

[nex81] Stable probability distributions

Consider N independent random variables X_1, \dots, X_N , each having the same probability distribution $P_X(x)$. If the probability distribution of the random variable $Y_N = X_1 + \dots + X_N$ can be written in the form $P_Y(y) = P_X(y/c_N + \gamma_N)/c_N$, then $P_X(x)$ is *stable*. The multiplicative constant must be of the form $c_N = N^{1/\alpha}$, where α is the *index* of the stable distribution. $P_X(x)$ is *strictly stable* if $\gamma_N = 0$.

Use the results of [nex19] to determine the indices α of the Gaussian and Lorentzian distributions, both of which are both strictly stable. Show that the Poisson distribution is not stable in the technical sense used here.

Solution: