[nex63] Causality property of response function.

The Kramers-Kronig dispersion relations

$$\chi'_{AA}(\omega) = \frac{1}{\pi}\,\mathrm{P}\int_{-\infty}^{\infty} d\omega' \frac{\chi''_{AA}(\omega')}{\omega' - \omega}, \quad \chi''_{AA}(\omega) = -\frac{1}{\pi}\,\mathrm{P}\int_{-\infty}^{\infty} d\omega' \frac{\chi'_{AA}(\omega')}{\omega' - \omega}$$

between the reactive part $\chi'_{AA}(\omega)$ and the dissipative part $\chi''_{AA}(\omega)$ of the generalized susceptibility $\chi_{AA}(\omega)$ are a direct consequence of the causality property of the response function $\tilde{\chi}_{AA}(t)$. Show that $\chi_{AA}(\zeta)$ for $\Im(\zeta) > 0$ can be expressed in terms of $\chi''_{AA}(\omega)$ as follows:

$$\chi_{AA}(\zeta) = \frac{1}{\pi}\int_{-\infty}^{\infty} d\omega \frac{\chi''_{AA}(\omega)}{\omega - \zeta}.$$

Solution: