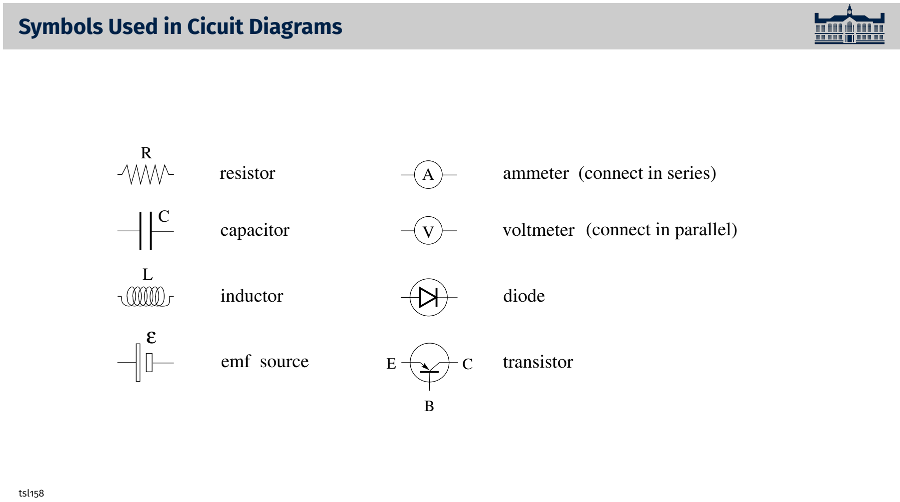
PHY204 Lecture 17 [rln17]



This first slide shows a list of symbols for devices of electric circuits. We are already familiar with the resistor and the capacitor. These two and the inductor will be center stage when we discuss ac circuits near the end of the course.

The emf source supplies power to a circuit by establishing a constant voltage between its terminals. That voltage is modified by its internal resistance when a current flows through it.

The ammeter is designed to measure the current in a circuit branch without impacting the circuit. This can be realized if the ammeter has negligible impedance (resistance) and is connected in series.

The voltmeter is designed to measure the voltage across a device or a group of devices without impacting the circuit. This can be realized if the voltmeter has very large impedance (resistance) and is connected in parallel.

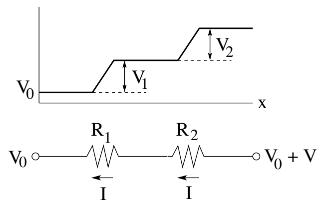
Earlier we have encountered the diode and pointed out its nonlinear current response to a voltage, which is in contrast to the linear responses of the resistor, the capacitor, and the inductor (attributes to be elucidated later).

What distinguishes the transistor from the other devices is its role as an active device. It has three terminals, named emitter, collector, and base. Its response to a voltage between two terminals can be controlled by input from the third terminal. This opens the door to many device functions.

Resistors Connected in Series

Find the equivalent resistance of two resistors connected in series.

- Current through resistors: $I_1 = I_2 = I$
- Voltage across resistors: $V_1 + V_2 = V$
- Equivalent resistance: $R \equiv \frac{V}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2}$
- $\Rightarrow \; R = R_1 + R_2$



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There are two distinct ways in which a pair of resistors can be connected, as a unit, to other parts of a circuit. The slide on this page shows a *series* connection and the slide on the next page a *parallel* connection.

The terminals, indicated by little rings, are the points where the unit connects to other parts of a circuit. In both cases, the unit can be replaced by a single resistor, named *equivalent resistor*, with the same function in the circuit.

The hallmark of two resistors connected in series is that the current through each is the same: $I_1 = I_2 = I$. The electric potential changes across each device and stays constant between devices. The voltage (potential difference) across the unit is the sum of the voltages across the individual devices: $V = V_1 + V_2$.

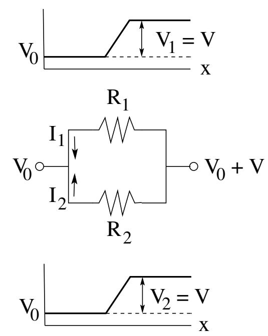
The equivalent resistance, constructed from the definition as shown on the slide, is the sum of the resistances pertaining tho the resistors connected in series:

$$R = R_1 + R_2.$$

Resistors Connected in Parallel

Find the equivalent resistance of two resistors connected in parallel.

- Current through resistors: $I_1 + I_2 = I$
- Voltage across resistors: $V_1 = V_2 = V$
- Equivalent resistance: $\frac{1}{R} \equiv \frac{I}{V} = \frac{I_1}{V_1} + \frac{I_2}{V_2}$
- $\Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$



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A parallel connection of two resistors involves two junctions. At the junction on the right, the current splits into two parts. The two parts join back into the original current at the junction on the left. We have $I = I_1 + I_2$

Given that the electric potential in a circuit only changes across devices, it is the same on the left of both resistors and the same again but at a different value on the right of both resistors. The voltage across resistors (or any devices, for that matter) connected in parallel is identical: $V_1 = V_2 = V$.

If we wish to replace the unit of two parallel resistors with resistances R_1 and R_2 by an equivalent resistor, what must its resistance R be? The answer is worked out on the slide.

In this case, we have to add up the inverse resistances to get the inverse equivalent resistance:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \Rightarrow \ R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}.$$

Here we add a few general remarks about series and parallel connections.

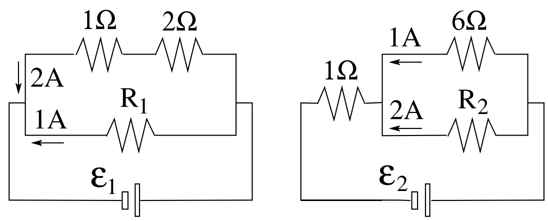
There are series and parallel connections between different devices (e.g. a resistor and a capacitor). Equivalences only apply to devices of the same kind (e.g. two resistors or two capacitors).

If we have a single resistor connected to an EMF source in a one-loop circuit, they can be said to be connected in series because the same current flows through both. We can also be say that they are connected in parallel because they have the same voltage across.

Resistor Circuit (1)

Consider the two resistor circuits shown.

(a) Find the resistance R_1.
(b) Find the emf $\mathcal{E}_1$.
(c) Find the resistance R_2.
(d) Find the emf $\mathcal{E}_2$.



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The questions asked on the slide of this page and the next test our understanding of what the notions *in parallel* and *in series* imply.

(a) We recognize that the 1Ω and 2Ω resistors are in series. Their equivalent resistance is 3Ω. Hence the voltage across both is $V = (2\text{A})(3\Omega) = 6\text{V}$. The resistor with resistance R_1 is connected in parallel. Hence it has the same voltage across. We know the current flowing through it. Hence we infer that $R_1 = 6\text{V}/1\text{A} = 6\Omega$.

(b) It is the battery that supplies the 6V across the resistor with resistance R_1. The two devices are connected in parallel. Therefore, ignoring internal resistance, we have $\mathcal{E}_1 = 6\text{V}$.

(c) We recognize the resistors with resistances 6Ω and R_2 to be connected in parallel. The voltage across the former is $(1\text{A})(6\Omega) = 6\text{V}$, which must be equal to the voltage across the latter. We conclude that $R_2 = 6\text{V}/2\text{A} = 3\Omega$.

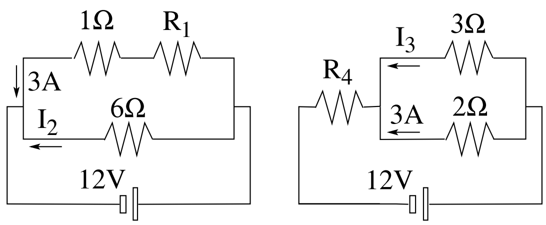
(d) The sum of the currents that flow into the junction must flow out of it on the other side. Hence the current through the 1Ω resistor is $1\text{A} + 2\text{A} = 3\text{A}$. The voltage across it is $(3\text{A})(1\Omega) = 3\text{V}$.

The parallel combination with 6V across and the 1Ω resistor with 3V across are connected in series for a total of 9V across. That must be the voltage supplied by the EMF, which is parallel to the series combination: $\mathcal{E} = 9\text{V}$.

Resistor Circuit (2)

Consider the two resistor circuits shown.

(a) Find the resistance R_1.
(b) Find the current I_2.
(c) Find the current I_3.
(d) Find the resistance R_4.



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The voltage across the parallel connection in the circuit on the left is,

$$12\text{V} = (3\text{A})(1\Omega + R_1) = (I_2)(6\Omega),$$

from which we infer (a) $R_1 = 3\Omega$ and (b) $I_2 = 2\text{A}$.

(c) In the circuit on the right, the voltage across the parallel connection is

$$(3\text{A})(2\Omega) = (I_3)(3\Omega) = 6\text{V},$$

from which we infer that $I_3 = 2\text{A}$.

(d) The resistor with resistance R_4 is connected in series with the parallel combination. Hence the voltages must add up to the voltage supplied by the battery:

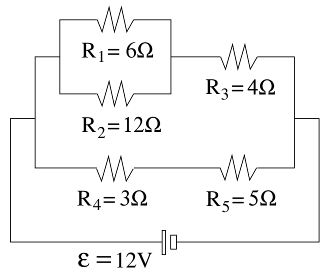
$$V_4 + 6\text{V} = 12\text{V} \quad \Rightarrow \; V_4 = 6\text{V}.$$

The junction rule says that the current that flows out of the parallel combination is $2\text{A} + 3\text{A} = 5\text{A}$. Hence we $R_4 = 6\text{V}/5\text{A} = 1.2\Omega$.

Resistor Circuit (8)

Consider the circuit of resistors shown.

- Find the equivalent resistance R_{eq}.
- Find the currents $I_1, \ldots, I_5$ through each resistor and the voltages $V_1, \ldots, V_5$ across each resistor.
- Find the total power P dissipated in the circuit.



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We begin be reducing this circuit into one with a single equivalent resistor in three steps:

$$R_{12} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{1}{6\Omega} + \frac{1}{12\Omega}\right)^{-1} = \left(\frac{3}{12\Omega}\right)^{-1} = 4\Omega.$$

$$R_{123} = R_{12} + R_3 = 4\Omega + 4\Omega = 8\Omega, \quad R_{45} = R_4 + R_5 = 3\Omega + 5\Omega = 8\Omega.$$

$$R_{eq} = \left(\frac{1}{R_{123}} + \frac{1}{R_{45}}\right)^{-1} = \left(\frac{1}{8\Omega} + \frac{1}{8\Omega}\right)^{-1} = \left(\frac{2}{8\Omega}\right)^{-1} = 4\Omega.$$

Now we reverse the reduction process, again in three steps:

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{12\text{V}}{4\Omega} = 3\text{A}, \quad V_{123} = V_{45} = \mathcal{E} = 12\text{V}.$$

$$I_{123} = \frac{V_{123}}{R_{123}} = \frac{12\text{V}}{8\Omega} = \frac{3}{2}\text{A}, \quad I_{45} = \frac{V_{45}}{R_{45}} = \frac{12\text{V}}{8\Omega} = \frac{3}{2}\text{A}.$$

$$I_{12} = I_3 = I_{123} = \frac{3}{2}\text{A}, \quad I_4 = I_5 = I_{45} = \frac{3}{2}\text{A}.$$

$$V_{12} = R_{12}I_{12} = 6\text{V}, \quad V_3 = R_3I_3 = 6\text{V}, \quad V_4 = R_4I_4 = \frac{9}{2}\text{V}, \quad V_5 = R_5I_5 = \frac{15}{2}\text{V}.$$

$$V_1 = V_2 = V_{12} = 6\text{V}, \quad I_1 = \frac{V_1}{R_1} = 1\text{A}, \quad I_2 = \frac{V_2}{R_2} = \frac{1}{2}\text{A}.$$

$$P = \sum_{i=1}^{5} I_i^2 R_i = I^2 R_{eq} = (3\text{A})^2 4\Omega = 36\text{W}.$$

Kirchhoff's Rules

Loop Rule

- When any closed-circuit loop is traversed, the algebraic sum of the changes in electric potential must be zero.

Junction Rule

- At any junction in a circuit, the sum of the incoming currents must equal the sum of the outgoing currents.

Strategy

- Use the junction rule to name all independent currents.
- Use the loop rule to determine the independent currents.

tsl159

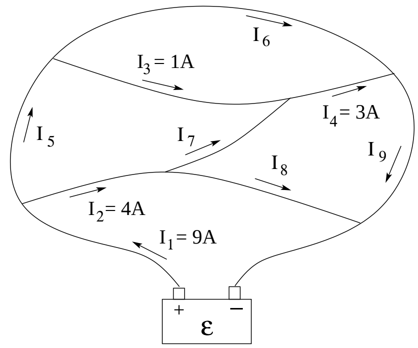
A systematic analysis of resistor circuits in a steady state employs Kirchhoff's rules, which we have already used off and on without naming them.

Here are a few key aspects that we must heed when embarking on the analysis of a resistor circuit.

- Attribute a current with direction to every branch in the circuit.
- Minimize the number of named currents by using the junction rule.
  - A one-loop circuit has one current. Give it a name and a direction.
  - A two-loop circuit has two junction and three branches. There are only two independent currents. Name them and assign a direction to each. The third current is a linear combination of the first two, dictated by the junction rule.
- All loops are paths closed in themselves. There are no loops across open switches. Any loop qualifies and produces a linear equation for the named currents.
- The number of loop equations to be established is equal to the number of named currents.

Applying the Junction Rule

In the circuit of steady currents, use the junction rule to find the unknown currents $I_5, \ldots, I_9$.



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This network of wires has nine branches and six junctions. One branch has two parts, each connected to a terminal of the battery. This network is equivalent to a circuit with nine resistors and steady currents in each branch.

There are four independent currents. In the slide, four of the currents, $I_1, \ldots, I_4$ are given specific values. By using the junction rule, we can express the currents $I_5, \ldots, I_9$ as linear combinations of the four known currents as follows:

$$I_5 = I_1 - I_2 = 5\text{A}, \quad I_6 = I_1 - I_2 - I_3 = 4\text{A}, \quad I_7 = I_4 - I_3 = 2\text{A},$$

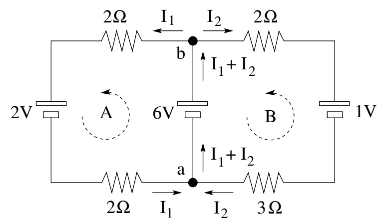
$$I_8 = I_2 + I_3 - I_4 = 2\text{A}, \quad I_9 = I_1 - I_2 - I_3 + I_4 = 7\text{A}.$$

If we were given the resistances $R_1, \ldots, R_9$ in the nine branches and the EMF $\mathcal{E}$ instead of the four currents, we would need to consider four loops to determine the four independent currents.

Applying Kirchhoff's Rules

Consider the circuit shown below.

- Junction a: I_1, I_2 (in); $I_1 + I_2$ (out)
- Junction b: $I_1 + I_2$ (in); I_1, I_2 (out)
- Two independent currents require the use of two loops.
- Loop A (ccw): $6\text{V} - (2\Omega)I_1 - 2\text{V} - (2\Omega)I_1 = 0$
- Loop B (ccw): $(3\Omega)I_2 + 1\text{V} + (2\Omega)I_2 - 6\text{V} = 0$
- Solution: $I_1 = 1\text{A}$, $I_2 = 1\text{A}$



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This slide is designed to serve as a template for the analysis of two loop circuits using Kirchhoff's rules.

We begin by naming to independent currents. The third current is determined by the junction rule. If we use junction a to determine the third current, the rule is automatically satisfied at junction b and vice versa.

Assigning a current to a branch means giving it a name and a direction indicated by an arrow as done on the slide. Recall that current directions are always a matter of choice. Whether the current in a chosen direction turns out to be positive or negative is a result of the analysis.

We need to consider two loops to determines the two independent currents. Loops A and B are simple choices. Part of choosing a loop is determining the direction (ccw) and the starting point (junction a).

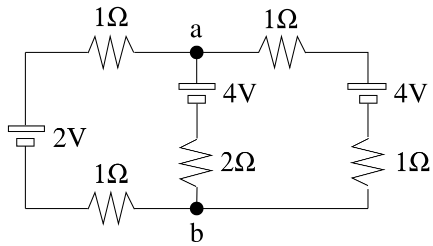
Recall that the change in potential across a resistor in current direction is $-IR$. The change in potential across an EMF source is $+\mathcal{E}$ if the loop moves from the negative terminal to the positive terminal.

In this simple case, each loop equation only involves one of the unknown currents. Hence they can be solved separately. Both currents come out positive.

Resistor Circuit (11)

Consider the electric circuit shown.

- Identify all independent currents via junction rule.
- Determine the independent currents via loop rule.
- Find the Potential difference $V_{ab} = V_b - V_a$.



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The analysis of this two-loop circuits is a bit more complex. We use the template from the previous page.

We begin by naming two currents and giving them directions. We choose current I_1 to flow from a to b along the branch on the left and I_2 to flow from a to b down the middle. The junction rule then demands that the a current $I_1 + I_2$ flows from b to a along the branch on the right.

Next we choose two loops to determine the two unknown currents I_1, I_2. The simplest choices are the physical loops (1) on the left and (2) on the right. We choose to go around ccw and start at the far lower corners.

Loop (1): $-(1\Omega)I_1 + (2\Omega)I_2 + 4\text{V} - (1\Omega)I_1 - 2\text{V} = 0$.

Loop (2): $-(1\Omega)(I_1 + I_2) + 4\text{V} - (1\Omega)(I_1 + I_2) - 4\text{V} - (2\Omega)I_2 = 0$.

Next we simplify the two equations by collecting and sorting terms:

(1): $(2\Omega)I_1 - (2\Omega)I_2 = 2\text{V}$, (2): $(2\Omega)I_1 + (4\Omega)I_2 = 0$.

Solving two coupled linear equations is simple enough:

$$I_1 = \frac{2}{3}\text{A}, \quad I_2 = -\frac{1}{3}\text{A}.$$

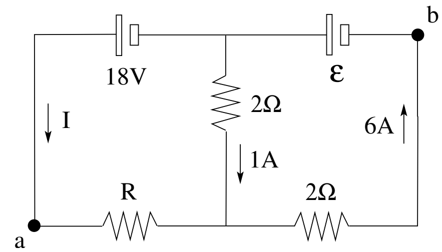
We calculate the voltage V_{ab} by starting at point a and add potential differences along one of the three branches until we reach point b.

$$V_{ab} = \underbrace{-4\text{V} - (2\Omega)(-\tfrac{1}{3}\text{A})}_{\text{center}} = \underbrace{-(1\Omega)(\tfrac{2}{3}\text{A}) - 2\text{V} - (1\Omega)(\tfrac{2}{3}\text{A})}_{\text{left}} = -\tfrac{10}{3}\text{V}.$$

Resistor Circuit (9)

Use Kirchhoff's rules to find

(a) the current I,
(b) the resistance R,
(c) the emf $\mathcal{E}$,
(d) the voltage $V_{ab} \equiv V_b - V_a$.



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Here we have another two-loop circuit, but its analysis requires a bit of creativity. We cannot just follow the template from two slides ago.

(a) Knowing the currents in two of the branches, we can determine the current in the third branch by employing the junction rule. Either junction will do. In the junction on top a 6A current goes in and currents $1\text{A} + I$ come out. Hence we have $I = 5\text{A}$.

(b) We use the loop on the left, starting at a and going ccw:

$$-(5\text{A})R + (2\Omega)(1\text{A}) + 18\text{V} = 0 \quad \Rightarrow \; R = \frac{20\text{V}}{5\text{A}} = 4\Omega.$$

(c) We use the loop on the right, starting at b and going ccw:

$$\mathcal{E} - (2\Omega)(1\text{A}) - (2\Omega)(6\text{A}) = 0 \quad \Rightarrow \; \mathcal{E} = 14\text{V}.$$

(d) We take any path from a to b and get the same answer:

First up, then right: $V_{ab} = -18\text{V} - 14\text{V} = -32\text{V}$.

First right, then up: $V_{ab} = -(5\text{A})(4\Omega) - (6\text{A})(2\Omega) = -32\text{V}$.

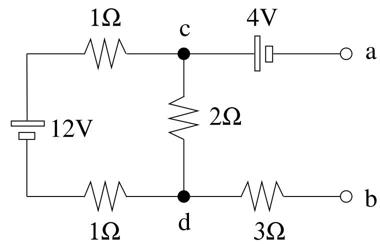
Right, up the middle, then right:

$$V_{ab} = -(5\text{A})(4\Omega) + (1\text{A})(2\Omega) - 14\text{V} = -32\text{V}.$$

Resistor Circuit (10)

Consider the electric circuit shown.

(a) Find the current through the 12V battery.
(b) Find the current through the 2Ω resistor.
(c) Find the total power dissipated.
(d) Find the voltage $V_{cd} \equiv V_d - V_c$.
(e) Find the voltage $V_{ab} \equiv V_b - V_a$.



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This is the quiz for lecture 17.

Keep in mind that a current only flows in branches that are part of closed loops. EMF sources, on the other hand, remain active in dead branches.