## [nex64] Reactive and absorptive parts of linear response.

In the framework of linear response theory for $H = H_0 - a(t)A$, the rate of energy transfer between the system and the radiation field is

$$\frac{d}{dt}\langle H_0\rangle = \int_{-\infty}^{\infty} dt' a(t)a(t')\frac{\partial}{\partial t}\tilde{\chi}_{AA}(t-t'), \tag{1}$$

where

$$\tilde{\chi}_{AA}(t-t') = \frac{i}{\hbar}\theta(t-t')\langle[A(t), A(t')]\rangle_0 \tag{2}$$

is the Kubo formula for the response function (see [nln38].)
(a) Evaluate this expression for a monochromatic perturbation,

$$a(t) = \frac{1}{2}a_m(e^{i\omega_0 t} + e^{-i\omega_0 t}) \tag{3}$$

and express it in terms of the reactive part, $\chi'_{AA}(\omega)$, and the absorptive (dissipative) part, $\chi''_{AA}(\omega)$, of the generalized susceptibility $\chi_{AA}(\omega)$ as defined in [nln26].
(b) Show that the time-averaged energy transfer depends only on the absorptive part of $\chi_{AA}(\omega)$:

$$\overline{\frac{d}{dt}\langle H_0\rangle} = \frac{1}{2}a_m^2\omega_0\chi''_{AA}(\omega_0). \tag{4}$$

**Solution:**