Acoustic fluctuations due to the temperature fine structure of the ocean

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The effect of oceanic temperature fine structure on sound transmission is investigated. The model used assumes that the layered fine structure is advected horizontally and vertically by the internal waves. Taylor's frozen turbulence hypothesis is then used to determine the space-time variations in sound speed. We compare our results to those of Ewart's Cobb Seamount experiment [J. Acoust. Soc. Am. 60, 46 (1976)]. The agreement between the calculated spectrum of the log-intensity fluctuations and the experiment is excellent except at low frequencies (ω < 0.3 cph) and extends, at high frequencies, even beyond the internal wave frequency range. Previous calculations based on the internal wave model of turbulence have consistently underestimated those fluctuations in this frequency range. The agreement between the observed and the previously calculated phase fluctuations is not affected; that is, the fine structure adds little to the phase fluctuations.

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INTRODUCTION

It has been pointed out by Ewart that in underwater sound transmission, the description of oceanic inhomogeneity by internal waves is not adequate when the acoustic wavelength is small (of the order of a few meters and less). The inadequacy of describing the small-scale ocean variability by higher mode internal waves is mentioned by others also. The Cobb Seamount experiments conducted by Ewart indicate that the spectrum of phase fluctuations is in good agreement with models based on internal waves alone, while the spectrum of intensity fluctuations is not. Measurements of the vertical and horizontal temperature and salinity fields in the ocean show that there are high gradient regions in the vertical direction where the scales of gradients are of the order of a few meters. It is reasonable to assume that the internal waves have a high wavenumber cutoff since the small-scale oceanic variability is not internal wave-related. Thus, the sound-speed fluctuations on scales larger than a few meters in the vertical direction are internal wave-related, but on a smaller scale, the fluctuations are due to the layered fine structure. The origin of these layers is unimportant; what we are interested in is how much energy they can scatter or reflect away, giving rise to intensity fluctuations. Ewart attempts to calculate the intensity fluctuations by calculating the reflected energy from each layer (scattering in a deterministic sense). Because of the complexity of the problem this is done by a computer-simulation method. However, the calculation gives a result which is in considerable agreement with the observations; the fine structure brings in a large difference in the log-intensity fluctuations, keeping the phase fluctuations unchanged.

We have made an analytic calculation of these fluctuations using the Rytov method as elucidated by Munk and Zachariasen. We define a spatial correlation function of the refractive index fluctuations, which we obtain from measured layer thicknesses and their probability of occurrence. We turn this into a time-correlation function by considering this spatial structure to be advected by internal waves. Since internal wave displacements are large compared to typical layer sizes, we can characterize the advection by a single average horizontal velocity (rms horizontal velocity of internal waves) and a single vertical velocity (rms vertical velocity of internal waves).

Our results, then, are the fluctuations due to the fine structure alone. For the Cobb Seamount experiment we find that the phase fluctuations we obtain are far below (by about 10⁴) those obtained by the previous calculations using internal wave models. Thus the inclusion of the fine structure effects will not disturb the agreement of the previously calculated phase fluctuations with observations. The log-intensity fluctuations due to the fine structure, however, are considerably larger than the internal wave-caused fluctuations, for frequencies 0.3 cph and above. This is exactly the range where the measured fluctuations are higher than previous predictions; our results agree very well with the measurements, in this range. On the low-frequency side the fine structure contribution is less than the internal wave contribution; internal waves dominate here, and again the already existing agreement with experiment is not destroyed. We have the conclusion, therefore, that fine structure is responsible for the higher frequency intensity fluctuations, while internal waves produce the lower-frequency intensity fluctuations.

I. STATISTICAL MODEL OF FINE STRUCTURE

The fine structure of the temperature/salinity field in the ocean has an irregular steppy nature with layers (within which the water is relatively well mixed), separated by regions of large vertical gradient, with scales in excess of a fraction of a meter. Figure 1 shows an example of the variation of the vertical temperature gradient with depth. Figure 2 shows a particular section of the layered ocean at different times. The thick-
ness of a layer, \( z \), and the temperature difference between two consecutive layers, \( T \), are found to be independent random variables. Thus we take the fine structure to be a steppy Poisson process in the vertical, as previously treated by others. The probability of encountering \( N \) steps in a vertical distance \( s \), \( p(N, s) \) is

\[
p(N, s) = \frac{s}{z_0 N!} e^{-z_0/s},
\]

where \( z_0 \) is the average thickness of the layer. The probability density function for the step thickness \( p(z) \) can be written as

\[
p(z) = \frac{1}{z_0} e^{-z/z_0},
\]

We can find a spatial correlation function for the sound-speed fluctuations as follows. The relation between the sound-velocity variations and the temperature and salinity variations can be written as

\[
\frac{\delta c}{c} = \alpha \frac{\delta T}{T} + \beta \frac{\delta S}{S},
\]

where \( \alpha = 3.19 \times 10^{-4} \text{ (}^\circ \text{C)}^{-1} \) and \( \beta = 0.96 \times 10^{-4} \text{ (}^0/00) \). From the measured vertical wavenumber spectra in the vertical gradients of temperature and salinity, the salinity effect on \( \delta c/c \) is only one tenth of the effect of the temperature. Therefore we ignore the second term in (3). We make the assumption that within a layer the temperature is constant, changing abruptly as we cross a layer. Thus the sound-speed fluctuations at two points are fully correlated if they are within the same layer and not correlated at all if they are in different layers. However, at a given location the layer thickness, \( z \), as well as the temperature difference with the neighboring layers, \( T \), keep changing with time (Fig. 2). The probability of occurrence of \( z \) is given by (2); a similar expression can be found for \( T \). Thus the correlation between two points is proportional to the probability of having a layer thickness larger than the distance between the two points. With \( \rho = (x, y) \) we can write

\[
B_{\rho}(\Delta z, \rho) = \left( \frac{\delta c}{c} \right) \left( \frac{\delta c}{c} \right) e^{-\gamma z_0^2} \int_{-\Delta z}^{\Delta z} p(z) dz,
\]

When an internal wave advects a fine structure layer in the horizontal direction, the mean horizontal distance between crossings of layers is the layer thickness divided by the slope of the wave (Fig. 3). Therefore the correlation length in the horizontal direction, \( \rho_0 \), is the ratio of the vertical correlation length to the rms slope of the internal waves. (McKean\textsuperscript{16} gives a value \( 2.7 \times 10^{-2} \) for typical internal wave slope.) The form of the correlation function in the horizontal direction will not affect our results much, since this correlation length is about 50 times larger than the vertical correlation length. For convenience we take this to have a Gaussian form, so that the complete spatial correlation function for the fine structure becomes

\[
B_{\rho}(z, \rho) = \left( \frac{\delta c}{c} \right)^2 e^{-\gamma z_0^2} \rho_0^2 \int_{-\Delta z}^{\Delta z} p(z) dz.
\]

II. ACOUSTIC FLUCTUATIONS

If we have a source at the origin of frequency \( \sigma \), the pressure at \( x \) due to this is

\[
p(x, t) = |p| \exp(-i\omega t) = |p| \exp(i qx - \omega t),
\]

where \( q \) is the wavenumber. We write

\[
p(x) = |p| \exp(iqx) = p e^{\chi}, \quad X = x + i\phi,
\]
where \( p_0 \) is the pressure amplitude in the absence of fluctuations. The phase is given by \( \phi \), and the log intensity is given by

\[
\langle i \rangle = \log_e [1 + 2 \delta x] \quad \text{and} \quad \langle i \rangle = \log_e [1 + 4 \delta x^2].
\]

We assume that the mean sound-speed fluctuation is zero, and hence \( \langle X \rangle \) is also zero, so that we have

\[
\langle i^2 \rangle = \langle \log_e |p|^2 \rangle = \langle i \rangle^2 + 4 \langle X^2 \rangle,
\]

Thus the log-intensity fluctuations become

\[
\langle i^2 \rangle = 4 \langle X^2 \rangle = 2 \langle |X|^2 \rangle + \text{Re} \langle X^2 \rangle,
\]

and the phase fluctuations are given by

\[
\langle \phi^2 \rangle = \frac{1}{2} \langle |X|^2 \rangle - \text{Re} \langle X^2 \rangle.
\]

Munk and Zachariasen\(^8\) have found expressions for \( \langle |X|^2 \rangle \) and \( \langle X^2 \rangle \), taking a Rytov approximation to the pressure. These are

\[
\langle |X|^2 \rangle = \frac{q_0^2}{2\pi^2} \int_0^R \int dk_x \int dk_y
\]

\[
\times \left[ - k_x \tan \theta (x, k_x, k_y) \right] \exp \left( i q \frac{x(R - x)}{R} + \frac{k_y^2}{A_{xx}} \right)
\]

\[
\langle X^2 \rangle = \frac{q_0^4}{2\pi^2} \int_0^R \int dk_x \int dk_y \left[ - k_x \tan \theta (x, k_x, k_y) \right] \exp \left( i q \frac{x(R - x)}{R} + \frac{k_y^2}{A_{xx}} \right)
\]

for a curved ray path. Here \( B(k) \) is the Fourier transform of the correlation function, \( R \) is the range, \( \theta \) is the angle the acoustic ray makes with the horizontal, and the quantity \( A_{xx}^{-1} \) is \( A_{xx}^{-1} = (1/K) \sin K x \sin KR / \sin K R \) for deep rays and \( A_{xx}^{-1} = (1/2) \) for near real rays, where \( 2\pi K \) is the wavelength of the sinusoidal ray path.

We now proceed to evaluate these expressions using the correlation functions and the advection process described earlier. For the correlation function given in (5) we can write

\[
\tilde{B}(k) = \frac{q_0^2}{c} \int d^3 r e^{-i x \cdot \mathbf{r}} \exp(-i k \cdot \mathbf{r})
\]

\[
= 2\pi \left( \frac{q_0^2}{c} \right) \frac{2\pi^2}{z_0^2} \frac{z_0^2}{4} \int d^3 k \exp(-i k \cdot \mathbf{r})
\]

so that we have

\[
B(r) = \frac{1}{2\pi} \int d^3 k \exp(i k \cdot \mathbf{r}) \tilde{B}(k).
\]

If all the time changes in the sound-speed fluctuations are caused by a simple translation of the spatial field distribution and no mixing is involved, we use Taylor's hypothesis and write

\[
\langle \delta c/c \rangle(r, t + \tau) = \langle \delta c/c \rangle(r - \mathbf{v} \tau, t),
\]

where \( \mathbf{v} \) is the mean velocity of transport. Then we have

\[
B(r, \tau) = \langle \delta c/c \rangle(r, t) \langle \delta c/c \rangle(r + \mathbf{v} \tau, t) + \langle \delta c/c \rangle(r - \mathbf{v} \tau, t) \langle \delta c/c \rangle(r + \mathbf{v} \tau, t)
\]

\[
= \langle \delta c/c \rangle(r - \mathbf{v} \tau, 0) \langle \delta c/c \rangle(r + \mathbf{v} \tau, t - \tau, 0)
\]

\[
= B(r - \mathbf{v} \tau).
\]

Writing both sides in terms of Fourier integrals\(^11\) as in (14)

\[
\frac{1}{(2\pi)^3} \int d^3 k \exp(i k \cdot \mathbf{r}) \int d\omega \exp(i \omega \tau) d\omega
\]

\[
= \frac{1}{(2\pi)^3} \int d^3 k \exp(i k \cdot \mathbf{r} - \mathbf{v} \tau) \tilde{B}(k).
\]

Thus Taylor's hypothesis can be written in the form

\[
w(k, \omega) = \tilde{B}(k)
\]

\[
= \tilde{B}(k),
\]

where \( u \) and \( v \) are the rms horizontal and vertical velocities of internal waves with which the fine structure is advected. To determine

\[
\int d^3 k \exp(i k \cdot \mathbf{r}) \tilde{B}(k)
\]

in the expressions for \( \langle |X|^2 \rangle \) and \( \langle X^2 \rangle \) we change the variable \( k_0 \) to \( k_0 \), the radial coordinate in the \( k \) space, and the quantity \( A_{xx}^{-1} \) is \( A_{xx}^{-1} = [x(R - x)/R] \) for deep rays and \( A_{xx}^{-1} = (1/2) \sin K x \sin KR / \sin K R \) for near real rays, where \( 2\pi K \) is the wavelength of the sinusoidal ray path.

We consider in detail deep rays for which the path is an arc of a circle. Let \( R_0 \) be the radius of the arc for such a ray. The path integral in (20) and (21) is simpler if we change the variable \( x \) to \( tan \theta \). The upper limit of the

\[
\langle X^2 \rangle = \frac{q_0^2}{\pi} \left( \frac{q_0^2}{c} \right) \int_0^{R_0} dx \sec \theta \int_0^\infty dw
\]

\[
\times \int_0^\infty \int_0^\infty \frac{dk_x}{k_x^2 + (\omega + k_x u)^2} \frac{\exp \left( i q \frac{x(R - x)}{R} + \frac{(\omega + k_x u)^2}{v^2 R} \right)}{\left[ v^2 + (\omega + k_x u)^2 \right]^{3/2}}
\]

\[
\frac{1}{q} \left[ \left( k_x^2 + (\omega + k_x u)^2 \right) \tan \theta \right]^{x(R - x)/v^2 R + (\omega + k_x u)^2} \right]^{1/2}
\]

The dependence of \( R_0 \) on \( k_x \) comes from the dependence of \( tan \theta \) on \( k_x \), viz. \( k_x > k_x tan \theta \), or \( tan \theta < \mathbf{v} k_x / (\omega + k_x u) \). For near axial rays \( \theta = 0 \).

We consider in detail deep rays for which the path is an arc of a circle. Let \( R_0 \) be the radius of the arc for such a ray. The path integral in (20) and (21) is simpler if we change the variable \( x \) to \( tan \theta \). The upper limit of the
The path integration result for \(|X|^2\) is:

\[
\langle |X|^2 \rangle = \left(\frac{\delta c}{c}\right)^4 q^4 R_0^2 \int \sum_{R/2R_0} \frac{dk_h}{(\omega + k\mu)^3} \left(\frac{R}{\omega + k\mu}\right)^{3/2} \left(\frac{e^{-k^2/4}}{\sqrt{\pi}}\right) \left\{ \int_0^{\pi} \cos^2 \theta \right\}^1/2
\]

For \(\langle X^2 \rangle\) we get:

\[
\langle X^2 \rangle = -\frac{2}{(\delta c/c)^4} q^4 R_0^2 \int \sum_{R/2R_0} \frac{dk_h}{(\omega + k\mu)^3} \left(\frac{R}{\omega + k\mu}\right)^{3/2} \left(\frac{e^{-k^2/4}}{\sqrt{\pi}}\right) \left\{ \int_0^{\pi} \cos^2 \theta \right\}^1/2
\]

In (24) the path \((\tan \theta)\) integral in the first part can be done analytically, if tediously, as follows. By putting:

\[
a = \frac{v_h}{(\omega + k\mu)}, \quad b = \frac{R_0}{(\delta c/c)^4} \left(\frac{R}{\omega + k\mu}\right)^{3/2}
\]

and \(\tan \theta = y\), we get:

\[
\int_0^{\pi} \cos \theta \left(\frac{e^{-k^2/4}}{\sqrt{\pi}}\right) \left\{ \int_0^{\pi} \cos^2 \theta \right\}^1/2
\]

By splitting the integral into two parts, \(0 \rightarrow (a/\sqrt{2})\) and \((a/\sqrt{2}) \rightarrow a\), and using methods of integration by parts we get a simpler form for the integral, which can be done for three different ranges of \(ba/\sqrt{2}\), using different approximations. This amounts to dividing the \(k_h\) integration region into three parts. The \(k_h\) integration and the \(\tan \theta\) integration in the second part of \(\langle X^2 \rangle\) have been done numerically. Note that if \(2R_0 \mu < u\mu\) the second part of \(\langle X^2 \rangle\) does not arise.

### III. RESULTS AND DISCUSSION

The measured value for the variance of the temperature jumps between two consecutive layers in the Sargasso Sea, \((T_2 - T_1)^2\) reported by Joyce and Desaubies,

\[
= 5 \times 10^{-10} C^2
\]

The sound-speed fluctuation corresponding to this is \((\delta c/c)^2 = 5 \times 10^{-10}\). The rms values of the velocities related to internal waves, computed from the measurements of temperature and salinity using GM spectrum are given by Flatté et al.

The rms vertical velocity \(u\) is 0.5 cm/s. The horizontal velocity is given by \(u = \frac{\omega}{2}\), where \(\omega\) is the Väisälä frequency at a depth \(z\) and \(u(0) = 4.7\) cm/s. The vertical correlation distance is found to be 2 m in the Sargasso Sea.

From the slope of internal waves of about 2.7 \times 10^{-6} the horizontal correlation distance can be found as 74 m. We use these values for the various parameters, to calculate log-intensity fluctuations for Ewart’s Cobb Seamount experiment since local data are unavailable.

The log-intensity spectrum for 4 kHz is shown in Fig. 4 and for 8 kHz, in Fig. 5. The phase-fluctuation spectrum is of the order of \(10^{-6}\) times the spectrum caused by internal waves. That is, the agreement of the internal wave-based calculations with the experiment is maintained even in the presence of fine structure.
FIG. 5. Spectrum of log-intensity fluctuations for an acoustic frequency of 8 kHz. Curves 1, 2, and 3 are as in Fig. 4.

The log-intensity spectra are below the internal wave prediction on the low-frequency side. On the high-frequency side, however, the internal wave prediction is much less than the fine structure results, and the fine structure results agree very well with the experimental results. Therefore, qualitatively we can say that the low-frequency fluctuations are caused by the internal waves while the fine structure is the main reason for the high-frequency fluctuations. However, the actual spectrum is not the simple sum of the internal wave results and the fine structure results of Figs. 4 and 5. In the internal wave model, even the smallest scales of turbulence are assumed to result from (high wavenumber) internal waves. It is necessary to use a high wavenumber cutoff in order not to calculate the effects of small-scale turbulence twice. Exactly where the cutoff should be and how to combine the results are problems that must be separately considered.

IV. CONCLUDING REMARKS

We have presented here an analytic calculation of acoustic fluctuations using a statistical model for the oceanic temperature fine structure. Even though the large-scale turbulence in the ocean is predominantly due to the internal waves, the intensity fluctuations are caused mainly by the small-scale oceanic variations. The treatment of the fine structure as a perturbation on the internal waves in the displacement spectrum is not adequate for this calculation. Considering the fine structure effects separately, we find over a considerable range of frequencies, the log-intensity fluctuations are almost entirely due to the fine structure.