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Citation: American Journal of Physics 30, 93 (1962); doi: 10.1119/1.1941953
View online: http://dx.doi.org/10.1119/1.1941953
View Table of Contents: http://scitation.aip.org/content/aapt/journal/ajp/30/2?ver=pdfcov
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acoustics, where physiological reactions should also be considered. The dimensions derived from the fundamental $[L]$, $[T]$, $[Q]$, and $[\phi]$ dimensions are very simple and explicit.

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ACKNOWLEDGMENT

The author wishes to thank Professor L. A. Kramer of Purdue University for proofreading the manuscript.

Classical Models of Radiative Decay*

CHARLES KAUFMAN AND ROLF G. WINTER

Department of Physics, The Pennsylvania State University, University Park, Pennsylvania
(Received July 12, 1961)

Classical models for radiative fundamental particle decay are examined. Calculations are presented for radiative nuclear beta processes, pion decay, and muon decay. At least some features of the photon spectra are obtained correctly for these examples.

Although only proper quantum calculations show all features of radiative transitions, classical models of these processes are of some interest. Such models help us form intuitively satisfactory pictures of complex processes and sometimes suggest new questions.

If a radiative transition is between discrete states, a sharp line spectrum results. The "correspondence" treatment of these spectra was a great aid before the development of a consistent quantum theory of radiation. We can still make pedagogically interesting pictures of such processes. For example, in $e^-\gamma^+$ annihilation, we might say that the two particles form a current as they rush together, that the size of the region of annihilation is $\lambda_\gamma = \hbar / mc$, that therefore the time during which the current is turned off is $\lambda_\gamma / c$, and that the angular frequency of the emitted radiation is "therefore" of the order of the reciprocal of this time, or $mc^2 / \hbar$. Most of such arguments are rather forced a posteriori constructions of little use.

If the transition leads to a state in the continuum, the spectrum of the radiation will generally contain all frequencies from zero up to some maximum. It is for such radiation, and particularly for the low-frequency part of the spectrum, that classical discussions are most sensible. For example, the low-energy part of the Coulomb scattering bremsstrahlung spectrum can be obtained very well classically. It is the purpose of this note to discuss classical models for several radiative fundamental particle transitions during which a continuum is emitted.

MODELS

We need expressions for the classical radiation from certain time-dependent current distributions. For each such distribution $j(r,t)$, the energy radiated into the solid angle $d\Omega$ at $\theta, \phi$ in the angular frequency band between $\omega$ and $\omega + d\omega$ is

$$w(\theta, \phi, \omega) d\Omega d\omega = \frac{\omega^2}{c^3}$$

$$\times \left| n \times \int j_w(r) \exp \left( \frac{i \omega r \cdot n}{\hbar} \right) dr \right|^2 d\Omega d\omega.$$  

The unit vector $n$ points from the origin to the

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* Largely based on a thesis submitted by C. Kaufman in partial fulfillment of the requirements for the M.S. degree in physics at The Pennsylvania State University, June, 1959. Supported by the National Science Foundation.


observer, and the Fourier transform of the current is given by
\[ j_\nu(r) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} j(t) e^{-i\omega t} dt. \]

To interpret the result in terms of photons, we use, for the number of photons emitted into the solid angle \( d\Omega \) in the angular frequency band \( d\omega \),
\[ N(\theta, \phi, \omega) d\Omega d\omega = \omega(\theta, \phi, \omega) d\Omega d\omega / \hbar c. \]

First, consider the creation of a particle with charge \( e \) and velocity \( v_0 \):
\[ j_r(r, t) = 0, \quad t < 0, \]
\[ = e v_0 \delta(x) \delta(y) \delta(z - vt) (1 - e^{-i\omega t}) e^{-i\omega t}, \quad t \geq 0. \]

The time \( \tau \) determines the rate at which the current rises to its final value. The factor \( \exp(-t/T) \) is introduced to give convergent integrals; we let \( T \to \infty \) after integration. With \( \alpha = e^2 / \hbar c \approx 1/137 \) and \( \beta = v/c, \) we obtain the spectrum radiated by a charged particle during its emission:
\[ N_e(\beta, \theta, \phi, \omega) = \frac{\alpha e^2}{4 \pi^2 \omega} \sin^2 \theta \]
\[ \times \left[ 1 - \frac{(1 - \beta \cos \theta)^2}{(1 - \beta \cos \theta)^2 + 1/\omega^2} \right]. \]

Integration over angles yields, for the total emission at \( \omega, \)
\[ N_e(\beta, \omega) = \frac{\alpha e^2}{\pi \omega} \left[ \frac{1}{1 - \beta} - 1 \right], \quad (1 - \beta) \]

where
\[ f = \frac{1 - \omega^2 \tau^2 - \beta^2 \omega^2}{2 \omega^2 \tau^2}. \]

For small \( \omega \tau, f \approx \omega^2 \tau^2 (\beta + \frac{1}{2}), \) so that it does not affect the low-energy part of the spectrum. Except for the introduction of the rise time \( \tau \) and the resultant \( f \) in the spectrum, this calculation is the same as that used by Chang and Falkoff.\(^2\)

If the created particle has a magnetic moment, there will be additional radiation. We choose as a model the creation of a loop of radius \( a \) carrying a current \( I, \) traveling in the \( z \) direction with its plane perpendicular to the \( z \) axis. In cylindrical coordinates,
\[ j_r(r, t) = 0, \quad t < 0 \]
\[ = I (\epsilon_x \sin \phi + \epsilon_y \cos \phi) \delta(r-a) \delta(z-vt) \]
\[ \times (1 - e^{-i\omega t}) e^{-i\omega t}, \quad t \geq 0. \]

With the choice
\[ \pi a^2 I / e = \mu e k / 2mc, \]
the current loop has the desired magnetic moment, chosen so that \( \mu = 1 \) for a particle of spin \( \frac{1}{2} \) without an anomalous moment. We obtain
\[ N_m(\beta, \theta, \phi, \omega) = \frac{\alpha e^2 k^2}{4 \pi^2 a^2 m c \omega} \frac{J_1^2(\omega \sin \theta / c)}{(1 - \beta \cos \theta)^2 \left[ 1 - \frac{(1 - \beta \cos \theta)^2}{(1 - \beta \cos \theta)^2 + 1/\tau^2} \right]^2}. \]

If the radius \( a \) of the loop is much less than the wavelength of the radiation, we can expand the Bessel function:
\[ J_1(\omega \sin \theta / c) \approx \frac{\omega \sin \theta}{2c}. \]

With this replacement, the angular distributions for \( N_m \) and \( N_e \) become the same, and
\[ N_m / N_e = \left[ (\mu \omega) / (2mc^2) \right]^2. \]

If we are interested in the disappearance at rest of a magnetic moment, as in electron capture, we consider a loop in which the current decays, as used by one of us previously to discuss radiative \( K \) capture.\(^3\) The results are
\[ N_K(\theta, \omega) = \frac{\alpha}{16 \pi^2 m c^2} \frac{\mu^2}{1 + \omega^2 \tau^2}, \]
\[ \frac{1}{6 \pi m e^2} \frac{\mu^2}{1 + \omega^2 \tau^2}. \]

As one should expect, the expressions are just the \( \beta \to 0 \) limit of \( N_m. \)

**CHOICE OF \( \tau \)**

One of the essential features of quantum theory is the instantaneous, discontinuous nature of transitions. To construct sensible classical models, however, we must treat transitions as if

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they evolved continuously; we have used the time $\tau$ above to describe the duration of the transitions. We choose $\tau = \hbar/E$, where $E$ is the total energy of the transition; measurement of the time of transition with an accuracy better than $\tau$ would disturb the system appreciably. While this choice has an aura of plausibility, it is made because it works, not because it can be derived. This arbitrariness is an inevitable consequence of any attempt to make classical models for quantum processes.

**ENERGY REDUCTION EFFECTS**

The probability of a radiative transition is not simply the product of the probabilities of the nonradiative transition and of radiation. The emission of the photon reduces the energy available to the other particles; even if the matrix elements are energy independent, the phase space volume available to the other particles is reduced. The closest classical analog of this phenomenon is the effect that radiative reaction has on the behavior of the system. We do not attempt to include such effects. Radiation for which $\hbar \omega$ is a small fraction of the total available energy should be predicted correctly, but the probability of high-energy radiation will be overestimated. The term $f$ in Eq. (1) and the magnetic radiation given by Eq. (3) should be viewed with distrust, since both are important only in the high-energy region.

**RADIATIVE BETA DECAY**

Radiative capture of $l=0$ electrons has been treated semiclassically by one of us before. We are concerned here with the shrinking of a spherically symmetric charge distribution, which gives no radiation, and the reduction of a magnetic moment. We use the expression for $N_K$ given by Eq. (6) above and obtain a satisfactory classical model. Both the total number of photons and the low-energy part of the spectrum are given correctly.

The situation is radically different for the radiative capture of $l=1$ electrons. There exists an intense, broad peak in $N(\omega)$ in the $K$ x-ray region, according to both experiment and accurate theory. There seems to be no classical model that shows this behavior; all such models yield an $N(\omega)$ that is small near $\omega = 0$. This failure may seem puzzling. We expect, from general correspondence arguments, that the low-energy region should be derivable classically, and this expectation is fulfilled in numerous examples.

The source of the disagreement becomes clear when we examine the quantum calculations. The essential process is not direct capture from the $p$ state. Since $l=1$, wave functions are zero at the origin; and since an overlap of the initial electron wave function with the nucleus is required, such direct transitions are very improbable. The dominant contribution comes from virtual transitions of $p$ electrons to an $s$ state with capture from the $s$ state. The intense broad peak should be considered the x rays emitted in the $p$ to virtual $s$ transitions, as pointed out by Glauber and Martin. There is no way in which our classical models allow for this feature.

For the emission of electrons or positrons from nuclei, the classical picture would give the radiation described by $N_s + N_n$, assuming that the two contributions should be added incoherently. The important term in the low-frequency region is $N_s$ given by Eq. (1), with $f$ omitted. It has already been shown by Chang and Falkoff that the main features of the radiation are given correctly by this expression. In the higher frequency region, the contribution of $N_n$ increases, while $f$ decreases the size of $N_s$. The shape of the spectrum is still qualitatively correct; detailed comparison is, of course, unreliable in this region.

**RADIATIVE PION DECAY**

The usual mode of charged pion decay is

$$\pi^\pm \rightarrow \mu^\pm + \nu$$

with the muon energy constrained to be 4.1 Mev by energy and momentum conservation. It has been observed, first by Fry, that occasionally a muon appears with appreciably less energy, apparently because of occurrence of the process

$$\pi^\pm \rightarrow \mu^\pm + \nu + \gamma.$$
the spectrum is qualitatively the same as that
given by the quantum mechanical calculation\(^7\); both show an "infrared
catastrophe" and a rapid decrease in the high-energy region.

Observations have been made by counting
muon tracks that have a range less than that corresponding to 4.1 Mev. In practice, only those
events in which the photon removes appreciable energy can be distinguished from nonradiative
decays. Since \(\beta\) for a 4.1-MeV muon is 0.273, the fraction of the decays that appears radiative is given by
\[
\int_{E_{\text{min}}}^{E_{\text{max}}} N_d E \alpha \frac{1}{\pi \beta} \ln \left(\frac{1+\beta}{1-\beta} \right) - 2 \ln \left(\frac{E_{\text{max}}}{E_{\text{min}}}\right)
\]
\[
= 1.2 \times 10^{-4} \ln \left(\frac{E_{\text{max}}}{E_{\text{min}}}\right).
\]

This fraction is the experimentally accessible number and is in very good agreement with the
quantum mechanical result of Huang and Low,\(^5\) who obtained
\(1.1 \times 10^{-4} \ln (E_{\text{max}}/E_{\text{min}}).\) The
maximum energy that can be removed by a photon is 29.8 Mev, the energy removed by the
neutrino in nonradiative decays. The energy \(E_{\text{min}}\) is, for experimental reasons, about 1 Mev. We expect then
\(4 \times 10^{-4}\) for the fraction of muons with noticeably less than the usual energy. The
experimental result is \((3.3 \pm 1.3) \times 10^{-4}.\)

**RADIATIVE MUON DECAY**

In the muon decay
\[
\mu^+ \rightarrow e^+ + \nu + \bar{\nu},
\]
the electron can be emitted with all energies up to about 52 Mev. If radiative corrections are ignored, the electron spectrum is given by

\[P(x, \rho) = \frac{4x^2}{\tau} \left[ 3(1-x) + \frac{2\rho}{3} (-4x - 3) \right],\]

where \(x = E/E_{\text{max}}, \tau\) is the mean life, and \(\rho\) is the
Michel parameter. The quantity \(\rho\) is a function of the coupling constants and is \(\frac{1}{2}\) in the unmodified form of the two-component neutrino theory.\(^8\)

According to the quantum calculation
\(^7\) T. Eguchi, Phys. Rev. 85, 943 (1952).
\(^8\) K. Huang and F. E. Low, Phys. Rev. 109, 1400 (1958).

of Kinoshita and Sirlin,\(^9\) radiative corrections reduce the value of \(\rho\) to 0.71. The experimental
situation is not clear; values of \(\rho\) ranging from
\(0.67 \pm 0.05\)\(^11\) to \(0.785 \pm 0.020\)\(^12\) have appeared
during the last three years.

We shall examine the sense in which the radiative corrections to muon decay can be understood
classically. First, we use the \(N_d\) of Eq. (1). The
\(f\) term only modifies the improbable high-energy radiation and will be neglected. We shall not use
the magnetic contribution \(N_m\) and shall discuss this omission later.

All electrons with energies \(E_2 > E_1\) can radiate
\(E_2 - E_1\) and appear finally as electrons of energy \(E_1\). The probability of such radiation is given by
\[P_+(x_1 = E_1/E_{\text{max}})
\]
\[= \int_{E_1 + 1 \text{Mev}}^{E_{\text{max}}} P(x = E_2/E_{\text{max}}, \rho)
\]
\[\times N_d[\beta = (1 - m^2 c^4 / E_2^4)^{\frac{1}{2}}, \omega = (E_2 - E_1) / h] dE_2
\]
\[= \frac{\alpha}{\pi} \int_{10/3 + 8x_1 - 26x_1^2 + 44x_1^3 / 3}^{(12 - 8x_1)x_1^2 \ln (E_{\text{max}} - E_1)}
\]
\[\times \left[ \ln (2(E_1 + E_{\text{max}}) - 1) \right].
\]

To obtain this expression in closed form, \(\ln (2E_1 / m \rho^2)\) has been replaced by its value at the
center of the range of integration. All energies in the result are in Mev. The radiation of energy
less than 1 Mev has been neglected as experimentally undetectable. The number of electrons
that appears at \(E_1\) is decreased because of the possibility of radiative transitions to lower
energies. This probability is given by
\[P_-(x_1) = P(x_1, \rho)
\]
\[\times \int_{1 \text{Mev}}^{E_1} N_d[\beta = (1 - m^2 c^4 / E_2^4)^{\frac{1}{2}}, \omega] h d\omega
\]
\[= \frac{\alpha}{\pi} \left( 12x_1^2 - 8x_1^3 \right) \ln \left( \frac{E_1}{1 \text{ Mev}} \right)
\]
\[\times \left[ \ln \left( \frac{2E_1}{m \rho^2} \right) - 1 \right].
\]

\(^12\) R. J. Plano, Phys. Rev. 119, 1400 (1960).
The corrected spectrum is given by

\[ P_\epsilon(x) = P(x, \rho) + P_+ (x) - P_-(x). \]

The result can be expressed as a modification of the Michel parameter by the method of Kinoshita and Sirlin.\textsuperscript{18} We obtain that value of \( \rho' \) which makes \( P(x, \rho') \) approximate \( P_\epsilon(x) \) as well as possible and obtain \( \rho' = 0.72 \). This result agrees with that of Kinoshita and Sirlin,\textsuperscript{4} who found through a somewhat different approach that essentially classical effects account for about half of the radiation.

We return now to the question of the magnetic contribution. Without it, we get fair agreement. If, however, we include it with the \( a \alpha / c \ll 1 \) approximation of Eq. (4), we get vastly too much high-energy radiation. The least artificial suppression of this contribution is accomplished by assuming that the radius \( a \) of the current loop must not be taken to be small compared to the wavelengths involved. Since the Bessel function \( J_1 \) represents a fairly rapidly damped oscillation, Eq. (5) would then be a large over-estimate of \( N_m / N_e \). This observation suggests that, for purposes of classical model-making, the magnetic moment of the electron should be considered as spread over a region of the order of the Compton wavelength \( \hbar / mc \), rather than over a region of the order of the "classical electron radius" \( e^2 / m c^2 = \alpha \hbar / mc \). This result is consistent with viewing the magnetic moment as caused by the Zitterbewegung.\textsuperscript{14} If we attempt to use the exact expression (3), we cannot express the integral over \( \theta \) in closed form, but it is clear that, by choosing \( a \) suitably, we can produce a range of values for the radiative correction.

\textsuperscript{18} T. Kinoshita and A. Sirlin, Phys. Rev. 107, 593 (1957).

\textsuperscript{14} K. Huang, Am. J. Phys. 20, 479 (1952).

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The Macroscopic Nature of Space-Time

E. J. ZIPPERMAN

Department of Physics, University of Nebraska, Lincoln, Nebraska

(Received May 13, 1960; Revised manuscript received October 31, 1961)

Current interpretations of quantum mechanics suggest that the classical concepts of space and time are not applicable to microscopic systems. Salecker and Wigner have recently proved that these concepts have no operational meaning for microsystems. Therefore, space-time descriptions may be valid only for macroscopic systems. It is here suggested that space and time themselves arise from, but do not have analogs in, the properties of microscopic particles, in the same way that thermodynamic properties arise as a result of interactions among the many actually existing particles of the universe. Neither the particles nor the interactions need to be described in spatial-temporal terms. This macroscopic interpretation of space-time seems compatible with the known properties of the physical world, suggests a more direct interpretation of the statistical nature of microscopic events, and offers a new approach to some physical problems.

I. INTRODUCTION

The fundamental concepts of physical science do not have fixed meanings, but evolve as our understanding of the physical world deepens. It is therefore always appropriate to re-examine the current concepts to see whether some modification will better describe our experience of the external world. The purpose of this paper is to discuss critically some current ideas about the nature of space and time. The discussion leads to a somewhat novel point of view which seems consistent and which may merit further consideration.

II. CURRENT CONCEPTS OF SPACE-TIME

Our intuitive feeling for space and time is so primitive and so strong that it is difficult to discuss these concepts in more fundamental terms. Space and time appear to us almost as prerequisites for the comprehending of physical experience. They set the stage or form the background against which all that happens will