

PHY204 Lecture 1 [rln1]

Mechanics



Kinematics:

- **position:** $\vec{r} = \int \vec{v} dt$
- **velocity:** $\vec{v} = \frac{d\vec{r}}{dt} = \int \vec{a} dt$
- **acceleration:** $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

Dynamics: $\vec{F} = m\vec{a}$ (cause and effect)

Modes of motion: translation, rotation, oscillation

Conservation laws: energy, momentum, angular momentum

Effective forces: elastic, contact, friction, weight, ...

Fundamental interaction: gravitational, electromagnetic, strong, weak

ts1

This first slide with course content aims to recall key concepts learned in *Physics One*, which is about mechanics.

Kinematics relates quantities used to describe motion. Position \vec{r} , velocity \vec{v} , and acceleration \vec{a} are vectors and functions of time. They are related to each other via derivatives and integrals. Derivatives and integral are operations carried out on functions.

Dynamics connects motion to its causes. Newton's second law, $\vec{F} = m\vec{a}$, is a statement of cause and effect. A force \vec{F} applied to an object of mass m causes an acceleration \vec{a} of that object. Its velocity \vec{v} and its position \vec{r} can then be inferred via kinematic relations.

We shall see that the notion of cause and effect is of limited usefulness. It is much more productive to speak of interactions. Causes come around and affect the agent of cause directly and indirectly. Newton's third law about action and reaction really speaks of interaction.

We recall special quantities employed to describe different modes of motion: e.g. angular velocity and centripetal force for rotational motion, or angular frequency and period for oscillatory motion.

We also recall conserved quantities such as energy, momentum, and angular momentum, applicable to dynamical problems in different combinations. The use of conservation laws simplifies calculations.

All familiar forces used in dynamics problems can be traced back to the four fundamental forces listed on the slide. In this course, our focus will be on electromagnetic forces.

**Particles generate fields:**

- Massive particle generates gravitational field.
- Charged particle generates electric field.
- Moving charged particle generates electric field and magnetic field.

Fields exert force on particles:

- Gravitational field exerts force on massive particle.
- Electric field exerts force on charged particle.
- Magnetic field exerts force on moving charged particle.

Dynamics:

- Cause and effect between particles and fields and among fields.

Energy and momentum:

- Particles carry energy (kinetic, potential) and momentum.
- Fields carry energy (electric, magnetic) and momentum.

tsl2

The level of abstraction goes up a notch when we introduce (positive and negative) electric charges and discuss what their effects are. They are the source of electric fields and magnetic fields. Fields are even less tangible quantities.

We shall see that charges generate fields and fields exert forces on charges. It gets more complicated. Electric fields can generate magnetic fields and magnetic fields can generate electric fields.

We are familiar from mechanics with kinetic energy and potential energy of tangible objects. As it turns out, electric fields and magnetic fields carry energy as well.

We know that moving massive objects carry kinetic energy and momentum. Fields can do that too. Electric and magnetic fields can perform a dance of mutual generation while traveling through space at the speed of light. In the process they transport energy and momentum.

When you are hit by an electromagnetic wave, e.g. by sunlight, you are likely to feel how your skin absorbs some of its energy. You also experience the momentum of the wave as pressure and recoil even though that effect is imperceptibly small.

One key to success in this course is that you embrace unfamiliar concepts with no reservations and without hesitation. This includes higher levels of abstraction needed to understand them.



Periodic table: ~ 100 elements.

Building blocks of atoms: fundamental particles.

particle	charge	mass
electron	$q_e = -e$	$m_e = 9.109 \times 10^{-31} \text{ kg}$
proton	$q_p = +e$	$m_p = 1.673 \times 10^{-27} \text{ kg}$
neutron	$q_n = 0$	$m_n = 1.675 \times 10^{-27} \text{ kg}$

- SI unit of charge: 1C (Coulomb).
- Elementary charge: $e = 1.602 \times 10^{-19} \text{ C}$.
- Atomic nuclei (protons, neutron) have a radius of $\sim 1 \text{ fm} = 10^{-15} \text{ m}$.
- Atomic electron shells have a radius of $\sim 1 \text{ \AA} = 10^{-10} \text{ m}$.
- Atoms are electrically neutral (equal numbers of electrons and protons).
- Ions: atoms with one or several electrons added or removed.
- Isotopes: atoms differing in the number of neutrons.
- Positively (negatively) charged objects have a deficiency (surplus) of electrons.

tsls

Electric charge, which comes in units of the elementary charge e , is an attribute of some elementary particles, including two of the three that are constituents of atomic matter. The proton carries a positive elementary charge $+e$ and the electron a negative elementary charge $-e$. The neutron, as its name suggests, carries no electric charge.

The SI unit of electric charge is named after one of the pioneers whose major accomplishment will be portrayed below.

All three constituent particles have a mass. The mass of the electron is much smaller than the (roughly equal) masses of the proton and the neutron.

Atoms consist of a nucleus (protons and neutrons), surrounded by electron shells. The nucleus is tiny compared to the size of the shells. Almost all mass is concentrated in the nucleus.

The nucleus is positively charged and the shell negatively. Atoms are electrically neutral. They contain the same numbers of protons and electrons.

There is nothing truly negative about the negative charge of electrons. The assignment of negative charge to electrons and positive charge to protons is a convention.

In some atoms, an additional electron is easily accommodated, in others an electron is easily removed. Such atoms are said to be ionized, negatively or positively.

Periodic Table of Elements



1																	18																		
1	H																																		
	2																																		
3	Li	4	Be											5	B	6	C	7	N	8	O	9	F	10	Ne										
11	Na	12	Mg	3	4	5	6	7	8	9	10	11	12	13	Al	14	Si	15	P	16	S	17	Cl	18	Ar										
19	K	20	Ca	21	Sc	22	Ti	23	V	24	Cr	25	Mn	26	Fe	27	Co	28	Ni	29	Cu	30	Zn	31	Ga	32	Ge	33	As	34	Se	35	Br	36	Kr
37	Rb	38	Sr	39	Y	40	Zr	41	Nb	42	Mo	43	Tc	44	Ru	45	Rh	46	Pd	47	Ag	48	Cd	49	In	50	Sn	51	Sb	52	Te	53	I	54	Xe
55	Cs	56	Ba	Rare Earths	Hf	72	73	Ta	74	W	75	Re	76	Os	77	Ir	78	Pt	Au	80	Hg	81	Tl	82	Pb	83	Bi	84	Po	85	At	86	Rn		
87	Fr	88	Ra	89-103 Actinides	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg																							

* The 1-18 group designation has been recommended by the International Union of Pure and Applied Chemistry (IUPAC).
Elements with atomic numbers 112, 114, and 116 have been reported but not fully authenticated as of September 2010.
From http://www.iupac.org/reports/periodic_table/IUPAC_Periodic_Table-Short.pdf

ts1368

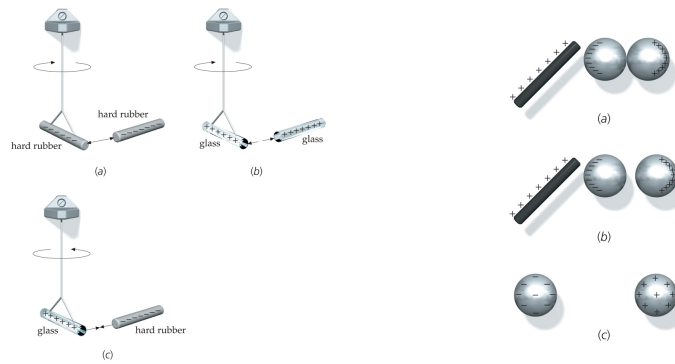
The periodic table lists the known elements. There are a few late additions with shorter-lived nuclei, identified as products of nuclear collisions.

The number inside each box is the *atomic number*. It represents the number of protons in the nucleus and, at the same time, the number of electrons in the surrounding shells.

The number of neutrons in the nucleus of a given element may vary. Atoms of the same element with different numbers of neutrons are named *isotopes*. Isotopes exhibit the same chemical properties but have different masses. Some isotopes have unstable nuclei. They are radioactive, meaning that they tend to decay and “radiate” decay products (including electrons, neutrons, smaller nuclei).

The elements are organized in 18 groups (columns) according to their electronic structure, which governs their chemical properties. Two sets of elements are listed separately due to complications in the architecture of electronic shells.

For this course we just need to appreciate the importance of electric charge as an attribute of atomic matter. We do not really know what electric charge is. However, we need to understand what it does and that there are two kinds of electric charge: positive charge and negative charge.



ts1369

What electric charge does, at face value, is to exert a repulsive or attractive force on other charges. This is readily demonstrated in the low-tech experiment shown on the left.

If we treat a rubber stick with fur, electrons from the hair of the fur are being pasted onto the surface of the rubber. This makes the rubber stick charged negatively. If, on the other hand, we treat a glass stick with silk, electrons are being rubbed off the glass surface. This makes the glass stick charged positively.

The experiment with the torsion balance demonstrates that two rubber sticks or two glass sticks exert a repulsive force on each other, whereas a rubber stick and a glass stick exert an attractive force on each other.

Rubber and glass are materials categorized as *electric insulators*. Excess electric charges in such materials are immobile and located near the surface, for the most part.

In materials that are *electric conductors*, some charge carriers are mobile, free to move through the interior and across interfaces with other conductors. In metals, the mobile charge carriers are a subset of electrons, named conduction electrons, which are shared between atoms.

The experiment on the right illustrates how two (conducting) metal spheres can be charged up oppositely in a low-tech demonstration. When the two spheres are in contact and being approached from one side by a (positively charged) glass stick from the previous experiment, the mobile conduction electrons are being attracted, which results in a surplus on the near side and a deficiency on the far side.

When the spheres are separated, the excess electrons are trapped in one sphere, leaving a deficiency on the other sphere. The two spheres now exert an attractive force on each other because they are oppositely charged.

Coulomb's Law (1)

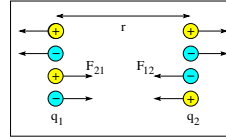


Electrostatic force between two charged particles:

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} = k \frac{|q_1 q_2|}{r^2}$$

Permittivity constant: $\epsilon_0 = 8.854 \times 10^{-12} \text{C}^2 \text{N}^{-1} \text{m}^{-2}$

Coulomb constant: $k = 8.99 \times 10^9 \text{Nm}^2 \text{C}^{-2}$



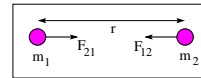
Action-reaction pair of forces: $\vec{F}_{21} = -\vec{F}_{12}$.

Newton's law of gravitation (for comparison)

Gravitational force between two massive particles:

$$F = G \frac{m_1 m_2}{r^2}$$

Gravitational constant: $G = 6.673 \times 10^{-11} \text{Nm}^2 \text{kg}^{-2}$



tsll

The French scientist Coulomb described the force between electric charges quantitatively based on empirical evidence. The expression of that force is now named Coulomb's law.

The expression shown on the slide is for the magnitude of the force between two small charged objects a distance r apart. The vertical bars in the numerator indicate magnitude, meaning that the minus signs of negative charges are ignored.

The gist of Coulomb's law is that the strength of the force is proportional to the product of the charges and inversely proportional to the square of the distance between them. We are dealing with an action-reaction pair of forces, i.e. with two forces that are equal in magnitude and opposite in direction as shown on the slide.

The overall strength of this electrostatic force is determined by a constant k , named *Coulomb constant*. We shall see later that it will be more useful to express this constant differently, in terms of the *permittivity constant* ϵ_0 ("epsilon naught").

The SI units of both constants are such that if charges are expressed in units of [C] and the distance in units of [m], then the force comes out in units of [N]. We do not have to memorize the SI units of k or ϵ_0 , but it is useful and easy to remember that the numerical value of k is very close to 9×10^9 .

For comparison, the slide also shows Newton's law of gravitation for two massive objects whose extensions are small compared to the distance between them. The structure of the law is the same but important differences must be noted: (i) only positive masses are known to exist, (ii) the force is always attractive, (iii) the overall strength (as reflected in the constant G) is much smaller.

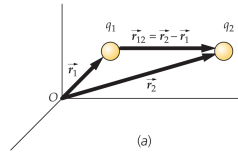
Coulomb's Law (2)



Coulomb's law for electrostatic force in vector form

$$\vec{F}_{12} = k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12},$$

$$\vec{r}_{12} \doteq \vec{r}_2 - \vec{r}_1, \quad \hat{r}_{12} \doteq \frac{\vec{r}_{12}}{r_{12}}.$$

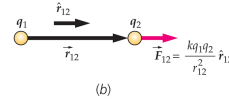


Electric force in hydrogen atom:

Average distance: $r = 5.3 \times 10^{-11} \text{ m}$.

Elementary charge: $e = 1.60 \times 10^{-19} \text{ C}$.

$$\begin{aligned} F &= k \frac{|q_1 q_2|}{r^2} \\ &= \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(5.3 \times 10^{-11} \text{ m})^2} \\ &= 8.2 \times 10^{-8} \text{ N}. \end{aligned}$$



ts1370

If we wish to express the Coulomb force in vector form as done in the top part of the slide, we must pick one or the other force of the action-reaction pair. \vec{F}_{12} is the force particle 1 exerts on particle 2. The direction is indicated by the unit vector \hat{r}_{12} as constructed from the distance vector $\vec{r}_{12} \doteq \vec{r}_2 - \vec{r}_1$.

Note the absence of the vertical bars around the product of charges. If both charges are positive or both negative, then that product is positive and the force is repulsive. The direction of \vec{F}_{12} is the same as that of \hat{r}_{12} . If, on the other hand, one charge is positive and the other negative, then the product $q_1 q_2$ is negative, which makes the force attractive. The direction of \vec{F}_{12} is opposite to that of \hat{r}_{12} .

The force \vec{F}_{21} exerted by particle 2 on particle 1 has the same magnitude because $r_{21}^2 = r_{12}^2$ but opposite direction because $\hat{r}_{21} = -\hat{r}_{12}$.

The bottom part of the slide examines the force between the proton and the electron in a hydrogen atom. It is an attractive force because the proton charge is positive and the electron charge negative.

Quantum mechanics insists that the distance between the proton and the electron remains uncertain but allows us to measure an average distance with certainty. When we use this distance as calculated in the framework of quantum mechanics, in units of [m], the elementary charges for the two particles in units of [C], and the Coulomb constant as given on the previous page, we obtain the force in units of [N].

Coulomb Force in One Dimension (1)



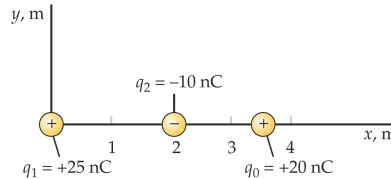
Find net force on charge q_0 due to charges q_1 and q_2 .

Consider x -component of force.

$$F_0 = +k \frac{|q_1 q_0|}{(3.5\text{m})^2} - k \frac{|q_2 q_0|}{(1.5\text{m})^2} = +3.67 \times 10^{-7}\text{N} - 7.99 \times 10^{-7}\text{N} = -4.32 \times 10^{-7}\text{N}.$$

Find net force on charge q_2 due to charges q_1 and q_0 .

$$F_2 = -k \frac{|q_1 q_2|}{(2.0\text{m})^2} + k \frac{|q_2 q_0|}{(1.5\text{m})^2} = -5.62 \times 10^{-7}\text{N} + 7.99 \times 10^{-7}\text{N} = +2.37 \times 10^{-7}\text{N}.$$



ts15

We must always keep in mind that force is a vector: it has magnitude and direction or, equivalently, vector components. When we use components, we can often simplify the notation depending on the case at hand.

In all scenarios considered in the remainder of this lecture, all electric charges are placed along a straight line, which we can declare to be the x -axis. All forces between them are then either directed right (positive x -component) left (negative x -component). In effect, we are declaring that positive forces are directed right and negative forces left.

This convention is introduced in the example worked out on the slide. The force F_0 is the x -component of a vector \vec{F}_0 . There are no y - or z -components in this example.

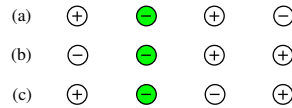
The force F_0 on particle 0 is the sum of two forces, a repulsive force (to the right) exerted by particle 1 and an attractive force (to the left) exerted by particle 2. The negative resultant force means that it is directed left.

Likewise the force F_2 experienced by particle 2 has two parts: an attractive force (to the left) exerted by particle 1 and another attractive force (to the right) exerted by particle 0. The positive resultant force means that it is directed right.



Four point charges equal magnitude are lined up in three different configurations.

The Coulomb force between nearest neighbors is 4N.



Find direction and magnitude of the net force experienced by the green particle in each configuration.

tslz

The green particle in each row experiences three forces, one from each neighbor. Some forces are to the right (positive x -direction) and others are to the left (negative x -direction).

The force exerted by a nearest neighbor has a magnitude of 4N, says the problem statement. Next-nearest neighbors have twice the distance between them. Therefore, the force between them is 1N, one fourth of the force between nearest neighbors.

Forces between like charges are repulsive and forces between unlike charges attractive. Attractive sometimes mean to the right and sometimes to the left. Same with repulsive.

(a) The green particle experiences a strong attractive from its neighbor to the left, a strong attractive force from its nearest neighbor on the right, and a weak repulsive force from its next-nearest neighbor on the right. The two attractive force cancel and the repulsive force is directed left:

$$F = -4\text{N} + 4\text{N} - 1\text{N} = -1\text{N}.$$

(b) Here we have one strong repulsive force to the right and two attractive forces, a weak one and a strong one, also to the right:

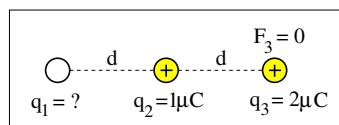
$$F = +4\text{N} + 4\text{N} + 1\text{N} = +9\text{N}.$$

(c) Here one of the two strong forces is attractive and one is repulsive. Both are directed left. The weak force is attractive and directed right.

$$F = -4\text{N} - 4\text{N} + 1\text{N} = -7\text{N}.$$



Three charged particles are positioned along a straight line with two equal spacings .
The net Coulomb force on charge q_3 happens to vanish.



What is the value of q_1 ?

ts18

Each of the three particles experiences two Coulomb forces. The distance between nearest neighbors is d . The distance between next-nearest neighbors is $2d$.

Given the charges q_2 , q_3 , and the fact that the net force on particle 3 vanishes, we can determine the unknown charge q_1 .

The force F_{13} that particle 1 exerts on particle 3 must be equal in magnitude and opposite in direction to the force F_{23} that particle 2 exerts on particle 3. Recall that both forces have only one component, which can either be positive (directed right) or negative (directed left).

F_{23} is repulsive, i.e. directed to the right (positive). Therefore F_{13} must be to the left (negative), which means attractive. We conclude that q_1 must be negative. But what is its value?

We equate the magnitudes of F_{23} and F_{13} :

$$|F_{23}| = |F_{13}| \quad \Rightarrow \quad \frac{k|q_2q_3|}{d^2} = \frac{k|q_1q_3|}{(2d)^2}.$$

Obvious simplifications in the last equation yield the relation,

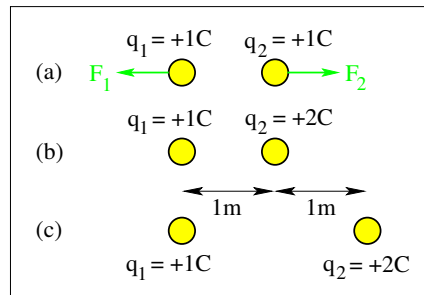
$$|q_1| = 4|q_2| = 4\mu\text{C}.$$

Recalling that particle 1 must be negatively charged, we arrive at the final result:

$$q_1 = -4\mu\text{C}.$$



How are the forces \vec{F}_1 and \vec{F}_2 in (a) affected by the changes made in (b) and (c)?



What changes if the charge q_2 is made negative?

ts1347

The purpose of this exercise is to improve your practical understanding of the Coulomb force.

The first thing to keep in mind is that the Coulomb force is an action-reaction pair of forces, i.e. two forces that are equal in magnitude and opposite in direction. Here they are named \vec{F}_1 and \vec{F}_2 .

Then we recall that the magnitude of both forces is,

$$F_1 = F_2 = k \frac{|q_1 q_2|}{r^2}.$$

Line (a) shows two positive charge. Therefore the Coulomb force is repulsive, which determines the directions of \vec{F}_1 and \vec{F}_2 as shown.

In line (b) we have doubled one of the charges. This has no effect on the directions of the two forces, but *both* forces increase in magnitude by a factor of two.

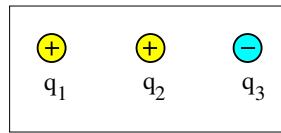
In line (c) we move one charge to double the distance between them. This again has no effect on the directions of the two forces, but *both* forces decrease in magnitude by a factor of four.

What happens when we switch q_2 into $-q_2$? The strength of either force is not affected, but both forces switch direction. The Coulomb force is now attractive. The result is the same if instead we switch the sign of q_1 .

When we switch the sign of both charges, the force remains repulsive. No change in magnitude and direction of either force.



Three particles with charges of magnitude 1C are positioned on a straight line with two equal spacings.



(a) Find the direction (left/right) of the net forces $\vec{F}_1, \vec{F}_2, \vec{F}_3$ on each particle.

(b) Which force is the strongest and which force is the weakest?

ts16

This is the quiz for lecture 1.

The magnitude of all three charges is the same and the distance between nearest-neighbors as well. Each particle experiences two forces, either strong or weak, either repulsive or attractive, either to the left or to the right.

The resultant force for each particle is either to the *left* or to the *right* and it is either *strong*, *intermediate*, or *weak*. Which is it for each of the three particles? Left or right? Strong or intermediate or weak?

Two strong forces in the same direction produce a strong resultant force. A strong and a weak force in the same (opposite) direction produce an intermediate (weak) resultant force.