Transforming a pair of random variables

Consider two independent random variables $X_1, X_2$ that are uniformly distributed on the intervals $0 \leq x_1, x_2 \leq 1$. Show that the transformed variables

$$Y_1 = \sqrt{-2 \ln X_1} \cos 2\pi X_2, \quad Y_2 = \sqrt{-2 \ln X_1} \sin 2\pi X_2$$

obey independent normal distributions:

$$P_Y(y_1, y_2) = \frac{1}{\sqrt{2\pi}} e^{-y_1^2/2} \frac{1}{\sqrt{2\pi}} e^{-y_2^2/2}.$$ 

Solution: