Propagation of transverse zero sound in He\textsuperscript{3} and of spin waves in He\textsuperscript{3}–He \textsuperscript{4} solutions

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The conditions of propagation of high-frequency Fermi-liquid modes are determined by taking into account both Landau and collisionless damping. Cherenkov absorption may dominate if the wave-propagation velocity is close to the Fermi value. It is shown that the propagation of transverse zero sound in He\textsuperscript{3} is possible even if the wave velocity is somewhat smaller than the Fermi velocity. The dispersion law for transverse zero sound is found without resorting to model concepts regarding the type of the f function, and all dispersion characteristics are expressed in terms of the propagation velocity of the oscillations. Due to collisionless absorption, the observation of spin waves in an He\textsuperscript{3}–He\textsuperscript{4} solution is possible only at sufficiently low temperatures. In a magnetic field, the transverse spin waves with small wave vectors become weakly damped even in the Boltzmann temperature range.

PACS numbers: 67.50.Fi, 67.60.Fp

According to the Landau theory of a Fermi liquid,\textsuperscript{4} weakly damped high-frequency $\omega T > 1$ ($\omega$ is the frequency of oscillation, $T$ is the characteristic relaxation time) zero-sound and spin waves can propagate in normal He\textsuperscript{3} and in degenerate solutions of He\textsuperscript{3} in superconducting He\textsuperscript{4}. The absorption of these modes is usually determined by collisions of the quasiparticles of the Fermi liquid with one another and the absorption falls off in proportion to $T^2$ upon decrease in the temperature $T$ (see, for example, Ref. 2).

There is also a collisionless damping mechanism for the high-frequency oscillations, connected with the Cherenkov absorption of the wave by the quasiparticles. The probability of such processes depends on the number of quasiparticles moving in phase with the wave. If the velocity of propagation of the high-frequency modes significantly exceeds the Fermi velocity $v_F$, then at $T < T_F$ ($T_F$ is the degeneracy temperature) the collisionless damping is exponentially small. The described Landau damping mechanism turns out to be dominant in those cases in which the wave velocity is so close to the Fermi that the energy of the quasiparticles moving with such velocities lies in the region of thermal smearing out of the Fermi step.

Just such a situation exists for transverse zero sound in normal He\textsuperscript{3} and spin waves in weak He\textsuperscript{3}–He\textsuperscript{4} solutions. The possibility of propagation of zero sound in He\textsuperscript{3} (high-frequency, nonsymmetric oscillations with azimuthal number $m = 1$), observed experimentally by Roach and Ketterer,\textsuperscript{3} Lea, Butcher and Dobbs,\textsuperscript{4} is roughly determined by the inequality at the first harmonic of the Fermi-liquid function $F_{1} > 6$. It is known from thermodynamic measurements (see, for example, Ref. 5) that the quantity $F_{1} > 6$ is very small at low pressures. The small difference $F_{1} - 6$ determines the closeness of the propagation velocity of the transverse sound to the Fermi velocity. The experimental difficulties in the study of transverse sound are of a significant degree associated with this circumstance. There are also some grounds\textsuperscript{5} for assuming that at very low pressures the account of the principal harmonics of the $f$ function leads to the result that the velocity of the wave would turn out to be smaller than the Fermi, which would generally make the propagation of undamped oscillations impossible. This also confirms the fact that there exists a region of pressures at which the velocity of transverse sound is close to the Fermi velocity. It is in just this region that the role of the collisionless damping studied in this paper is important.

The Cherenkov absorption is the basic mechanism of dissipation also in the case of spin waves in a weak He\textsuperscript{3} –He\textsuperscript{4} solution, the velocity of propagation in which is exponentially close to $v_F$ in the expansion in terms of the He\textsuperscript{3} concentration raised to the $\frac{1}{2}$ power.\textsuperscript{5} There is great interest in the study of collisionless damping of the high-frequency modes in an He\textsuperscript{3}–He\textsuperscript{4} solution in a constant magnetic field: the onset of this damping changes greatly the dispersion laws and the types of possible oscillations.\textsuperscript{4} In a weak He\textsuperscript{3}–He\textsuperscript{4} solution, it turns out to be possible to determine the velocity of propagation and the absorption coefficients of the spin waves at arbitrary temperatures, including the Boltzmann region $T > T_F$.

The basic experimental difficulty in the study of high-frequency oscillations that propagate with a velocity $u$ close to the Fermi velocity lies in the identification of the collective mode against the background of the signal of free quasiparticles of the Fermi liquid. Time-of-flight measurements are the most accurate method for the determination of the wave velocity. To resolve the signals of the wave and of the free fermions it is necessary that the time difference between the signals $\Delta t = L/\left(\omega - v_F\right)$ ($L$ is the distance between the receiver and the radiator) be large in comparison with the width of the pulse of high-frequency oscillations $\Delta t = L/\omega$, i.e., $\omega > L^2/\Delta t$. On the other hand, for the receiver to record a pulse it is necessary that the signal be not damped over a distance $L < c_L/\omega$, where $\omega$ is the absorption coefficient of the high-frequency mode. It is then seen that to separate experimentally at the wave contribution it is necessary to satisfy the condition

$$\Delta t > \frac{L}{\omega} \left(\omega^2 - v_F\right)^{\frac{1}{2}},$$

which is stronger than the simple condition of weak damping $\omega < v_F$.

Another method of determination of the signal of the

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collective mode is possible in principle. It is connected
with the fact that incoherence in the radiation of free
quasiparticles leads to a decrease in the amplitude of
their signal in proportion to $1/L^2$ even upon neglect of
the relaxations. This circumstance can turn out to be
important in the case in which the velocity of transverse
zero sound in He^3 becomes somewhat smaller than the
Fermi velocity (see below).

1. TRANSVERSE ZERO SOUND

In the study of transverse sound in liquid He^3 we are
usually limited to the first two harmonics of the Fermi-
liquid function, while to take account of the damping we
use the collision integral in the $r$ approximation. Let us
fix this limitation is connected with the fact that for trans-
verse oscillations, in contrast to the known case of a
symmetrical (n = 0) model it has not been possible to ex-
press the propagation velocities of the wave in terms of
the $f$ function in explicit form, even with logarithmic
accuracy under the condition of closeness of the veloc-
ity of zero sound to the Fermi velocity. This is due to
the same cause as those for which, in an almost ideal
Fermi gas, allowance for the factor of the exponential
in the expression for the velocity of the symmetric
wave is an exaggeration of accuracy. Nevertheless,
there exists a general method of determining the dis-
ersion of a transverse wave, similar to that proposed
by the authors earlier, in which all the characteristic
quantities are expressed in terms of the value of the
propagation velocity of the oscillations in the absence of
damping $T = 0$ and of magnetic field $H = 0$.

The collisionless kinetic equation, which describes
the propagation of zero sound in the absence of a mag-
netic field, reduces to the form

\[ \frac{d}{dt} n(p) = -k \frac{\partial}{\partial p} \left( \int dp' \frac{\partial}{\partial p'} \int dp'' \frac{\partial}{\partial p''} \frac{1}{E(p') - E(p'')} \right) \]

where $k$, $\omega$, and $\Delta$ are the wave vector, fre-
frequency, and energy of the oscillations, the change
in the distribution function from the equilibrium Fermi function $n(t, p) = \frac{1}{2\pi} \int dp' \frac{\partial}{\partial p'} \int dp'' \frac{\partial}{\partial p''} \frac{1}{E(p') - E(p'')} \] for the Fermi-liquid function averaged over the
space, and $d\omega = dp dp' dp''$. For an isotropic liquid, the
function $\beta$ depends only on the value of the momenta
$|p|$ and $|p'|$ and on the angle between the vectors
$p$ and $p'$. Since the propagation velocity of zero sound $v = \omega/k$ in degenerate He^3 at $T/T_F \ll 1$ is close to
the Fermi velocity $0 \leq (\omega - \nu) / \nu_a \leq a \ll 1$, the functions
$n(\rho, \theta)$ and $n(\rho', \theta')$ differ from zero only in a narrow
range of momenta near the Fermi momentum $p_F$ and of
small angles $\theta$. The accuracy of the further calcula-
tions corresponds to the neglect of terms of the order of $\alpha$ in comparison with the terms $\alpha \ln \alpha$. This allows
us to set $p = p' = \rho \theta$ in the argument of the function $\beta$,
which is a smooth function of the momenta $\rho$ and $\rho'$.
Equation (1.2) is then considerably simplified:

\[ \rho(\gamma = \gamma_0) = \frac{1}{2 \pi} \int dp \frac{\partial}{\partial p} \int dp' \frac{\partial}{\partial p'} \int dp'' \frac{\partial}{\partial p''} \frac{1}{E(p') - E(p'')} \]

We are interested in the case of small damping (1):

\[ \left| \frac{\omega}{1 - \nu} \right| < 1, \quad \nu = \frac{\omega}{\omega_a} \approx 0, \quad \nu = \frac{\omega}{\omega_a} \approx 0 \]

Only under such conditions can the zero sound signal be distinguished
from the signal of the free particles. Here the propaga-
tion velocity of the oscillations $\nu$ changes only insigni-
ficantly as a function of the temperature within the limits
of accuracy, $\omega(1) = \omega(0) = \omega_a$, i.e., $\nu = \omega/\omega_a$. Here $\alpha = \omega/\omega_a - 1 \ll 1$, $|\nu|^2 \ll \alpha_\nu$, and the main contribution is
made by the region of small angles $\theta$ and $\theta'$, while Eq.
(1.4) can be solved by expanding the function $\beta$ in pow-
sers of $(1 - \nu)^2$ and $(1 - \nu)^3$. The integral equation
(1.4) reduces to a system of linear algebraic equations for
the quantities

\[ \nu_\nu = \frac{1}{2 \pi} \int dp \frac{\partial}{\partial p} \int dp' \frac{\partial}{\partial p'} \int dp'' \frac{\partial}{\partial p''} \frac{1}{E(p') - E(p'')} \]

with coefficients containing integrals of the form

\[ I_{\nu, \nu} = \int_{-\nu_\nu}^{\nu_\nu} \frac{1}{2 \pi} \int dp \frac{\partial}{\partial p} \int dp' \frac{\partial}{\partial p'} \int dp'' \frac{\partial}{\partial p''} \frac{1}{E(p') - E(p'')} \]

The integrals $I_{\nu, \nu}$ do not enter into the equation since,
according to (1.3), $\nu(1, 1) = 0$. Since we are interested in
terms of the order $\alpha$ in all the equations, we have this accuracy,

\[ I_{\nu, \nu} = \int_{-\nu_\nu}^{\nu_\nu} \frac{1}{2 \pi} \int dp \frac{\partial}{\partial p} \int dp' \frac{\partial}{\partial p'} \int dp'' \frac{\partial}{\partial p''} \frac{1}{E(p') - E(p'')} \]

while the value of $I_{\nu, \nu}$ is easily calculated with the help of
the usual rule of passing around the poles:

\[ \int_{-\nu_\nu}^{\nu_\nu} \frac{1}{2 \pi} \int dp \frac{\partial}{\partial p} \int dp' \frac{\partial}{\partial p'} \int dp'' \frac{\partial}{\partial p''} \frac{1}{E(p') - E(p'')} \]

In the principal value of the integral, it suffices to set $\nu_\nu = 0$ or $\nu_\nu = \infty$. Correspondingly,

\[ I_{\nu, \nu} = \int_{-\nu_\nu}^{\nu_\nu} \frac{1}{2 \pi} \int dp \frac{\partial}{\partial p} \int dp' \frac{\partial}{\partial p'} \int dp'' \frac{\partial}{\partial p''} \frac{1}{E(p') - E(p'')} \]

After integration by parts and the substitution $\varepsilon = (2\nu_\nu / T)(\nu/\omega_\nu - 1)$ the integral in (1.7) reduces to the form

\[ I_{\nu, \nu} = \int_{-\nu_\nu}^{\nu_\nu} \frac{1}{2 \pi} \int dp \frac{\partial}{\partial p} \int dp' \frac{\partial}{\partial p'} \int dp'' \frac{\partial}{\partial p''} \frac{1}{E(p') - E(p'')} \]

where $\alpha_\nu = \nu_\nu^2 - 1$. Since the imaginary parts of both integrals $I_{\nu, \nu}(n > 1)$ are small in comparison with the imaginary part of $I_{\nu, \nu}$, the collisionless damping of the oscillations is deter-
mined by the equation

\[ \frac{\nu_\nu}{2 \pi} \int dp \frac{\partial}{\partial p} \int dp' \frac{\partial}{\partial p'} \frac{1}{E(p') - E(p'')} \]

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For the determination of the propagation velocity of the oscillations, all the coefficients of (1.6) and (1.8) are important, and the quantity \( \alpha_n \) is given with the specified accuracy by an equation of the form

\[
\Delta \propto \ln(1 + |a_n|).
\]

(1.10)

where the constant \( A \) is a combination of derivative of all orders of the Fermi-liquid function at the point \( \chi = 0 \) and the constants \( f_{\alpha} \) [11,6, (1.8)] at \( \varepsilon = 1 \). The calculation of the combination in general form is impossible.

The proposed method of determination of the dispersion of transverse sound is valid for a wave propagation velocity close to the Fermi velocity is formally equivalent to the solution of the kinetic equation in the single harmonic approximation, in which the role of the coefficient \( F_1 \), of the first harmonic of the \( f \) function is played by the quantity \( 6(1.3A) \). The value of \( A \) can be found with the help of (1.10) from experimental data on the velocity of transverse oscillations in He.3.

We note that the collisionless damping (1.9) can turn out to be small even in the case in which the velocity of wave propagation \( \nu \) is a little less than the Fermi velocity \( \nu_f \), i.e., at \( 0 < \nu < \nu_f \). This fact, which is connected with the necessity of satisfaction of the law of conservation of the angular momentum in the case of absorption of quanta of transverse zero sound by the quasiparticles of the Fermi liquid, does not occur in the propagation of the symmetrical modes with \( m = 0 \), when at \( \nu < \nu_f \) the damping is always large.

In the case when the velocity of transverse sound is greater than the Fermi velocity, \( \nu > \nu_f \), the condition of smallness of the collisionless damping (1) \( \alpha_n \propto \alpha_n \), the condition of smallness of the collisionless damping (1) \( \nu > \nu_f \), has the form

\[
\nu_T \propto \ln(1 + |a_n|),
\]

and the restriction it imposes on the temperature at which observation of the signal of high-frequency oscillations is possible is weaker than the condition \( T/T_0 \propto \alpha_n \), which is necessary for isolation of the symmetric waves propagating with a velocity close to the Fermi velocity. At low temperatures \( T/T_0 \propto \alpha_n \), the collisionless damping (1.9) is exponentially small, as noted in the Introduction:

\[
\Delta \nu = \nu_0 \nu F_1 \exp \left( \frac{-2\pi n T}{\nu_0} \right).
\]

(1.11)

At higher temperatures, \( \alpha_n \propto \ln(1 + \alpha_n) \), the damping is equal to

\[
\Delta \nu \propto \ln(1 + \alpha_n).
\]

If the velocity of transverse sound turns out to be less than the Fermi velocity, \( \nu < \nu_f \), then, at low temperatures, \( T/T_0 \propto \alpha_n \), the absorption coefficient

\[
\Delta \propto \ln(1 + |a_n|).
\]

(1.11)

does not depend on the temperature and is small upon approach of the wave velocity to the Fermi velocity \( \ln(1 + |a_n|) \). At higher temperatures, \( \alpha_n \ln(1 + |a_n|) \), the absorption is still small and the expression for \( \Delta \) has a form similar to (1.11).

The expressions obtained take into consideration in implicit form all the harmonics of the Fermi-liquid function, and the measurement of \( u_n \) can give information on the principal harmonics of the \( f \) function. If we limit ourselves to the two-harmonic approximation, then, with our accuracy, at \( T/T_0 \propto \alpha_n \), the absorption coefficient is equal to

\[
F_2 + \frac{2\nu_0(1 + F_2 T_0)}{2\nu_0(1 + \nu_0 T_0)}.
\]

(1.12)

We emphasize that the propagation of transverse zero sound turns out to be possible if the quantity on the right side of expression (1.12) is positive, i.e., in fact if \( F_2 > 6 \). In this case the oscillations are weakly damped at low temperature, to the extent of the smallness of the quantity \( \ln(1 + F_2/T_0) \).

We now consider the role of collisional damping in the case of closeness of the wave velocity to the Fermi velocity. The coefficient of collisional damping of the high-frequency mode, with account of the quantum properties of the zero-sound excitation, is expressed in terms of the relaxation time \( \tau \) in the following manner:

\[
w_e = \left( \frac{\tilde{\alpha}_e}{2 \pi} \right) \tau.
\]

(1.13)

For observation of the signal of high-frequency sound, satisfaction of the condition (1) is necessary. In the case of collisional absorption (1.13), this condition is satisfied at frequencies

\[
w_e \propto \ln(1 + |a_n|),
\]

where it is taken into account that \( \tau \propto \nu^{1/2} \) in the normal Fermi liquid. By use of experimental data,1 we can obtain the numerical estimate

\[
\tau \propto \ln(1 + |a_n|),
\]

and

\[
\tau \propto \nu^{1/2} \nu_0 T.
\]

It is easy to show that in this range of frequencies and temperatures, in which the oscillations are weakly damped and the condition of the expansion (1) is satisfied, the collisional absorption of zero sound is more significant than the Landau damping in the case of a wave velocity much greater than the Fermi velocity. Collisionless damping can turn out to be important dissipative mechanism in the case \( \nu < \nu_f \).

Transverse zero sound in a magnetic field

As a consequence of the closeness of the oscillation velocity to the Fermi velocity, the conditions of propagation of transverse zero sound turn out to be sensitive to the application of even a weak magnetic field \( B \). In He3 in a magnetic field, the Fermi surfaces of the quasiparticles with different spin orientations are two Fermi spheres, the radii of which \( r_0 \) in a weak magnetic field

\[
h = \frac{\nu}{2\pi|z|} < 1
\]

\[\beta \]

(1.13)

\[\beta \]

is the magnetic moment of the He3 nucleus, \( Z_e \) is the
zeroth harmonic of the exchange part of the f function and determines the static susceptibility of He$^3$ in weak fields) are equal to

$$\omega_0 = \omega_0 + \alpha h/2.$$  

In an isotropic Fermi liquid in the exchange approximation, the f function in a weak magnetic field can always be represented in the form of the following linear combination of spin operators $\sigma$ (see, for example, Ref. 7):

$$f(p, p') = (p \cdot p') \langle \sigma(p, p') \sigma(p, p')^\dagger \rangle \rho(p, p').$$

The propagation of high-frequency oscillations is determined by the collisionless kinetic equation for the single particle density matrix $\rho$. In the given case, we are interested in two coupled equations, for the scalar distribution function $n = \rho_0 \bar{\rho}$ and for the longitudinal magnetization $\rho_{\sigma z} \rho_{\sigma z}^\dagger$ (cf. Refs. 8 and 9):

$$\begin{align*}
\frac{\partial}{\partial t} \rho_\sigma &+ \nu \frac{\partial}{\partial p_\nu} \rho_\sigma = \frac{1}{2} g \left( p_\nu - p_\nu' \right) \left( \rho_{\sigma z} \rho_{\sigma z}^\dagger - \rho_\sigma ^\dagger \rho_\sigma \right), \\
\frac{\partial}{\partial t} \rho &+ \nu \frac{\partial}{\partial p_\nu} \rho = \frac{1}{2} g \left( p_\nu - p_\nu' \right) \left( \rho_{\sigma z} \rho_{\sigma z}^\dagger - \rho_\sigma ^\dagger \rho_\sigma \right).
\end{align*}$$

In the limits of accuracy, the differences of the momentum dependence of the sound absorption coefficient in the magnetic field (1.16) with account of (1.17) is carried out in similar fashion and leads to similar results as for the absorption coefficient in the absence of a field (1.9).

Thus, for example, at $T = 0$ and $\rho_{\sigma z} \rho_{\sigma z}^\dagger$ vanishes if $h - h_c$ becomes smaller.

The determination of the magnitude and temperature dependence of the sound absorption coefficient in the magnetic field (1.16) with account of (1.17) is carried out in similar fashion and leads to similar results as for the absorption coefficient in the absence of a field (1.9).

All the dispersion characteristics of the transverse zero sound in He$^3$ given above were expressed in terms of an observed quantity—the velocity of propagation of the oscillations $\omega_0$. In a real experiment, it may be simpler to determine not $\omega_0$ but the critical magnetic field $h_c$. The relation (1.18) makes it possible to express the dispersion of the oscillations in terms of $h_c$ also. With the help of experimental data on He$^3$ and the relations (1.12) and (1.18) for the value of the critical field at zero pressure we get a value of the order of 4 kOe.$^6$

2. SPIN WAVES

The collisionless kinetic equation which describes the propagation of spin waves in a paramagnetic Fermi liquid in the absence of an external field at $h = 0$ can be represented in form

$$\begin{align*}
\frac{\partial}{\partial t} \rho &+ \nu \frac{\partial}{\partial p_\nu} \rho = \frac{1}{2} g \left( p_\nu - p_\nu' \right) \left( \rho_{\sigma z} \rho_{\sigma z}^\dagger - \rho_\sigma ^\dagger \rho_\sigma \right), \\
\frac{\partial}{\partial t} \rho &+ \nu \frac{\partial}{\partial p_\nu} \rho = \frac{1}{2} g \left( p_\nu - p_\nu' \right) \left( \rho_{\sigma z} \rho_{\sigma z}^\dagger - \rho_\sigma ^\dagger \rho_\sigma \right).
\end{align*}$$

The velocity of propagation of transverse zero sound, given by the first of the relations (1.16), does not depend on the magnetic field in weak fields at the logarithmic accuracy with which the expressions (1.16) themselves are valid:

$$\omega_0 = \omega_0 (1 + \alpha h/2, \omega_0 = \omega_0 - \alpha h/2).$$

The condition of collisionless propagation of zero sound (at $T = 0$) $\nu_0 \omega_0$ ceases to be satisfied in a critical field $h_c$ such that $\alpha_0 (h_c) = 0$ and $\alpha_0 (h_c) = h_c$. Correspondingly, the critical field (at $\nu_0 \omega_0$)

$$h_c \sim 2\nu_0 t$$

turns out to be very small, which allows us to assume a weak magnetic field in all the calculations. In fields greater than $h_c$, damping arises even at $T = 0$. The wave is ever more weakly attenuated as $h - h_c$ becomes smaller.

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the He\textsuperscript{3} quasiparticles by one another), which is proportional to the concentration raised to the \( 3/2 \) power. The expression for the velocity of spin waves (2.2) was written with logarithmic accuracy. In the calculation of the pre-exponential factor, it is necessary to take into account the values of the derivatives of the \( f \) function at \( \chi = 0 \). The exponential closeness of the propagation velocity of the oscillations to \( v_\perp \) is the cause of the importance of the collisionless Landau damping even at very low temperatures, \( T \ll \omega_\perp \).

In the Born approximation, Eq. (2.1) is equivalent to the following dispersion equation:

\[
\gamma + \int \frac{d^3v}{(2\pi)^3} \int \frac{d^3v'}{(2\pi)^3} w_{vk} = 0,
\]

where \( \gamma \equiv \gamma'(P, P') = \frac{2\pi \hbar^2}{\beta M} \), \( P = 2.3\hbar \omega_0 \) is the effective mass of the bare He\textsuperscript{3} excitation, \( \omega_0 \) is the mass of the He\textsuperscript{3} atom. We bypass the pole of the integrand in (2.3) in the usual fashion, just as in the previous part of the work:

\[
\int \frac{d^3v}{(2\pi)^3} \int \frac{d^3v'}{(2\pi)^3} w_{vk} = \int \frac{d^3v}{(2\pi)^3} w_{vk} \delta(v - v') = \int \frac{d^3v}{(2\pi)^3} \delta(v - v').
\]

For the determination of the spectrum of weakly damped oscillations, we seek a solution of Eq. (2.3) in the form \( w = W_0 v_0 \), \( 0 \leq \gamma = w' \). In the case of small damping, it follows from the dispersion relation (2.3) that the real part of the propagation velocity \( w' / k \) and the absorption coefficient \( \gamma \) of the spin wave are determined by the relations:

\[
\Re \gamma(w', k) = 0,
\]

where \( \gamma = \ln(\gamma(w', k) / \eta_0) \). At low temperatures \( T/T_0 \ll 1 \), the derivative \( \partial \Re \gamma / \partial \epsilon \) can be represented in the form:

\[
\frac{d \Re \gamma}{d \epsilon} = - \int_0^\infty \epsilon^{-1} \frac{d \epsilon}{\epsilon} \ln \left( \frac{\Re \gamma}{\gamma_0} \right),
\]

where \( \mu \) is the chemical potential of He\textsuperscript{3} in the solution. The dispersion equation is given here in the following form:

\[
\Re \gamma(w', k) = - \int_0^\infty \epsilon^{-1} \frac{d \epsilon}{\epsilon} \ln \left( \frac{\Re \gamma}{\gamma_0} \right).
\]

In the expressions (2.7), \( s_0 = w/\hbar k_0 = w'/v_0 \), \( s_0 \eta_0 \) is the value of the distribution function for the He\textsuperscript{3} quasiparticles moving with a velocity equal to the velocity of the spin wave, i.e., at \( v = v' / k \) we have:

\[
\Re \gamma(w', k) = \frac{s_0^2}{2}, \quad \Re \gamma_0 = - \frac{s_0^2}{2} w/v_0.
\]

Since \( |\lambda| \ll 1 \) in the degenerate He\textsuperscript{3}–He II solutions at sufficiently low concentrations of impurity atoms (in practice, below \( 3 \times 10^{-1} \)), then the basic contribution to the function \( \omega_0(\lambda) \) is made by the large logarithmic \( \ln |\lambda| \). With account of this fact, we determine the collisionless absorption of the symmetric spin mode with logarithmic accuracy from the formulas (2.6) and (2.7):

\[
\omega_0(\lambda) = \omega_0(1) \exp \left( - \frac{1}{\lambda} \right)
\]

and the temperature corrections to the wave propagation velocity at \( T = 0 \):

\[
u' = \frac{\nu'-\mu}{\epsilon} - \frac{\nu'-\mu}{\epsilon} \exp \left( - \frac{1}{\lambda} \right),
\]

where \( \epsilon = \frac{1}{\lambda} \). We note that the second term in (2.10) is small in comparison with the difference between the real value of \( \omega_0 \) and that given by formula (2.2) with logarithmic accuracy. However, this term at \( T/T_0 = \exp(-1/|\lambda|) \) completely determines the temperature dependence of the spin wave velocity.

The condition of the smallness of the damping, which was used in the derivation of the equalities (2.5) in the given case, implies, as a consequence of the closeness of the propagation velocity of the spin wave to the Fermi velocity, that \( \gamma = w' \ll \hbar v_0 \exp(-1/|\lambda|) \). This means that the region of applicability of the expression for \( \gamma \) (2.5) and (2.9) is restricted by the condition:

\[
\eta_0(\lambda) = \exp \left[ - \frac{2 \ln(T_0/T)}{\lambda} \right] \ll 1.
\]

The dispersion characteristics of the symmetric spin mode can be expressed in terms of \( \omega_0 \), with the help of a method similar to that used in the previous section of the paper. In the corresponding notation, the dispersion equation has the form:

\[
\omega_0 = \int \frac{d^3v}{(2\pi)^3} \frac{(v' - \mu)}{v_0} = \eta_0(\lambda) \exp \left[ - \frac{2 \ln(T_0/T)}{\lambda} \right].
\]

where \( \omega_0 = \omega_0(1) \). In the case of weak damping \( |\lambda| \ll 1 \), the absorption coefficient is determined by Eq. (2.9). At higher temperatures \( \omega_0^2/\epsilon = 1 \), the quantity \( \omega_0 \) and the condition (1) is violated. Thus, the spin waves in the He\textsuperscript{3}–He II solution can be recorded only at sufficiently low temperatures \( T = T_0 \exp(1/|\lambda|) \). This is equivalent the numerical inequality:

\[
T[K] \ll 4 \times 10^{12} \cdot 10^{27} \times \exp(-2.4/s''),
\]

where \( s'' \) is the molar concentration of the He\textsuperscript{3} in the solution.

For collisional absorption (1.13), the resolution condition (1) is satisfied in the given case in the range of frequencies (1.13) with \( \omega_0 \ll \omega_\perp \) and the condition on the temperature, \( T[K] \ll \omega_0 \). This restriction on the temperature is weaker than that obtained above for the Landau damping. From a comparison of the coefficient of Cherenkov absorption (2.9) with \( \omega_0 \) (1.15), with account of (1), it is easy to determine the range of frequencies included in the interval between \( \omega_0 \) and \( \omega_\perp \), and the condition on the temperature, \( T[K] \ll \omega_0 \). This at which the collisionless damping dominates.
Spin waves in a magnetic field

The action of an external magnetic field \( \mathbf{H} \) leads to a change in the spectrum of the spin waves in the degenerate He\(^3\)-He\(^4\) solution. Two transverse slit modes appear here for the oscillations of the magnetic-moment components perpendicular to the direction of the field \( \mathbf{H} \), and a longitudinal mode of coupled high-frequency spin–sound oscillations. Since the longitudinal spin mode in the solution appears even in an exponentially weak magnetic field and at very low temperatures, we shall in what follows be interested in the conditions of propagation only of transverse oscillations of the magnetization.

The equation for the circular component of the magnetic moment \( M_z + i M_x \) can be represented in the following form:

\[
(\omega - kv) T_2 \left[ \frac{\partial w(s, p')}{\partial s} - \frac{2}{A} (s - s') \right] + \frac{2 M H}{A} \nu = 0, \tag{2.13}
\]

where \( \nu \) are the equilibrium distribution functions of quasiparticles with spin orientations along and counter to the direction of the magnetic field \( \mathbf{H} \), and \( s \) the thermal velocity of the quasiparticles of the Fermi liquid.

After simple transformations of (2.13) we find the dispersion equation for the spectrum of a transverse spin wave:

\[
(\omega - kv)^2 \left[ \frac{\partial w(s, p')}{\partial s} - \frac{2}{A} (s - s') \right] + \frac{2 M H}{A} \nu = 0, \tag{2.14}
\]

We seek the weakly damped solution of Eq. (2.14) in the form \( \omega = \omega' + i \gamma \) and \( k = k - i \kappa \), respectively. In Eqs. (2.13) we have used the value of the exchange part of the Fermi-liquid function along with the Bose-condensed state. We turn to a consideration of the solutions of the dispersion equation (2.13) limited by the condition of applicability of the quasi-classical kinetic equation used in the derivation of (2.13).

At higher temperatures of the solution, \( T \approx T_{c0} \), the damping connected with the finite lifetime of the quasiparticles of the Fermi liquid is important. However, thanks to the low concentration of the solution, the damping of the excitation of He\(^3\) is small to the extent of the smallness of the gas parameter \( \lambda \). In the Born approximation, the damping of the quasiparticles is generally absent. This allows us to solve the problem in first order perturbation theory of the propagation of waves of magnetization in the weak He\(^3\)-He\(^4\) solution at arbitrary temperatures. At \( T \approx T_{c0} \), the impurity atoms of He\(^3\) obey Boltzmann statistics. In the Boltzmann region \( T \approx T_{c0} \), the chemical potential of He\(^3\) in the solution due to interaction of the impurity excitations with one another, \( \nu = (T/\hbar) \nu_0 = (T/\hbar) \nu_0 \), is the total number of impurity atoms of He\(^3\) with different spin orientations per unit volume of solution, and \( N = N_+ + N_- \), is the total number of He\(^3\) atoms per unit volume.

In the region of small wave vectors, the magnon spectrum is quadratic in \( k' \):

\[
\omega = \frac{2 M H}{A} (\Delta s) (s) + \frac{2 M H}{A} (s - \delta s), \quad \omega = \frac{2 M H}{A} (\Delta s) \tag{2.17}
\]

Here the Cherenkov damping of the spin wave is exponentially small:

\[
\epsilon = \left( \frac{\Delta s}{k_{0v}} \right) \exp \left( -\frac{T_{c0}}{T_{c0}} \right), \quad \omega_{\text{sc}} - \frac{2 M H}{A} (s - \delta s) \tag{2.18}
\]

and the fundamental dissipation mechanism is the collisional relaxation absorption.

In the shortwave region \( k_{0v} \gg \omega_{\text{sc}} \), the dispersion equation (2.13) takes a form similar to (2.3). Therefore, the results of the previous section of the paper can be extended directly to this case. It should be noted that the region of existence of the shortwave solution of Eq. (2.13) is limited by the condition of applicability of the quasi-classical kinetic equation used in the derivation of (2.13).
tion functions $n_+$ and $n_-$, the dispersion equation (2.14) for the transverse oscillations of the magnetization takes the form

$$\omega + i \frac{1}{\frac{\omega}{c_B^2}} \left( \frac{\omega}{c_B^2} + \omega_m \right) = \omega_m^0,$$  \hspace{1cm} (2.21)

where $\Omega = \omega - 2BH/k + \omega_m$ and the functions $\omega_m$ and $J(\pi)$ are determined by the expressions (2.18) and (2.20), respectively.

We seek a long-wave weakly damped solution of Eq. (2.21) in the form of an expansion in powers of $kvd_w$.

In the zeroth approximation $N_+ - N_-$ corresponds to a uniform free precession of the nuclear spins of He$^3$. With the help of the asymptotic representation (2.20) of the function $J(x)$, for homogeneous solution $w' = w$, corrections that are quadratic in $kvd_w$:

$$\omega_{\text{m}} = \omega_{\text{m}}^0 - 2\omega_{\text{m}} \omega_{\text{m}}^0 + \frac{k^2}{k_B^2} \omega_{\text{m}}^0 \omega' \left( \frac{\omega_{\text{m}}^0}{\omega} \right).$$  \hspace{1cm} (2.22)

It was taken into account in (2.22) the fact that in the Born approximation, it is necessary to use for the difference in populations $N_+ - N_-$, which enters into $\omega_{\text{m}}$, its value in the ideal Boltzmann gas with neglect of the Fermi-liquid interaction:

$$N_+ - N_- = N \tanh (B\pi / T).$$

The absorption coefficient $\gamma$ of the wave is equal to

$$\gamma = \frac{2}{3} \frac{\omega_{\text{m}}^0}{k_B^2} \omega_{\text{m}}^0 \left( \frac{1}{\frac{\omega_{\text{m}}^0}{\omega}} \right)^2 \omega_{\text{m}}^0 \omega' \left( \frac{\omega_{\text{m}}^0}{\omega} \right).$$  \hspace{1cm} (2.23)

At not small vectors $k$ the Cherenkov damping (2.23) can turn out to be greater than the collisional damping $\gamma_{\text{coll}} = N\nu_k v$. The short-wave region, when the presence of a gap in the dispersion law becomes unimportant and the oscillations of the magnetization are described by an equation of the form (2.19), strong collisionless damping makes propagation of the spin wave impossible.

Thus, the observation of spin waves in the He$^3$–He II solution in the absence of a magnetic field required the obtaining of very low temperatures. The turning on of the magnetic field makes possible the propagation of weakly damped spin modes over a wide range of temperatures. The increase in the temperature decreases the width of the region of wave vectors in which undamped magnons exist.

We express our gratitude to A. F. Andreev, M. I. Kaganov and I. A. Fomin for useful discussions.