[nex60] Brownian harmonic oscillator VI: nonequilibrium correlations

Use the formal solution for the velocity from [nex59],

$$v(t) = v_0 e^{-\Gamma t} c(t) - \frac{\omega_0^2}{\Omega_1} x_0 e^{-\Gamma t} \sinh \Omega_1 t + \frac{1}{m} \int_0^t dt' f(t') e^{-\Gamma(t-t')} c(t-t'),$$

with $\Gamma = \gamma/2m$, $\Omega_1 = \sqrt{\Gamma^2 - \omega_0^2}$, $c(t) = \cosh \Omega_1 t - (\Gamma/\Omega_1) \sinh \Omega_1 t$ of the Langevin-type equation, $m\ddot{x} + \gamma\dot{x} + kx = f(t)$, for the overdamped Brownian harmonic oscillator with mass m, damping constant γ, spring constant $k = m\omega_0^2$, initial conditions $x(0) = x_0$ and $v(0) = v_0$, and white-noise random force $f(t)$ with intensity I_f to calculate the velocity correlation function $\langle v(t_2)v(t_1)\rangle$ for the nonequilibrium state. Then take the limit $t_1, t_2 \to \infty$ with $0 < t_2 - t_1 < \infty$ to recover the result of [nex58] for the stationary state.

Solution: