t)$. In [nex118] and [nex56] we have taken two different routes to calculate the mean-square displacement,

$$\langle x^2(t)\rangle = 2D\left[t - \frac{m}{\gamma}\left(1 - e^{-(\gamma/m)t}\right)\right], \tag{1}$$

from the Langevin equation. The task here is to derive (1) directly from the formal solution (obtained in [nex53]),

$$x(t) = v_0\frac{m}{\gamma}\left(1 - e^{-(\gamma/m)t}\right) + \frac{1}{\gamma}\int_0^t ds\left(1 - e^{-(\gamma/m)(t-s)}\right)f(s), \tag{2}$$

of the Langevin equation with a white-noise random force. That random force is uncorrelated, $\langle f(t)f(t')\rangle = I_f\delta(t-t')$ and has intensity $I_f = 2k_BT\gamma$. Use equipartition, $\frac{1}{2}m\langle v^2\rangle = \frac{1}{2}k_BT$, when taking the thermal average $\langle v_0^2\rangle$ of initial velocities v_0.

Solution: