

[nex71] Structure function of harmonic oscillator I.

Consider the quantum harmonic oscillator (for $\hbar = 1$),

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega_0^2 q^2 = \omega_0 \left(a^\dagger a + \frac{1}{2} \right),$$

where $q = (a^\dagger + a)/\sqrt{2m\omega_0}$, $p = i\sqrt{m\omega_0/2}(a^\dagger - a)$ relate the position and momentum operators ($[q, p] = i$) to the boson creation and annihilation operators ($[a, a^\dagger] = 1$).

Use the recursion method with inner product $\langle A|B\rangle = \frac{1}{2}(\langle AB\rangle + \langle BA\rangle)$ to calculate the structure function $S_{qq}(\omega)$ for the position variable at temperature T , where $\langle a^\dagger a\rangle = n_B = (e^{\beta\omega} - 1)^{-1}$, $\beta = 1/k_B T$.

Solution: