

**[nex78] Transforming a pair of random variables**

Consider two independent random variables  $X_1, X_2$  that are uniformly distributed on the intervals  $0 \leq x_1, x_2 \leq 1$ . Show that the transformed variables

$$Y_1 = \sqrt{-2 \ln X_1} \cos 2\pi X_2, \quad Y_2 = \sqrt{-2 \ln X_1} \sin 2\pi X_2$$

obey independent normal distributions:

$$P_{\mathbf{Y}}(y_1, y_2) = \frac{1}{\sqrt{2\pi}} e^{-y_1^2/2} \frac{1}{\sqrt{2\pi}} e^{-y_2^2/2}.$$

**Solution:**