EFFECTS OF HIGH SPIN POLARIZATION IN $^3$He-$^4$He MIXTURES

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We apply a generalized microscopic Landau theory of spin-polarized Fermi liquids to $^3$He-$^4$He mixtures. We describe spin dynamics, a zero temperature relaxation, and thermodynamic parameters including the retardation caused by the phonon part of $^3$He-$^4$He interaction.

I. INTRODUCTION

New techniques of spin polarization brought into being highly polarized Fermi liquids and gases [1-4]. It is known that the standard Fermi liquid theory cannot be applied to highly polarized Fermi liquids (see, e.g., [4]). Some of the reasons are the zero-temperature relaxation and non-locality in transverse spin dynamics [1,4,5]. Recently we developed an exact general non-local microscopic theory of transverse spin dynamics in polarized Fermi liquids [6]. We apply this theory to $^3$He-$^4$He mixtures including the retardation associated with a phonon part of the interaction.

II. GENERAL EQUATIONS

Microscopic equations of spin dynamics in highly polarized Fermi liquids do not reduce to any equation in a single-particle density. The exact equation in the mixed spin component of the Green's function, $G_{\uparrow\downarrow}$ reduces to two coupled non-local equations in transverse pseudo-densities $\delta n_{\uparrow\downarrow} = \delta n^{(\pi)}_{\uparrow\downarrow}$ and $\delta n_{\uparrow\downarrow} = \delta n^{(1)}_{\uparrow\downarrow}$:

$$ W_{\pi}(K,p)\delta n_{\pi}(p) = \frac{1}{2} \int \frac{d^3 p'}{(2\pi)^3} \left( \theta_p(p') - \theta_p(p) \right) \sum_{\pi=\uparrow,\downarrow} F_{\pi}(K,p,p') \delta n_{\pi}(p') $$

(1)

The indices $\uparrow, \downarrow$ stay for $\uparrow$ and $\downarrow$, the 4-vector $K=k+\alpha \hbar$, $\hbar$. The definitions of generalized Landau functions $F_{\pi}$ and generalized Liouville operators $W_{\pi}$ through the irreducible vertex and the mass operators are the same as in [6].

These equations can be explained in the following way. In equilibrium, all spins are aligned up or down. The transverse spin density is excited by tilting the spins. At low polarization, the interaction dresses spin-ups and spin-downs in the same way, and spin dynamics is described by an overall transverse spin density. At high polarization, tilted spin-ups and spin-downs are dressed differently. Then the spin dynamics is described by two coupled equations in the separate transverse pseudo-densities. The effective fields for spin-ups and spin-downs are different, and the spin-ups and spin-downs precess with different frequencies with a dephasing which leads to a zero-temperature attenuation.

Direct applications of this theory are limited to the extent to which mass operators and the irreducible vertex are known. Non-model calculations are possible for dilute systems like a $^3$He$^\uparrow$ subsystem in $^3$He-$^4$He mixtures.

III. A DILUTE GAS WITH LOCAL INTERACTION

In the first three orders in density, all quantities are expressed via the single interaction parameter - the scattering length $a = -0.97 \text{ a}_0$ [1,7]. In two lowest orders, Eqs.(1) collapse into a single equation which differs from the usual one mainly by the imaginary term in the spin wave spectrum $\alpha = \Omega + \gamma k^2$.

$$ \gamma = \frac{3\pi((1+\alpha)\hbar^2 -(1-\alpha)\hbar^2)}{40m^3c^2} + \frac{\pi F_1(\alpha)}{6m^3c^2} - \frac{\pi F_2(\alpha)}{105m^3c^2} $$

where $\alpha$ is the degree of spin polarization, $p_0$. 0921-4526/94/$307.00 \text{ © 1994 - Elsevier Science B.V. All rights reserved}$

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is the Fermi momentum. Functions $F_1(\alpha)$ and $F_2(\alpha)$ are plotted in Fig. 1. The first term is the same as elsewhere [1]. The imaginary term comes from the pole in the renormalized scattering operator near the real axis (similar to the collisionless Landau damping) and is missing in the standard theory.

NMR experiments [1,7,8] give the ratio of the quality factor, $\Omega_{Q} T_\varnothing$, and the transverse diffusion coefficient, $D_L$, which is equal to

$$\frac{\Omega_{Q} T_\varnothing}{D_L} = \frac{3 \rho m m}{\pi k_B T_F} (1 + \frac{2}{3} F_1(\alpha) p_\varnothing) T_\varnothing$$

Term with $F_1$ lowers the absolute value of the scattering length $\alpha$ [7].

IV. PHONON CONTRIBUTION AND EFFECTIVE MASSES

Non-local effects contribute to the transverse relaxation in higher orders in density. Non-locality has two sources: the retardation in a phonon-mediated part of $^3$He-$^8$He interaction, and the finite range of a direct interaction [9]. The phonon-mediated interaction contributes to Im$\gamma$ as

$$\gamma_{\rho\varnothing} = \frac{9 \pi \alpha^* \rho_\varnothing}{35 \pi \alpha^* c^2} F_5(\alpha)$$

where $\alpha^* = -0.1$ A, $c$ is the sound velocity, and the function $F_5$ is given in Fig. 1.

Our vertex function and self-energy can be also used thermodynamic calculations. We will give the expressions for the effective masses:

$$\delta m_1^* = \frac{m \rho_\varnothing}{\pi^2} F_4(\alpha)$$
$$\delta m_1^* = \frac{m \rho_\varnothing}{\pi^2} F_6(\alpha)$$

where $\pi^* = 2.34 m_1$ is the effective mass of an isolated $^3$He impurity, and the functions $F_4(\alpha)$ and $F_6(\alpha)$ are presented in Fig. 2.

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