[nex68] Zwanzig's kinetic equation.

In the derivation of Zwanzig's kinetic equation,

$$\frac{\partial}{\partial t}\rho_1(t) = -i\hat{P}\hat{L}\rho_1(t) - i\hat{P}\hat{L}e^{-i\hat{Q}\hat{L}t}\rho_2(0) - \int_0^t d\tau\, \hat{P}\hat{L}e^{-i\hat{Q}\hat{L}\tau}\hat{Q}\hat{L}\rho_1(t-\tau), \tag{1}$$

from two projections of the Liouville equation,

$$\hat{P}\,\frac{\partial\rho}{\partial t} = \frac{\partial\rho_1}{\partial t} = -i\hat{P}\hat{L}[\rho_1+\rho_2], \qquad \hat{Q}\,\frac{\partial\rho}{\partial t} = \frac{\partial\rho_2}{\partial t} = -i\hat{Q}\hat{L}[\rho_1+\rho_2], \tag{2}$$

we postulate the formal solution

$$\rho_2(t) = e^{-i\hat{Q}\hat{L}t}\rho_2(0) - i\int_0^t d\tau\, e^{-i\hat{Q}\hat{L}\tau}\hat{Q}\hat{L}\rho_1(t-\tau). \tag{3}$$

Verify that (3) is a solution of (2). The derivation involves one integration by parts.

Solution: