[nex6] Chebyshev's inequality

Chebyshev's inequality is a rigorous relation between the standard deviation $\sigma_X = \sqrt{\langle X^2 \rangle - \langle X \rangle^2}$ of the random variable X and the probability of deviations from the mean value $\langle X \rangle$ greater than a given magnitude a.

$$P[(x - \langle X \rangle)^2 > a^2] \leq \left(\frac{\sigma_X}{a}\right)^2$$

Prove Chebyshev's inequality starting from the following relation, commonly used for the transformation of stochastic variables (as in [nln49]):

$$P_Y(y) = \int dx\, \delta(y - f(x)) P_X(x) \text{ with } f(x) = (x - \langle X \rangle)^2.$$

Solution: