

[nex79] Exponential integral distribution

Consider two independent random variables X_1, X_2 , one exponentially distributed, $P_1(x_1) = e^{-x_1}$, $0 < x_1 < \infty$, and the other uniformly distributed, $P_2(x_2) = 1$, $0 < x_2 < 1$.

- (a) Determine the probability distribution $P_Z(z)$ of the random variable $Z = X_1X_2$ for $0 < z < \infty$.
- (b) Determine the asymptotic properties of $P_Z(z)$ for $z \rightarrow 0$ and for $z \rightarrow \infty$.
- (c) Calculate the moments $\langle z^n \rangle$ of $P_Z(z)$.
- (d) Plot $P_Z(z)$ for $0 < z < 6$.

Solution: