[nex97] Lindeberg condition for diffusion and Cauchy processes

Show that the Lindeberg condition for continuity of sample paths,

$$\lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_{|x-x_0|>\epsilon} dx\, P(x|x_0;\Delta t) = 0,$$

is satisfied by the diffusion process but violated by th Cauchy process. They are specified, respectively, by the conditional probability distributions,

$$P(x|x_0;\Delta t) = \frac{1}{\sqrt{4\pi D\Delta t}}\exp\left(-\frac{(x-x_0)^2}{4D\Delta t}\right), \qquad P(x|x_0;\Delta t) = \frac{1}{\pi}\frac{\Delta t}{(x-x_0)^2+(\Delta t)^2}.$$

The condition requires that the probability for the final position x to deviate a finite distance from the initial position x_0 vanishes faster than the time step Δt in the limit $\Delta t \to 0$.

Solution: