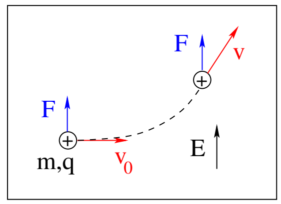
PHY204 Lecture 20 [rln20]

Charged Particle Moving in Uniform Electric Field

- Electric field $\vec{E}$ is directed up.
- Electric force: $\vec{F} = q\vec{E}$ (constant)
- Acceleration: $\vec{a} = \frac{\vec{F}}{m} = \frac{q}{m}\vec{E} = \text{const.}$
- Horizontal motion: $a_x = 0 \Rightarrow v_x(t) = v_0 \Rightarrow x(t) = v_0 t$
- Vertical motion: $a_y = \frac{q}{m}E \Rightarrow v_y(t) = a_y t \Rightarrow y(t) = \frac{1}{2}a_y t^2$
- The path is parabolic: $y = \left(\frac{qE}{2mv_0^2}\right)x^2$
- $\vec{F}$ changes direction and magnitude of $\vec{v}$.



tsl188

Here and on the next page we discuss how the motion of a charged particle is affected by the presence of a uniform electric or magnetic field.

In both cases we see Newton's second law, $\vec{F} = m\vec{a}$, in action. An applied force causes an acceleration, which is the rate of a velocity change. The velocity change can mean a change in speed (change in magnitude of velocity) or a turn (change in direction of velocity) or both.

Consider the electric force, $\vec{F} = q\vec{E}$. As the particle moves through a region of uniform electric field, the electric force remains unchanged, implying that the acceleration $\vec{a}$ is constant.

Motion with constant acceleration is a familiar matter from mechanics. The equation of motion,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{\vec{F}}{m},$$

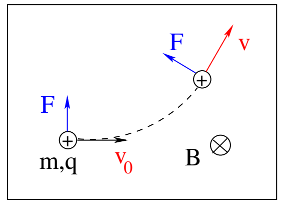
is solved separately for the x- and y-components of the velocity vector $\vec{v} = v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}}$. Integration then yields the position vector $\mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$.

If, as shown on the slide, the applied force acts in a direction perpendicular to the initial velocity $\vec{v}_0$ of the charged particle, then the path traced out by the particle has parabolic shape.

In the example shown the velocity changes in both magnitude and direction.

Charged Particle Moving in Uniform Magnetic Field

- Magnetic field $\vec{B}$ is directed into plane.
- Magnetic force: $\vec{F} = q\vec{v} \times \vec{B}$ (not constant)
- $\vec{F} \perp \vec{v} \;\Rightarrow\; \vec{F}$ changes direction of $\vec{v}$ only $\;\Rightarrow\; v = v_0$.
- $\vec{F}$ is the centripetal force of motion along circular path.
- Radius: $\dfrac{mv^2}{r} = qvB \;\Rightarrow\; r = \dfrac{mv}{qB}$
- Angular velocity: $\omega = \dfrac{v}{r} = \dfrac{qB}{m}$
- Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi m}{qB}$



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Here we consider the magnetic force, $\vec{F} = q\vec{v} \times \vec{B}$. As the particle moves through a region of uniform magnetic field with initial velocity $\vec{v}_0$ as shown, that force does not remain constant.

The cross product tells us that the magnetic force $\vec{F}$ is perpendicular to the velocity $\vec{v}$. We know from mechanics that when the applied force is perpendicular to the velocity, it changes only the direction of $\vec{v}$ but not its magnitude (the speed v).

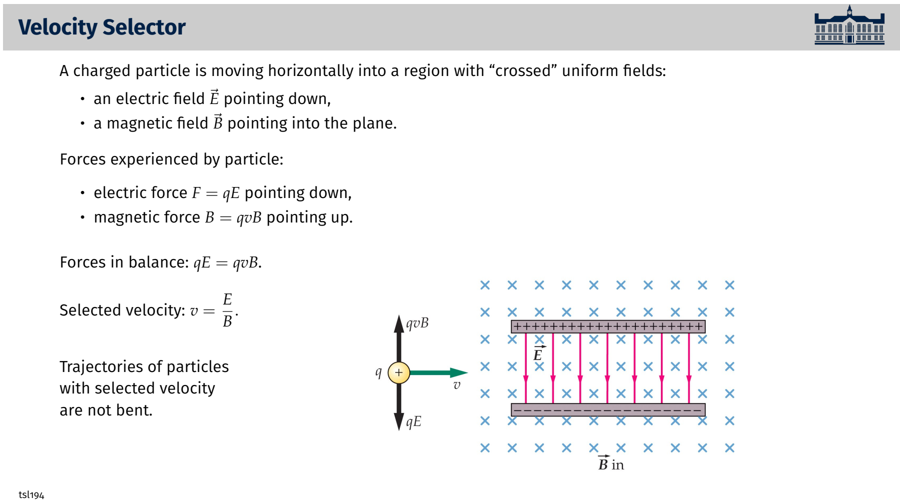
As the velocity $\vec{v}$ changes direction so does the force $\vec{F}$. Both quantities change direction at the same (constant) rate.

Motion in a plane where the velocity changes direction at a constant rate is circular in shape. The magnetic force in that situation plays the role of centripetal force.

The centripetal force, mv^2/r, is the force required to keep an object of mass m on a circular path of radius r. This force can be provided in different ways, e.g. by the tension in the string to which the object is attached or by the gravitational attraction to the earth in the case of the orbiting moon.

In the situation here, the centripetal acceleration is provided by the magnetic force of constant magnitude $F = qvB$. Hence we can equate centripetal force required and magnetic force provided. The radius of the orbit is then inferred from that equation.

Note that the period T of the circular motion is independent of the radius r of the orbit and the speed v of the particle.



Many experiments in physics involve the scattering of charged particles (electrons or protons) from all kinds of material samples. For that purpose it necessary to produce beams of particles that all have the same velocity $\vec{v}$.

Protons (with charge $q = +e$) can be collimated into a beam by having them pass through small holes in a pair of parallel shielding plates. The particles that make it through both holes then all move in the same direction.

The velocity selector portrayed on the slide is an important tool for trimming the beam down to particles with just one speed.

All particles coming in from the left initially move in the same direction but have different speeds. Fast particles experience a stronger magnetic force directed $\uparrow$ than slower particles. All particles experience the same electrical force directed $\downarrow$.

The two opposing forces are in balance for particles moving with one particular speed: $v = E/B$.

The paths of fast particles are bent upward and the paths of slow particles downward. A third parallel shielding plate with a small hole will only let particles pass that have the selected speed $v = E/B$.

The selected speed can be controlled by the experimenter via the electric or magnetic field.

Measurement of e/m_e for Electron

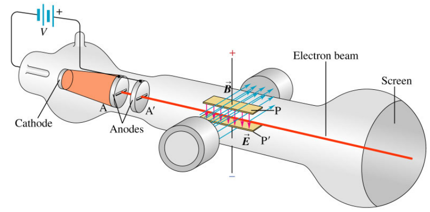
First experiment by J. J. Thomson (1897)

Method used here: velocity selector

Equilibrium of forces: $eE = evB \quad \Rightarrow \; v = \frac{E}{B}$

Work-energy relation: $eV = \frac{1}{2}m_e v^2 \quad \Rightarrow \; v = \sqrt{\frac{2eV}{m_e}}$

Eliminate v: $\frac{e}{m_e} = \frac{E^2}{2VB^2} \simeq 1.76 \times 10^{11}\text{C/kg}$



tsl431

Who discovered America? Columbus? The Vikings? The Chinese? Whoever got here first.

Who discovered that Jupiter has moons? Whoever saw them first. A telescope and the urge to look at Jupiter were needed. Galileo had both.

J. J. Thomson discovered the electron in 1897. He could not get there, he could not see it, how did he do it?

Thomson identified cathode rays as beams charged particles. He could not tell what their mass m or their charge e was but he could tell that e/m had a unique value.

A charged capacitor produces a glow in a vacuum tube when the cathode (negatively charged plate) is heated up. The glow continues past the anode (positively charged plate) when it has a hole or slit.

Something must be moving from cathode to anode: in all likelihood a massive particle with negative charge. When that particle moves across a potential difference V between cathode and anode, it picks up the amount eV of kinetic energy. If that accounts for most of its kinetic energy, the particle velocity is $v = \sqrt{2eV/m}$.

The velocity selector (see previous page) allows an independent determination of the velocity: $v = E/B$. Bingo! From the two relations, a value for e/m can be extracted.

It took more than a decade before the elementary charge e and the electron mass m were determined independently (see next page).

Measurement of e and m_e for Electron

First experiment by R. Millikan (1913)

Method used here: balancing weight and electric force on oil drop

Radius of oil drop: $r = 1.64\mu\text{m}$

Mass density of oil: $\rho = 0.851\text{g/cm}^3$

Electric field: $E = 1.92 \times 10^5\text{N/C}$

Mass of oil drop: $m = \frac{4\pi}{3}r^3\rho = 1.57 \times 10^{-14}\text{kg}$

Equilibrium of forces: $neE = mg$

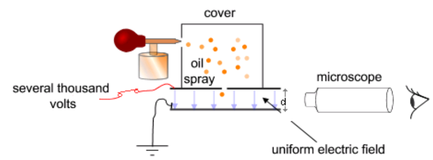
Quantized quantity: $\frac{mg}{E} = ne.$

Number of excess elementary charges: $n = 5$

Elementary charge: $e = \frac{mg}{nE} \simeq 1.6 \times 10^{-19}\text{C}$

Use result from J. J. Thomson: $\frac{e}{m_e} \simeq 1.76 \times 10^{11}\text{C/kg}$

Mass of electron: $m_e \simeq 9.1 \times 10^{-31}\text{kg}$



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Millikan's experiment, which produced the first determination of the electron mass, was also based on balancing two forces: the gravitational force and the electric force acting on a charged oil droplet.

The oil droplets are tiny and carry few elementary charges, created via some ionization technique that either knocks off or pastes on electrons. The exact number is not controllable. The key point is that the charge carried by the oil droplet is quantized, a multiple of the suspected elementary charge e.

The mass of the oil droplets can be determined from its size and the density of the material. The gravitational field g is given, whereas the electric field E is controllable.

Balancing the two forces yields a value for ne, where n is an integer. Data from several droplets makes it straightforward to determine the common unit, which is the elementary charge e.

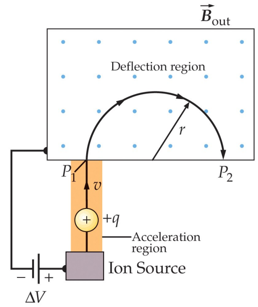
Once we know e, we can use the result from J. J. Thomson's experiment to determine the electron mass m_e.

The slide walks us through one (fictitious) set of data.

Mass Spectrometer

Purpose: measuring masses of ions.

- Charged particle is accelerated by moving through potential difference $|\Delta V|$.
- Trajectory is then bent into semicircle of radius r by magnetic field $\vec{B}$.
- Kinetic energy: $\frac{1}{2}mv^2 = q|\Delta V|$.
- Radius of trajectory: $r = \frac{mv}{qB}$.
- Charge: $q = e$
- Mass: $m = \frac{eB^2r^2}{2|\Delta V|}$.



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The mass spectrometer is an indispensable tool in chemistry and chemical engineering research. It allows for a quick determination of the molecular mass of reaction products.

The idea behind the mass spectrometer is to launch ionized molecules with given kinetic energy into a magnetic field directed perpendicular to its path and measure how that path is being bent into a circle.

Ionizing radiation (e.g. from a radioactive sample) knocks off an electron from molecules placed into the apparatus. The mass reduction is negligible because the electron mass is tiny compared the mass of the nuclei.

The positively charged ion is accelerated by the electric field across a potential difference ΔV and thus picks up a known amount of kinetic energy.

The ionized molecule thus accelerated then enters a region of magnetic field, where its path is curves into a semicircle as shown. A battery of detectors determines the radius r of the path.

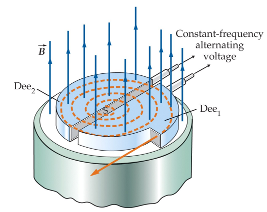
The important point is that the radius r is proportional to mv, not proportional to the kinetic energy, $\frac{1}{2}mv^2$, which is the same for all molecules, irrespective of mass. This has the consequence that molecules with different masses hit different detectors.

The result worked out on the slide shows that the mass is proportional to the square of the radius.

Cyclotron

Purpose: accelerate charged particles to high energy.

- Low-energy protons are injected at S.
- Path is bent by magnetic field $\vec{B}$.
- Proton is energized by alternating voltage ΔV between Dee_1 and Dee_2.
- Proton picks up energy $\Delta K = e\Delta V$ during each half cycle.
- Path spirals out as velocity of particle increases: Radial distance is proportional to velocity: $r = \dfrac{mv}{eB}$.
- Duration of cycle stays is independent of r or v: cyclotron period: $T = \dfrac{2\pi m}{eB}$.
- Cyclotron period is synchronized with alternation of accelerating voltage.
- High-energy protons exit at perimeter of $\vec{B}$-field region.



tsl200

A promising way of determining the make-up of an elementary particle is to accelerate it to high speed, smash it against other particles, and see what happens. That, in a nutshell, is a scattering experiment.

This slide describes how one of the earliest particle accelerator design works: the cyclotron. It only works for charged particles as do all other designs.

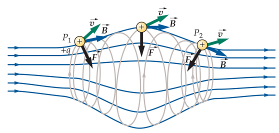
The cyclotron design uses the fact (identified at the end of page 2) that the period of a particle of mass m and charge e circling in a magnetic field B is $T = 2\pi m/eB$, independent of orbital radius r and particle velocity v.

If a potential difference ΔV between the D-shaped conductors is established that alternates with period T, then among all the particles that circle around when launched at low velocity, a subset always gets a kick in the rear when they cross the gap between the Dees.

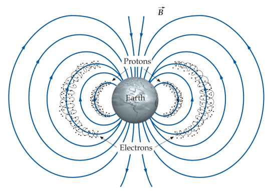
That kick increases both their speed and the radius of their orbit but not the period of their orbital motion. When their orbital radius reaches the edge of the region of magnetic field, the centripetal force stops and the particles fly off at high speed.

Things are, of course, not quite that simple in a real cyclotron.

Moving charged particle
confined by
inhomogeneous magnetic field.



Van Allen belt:
trapped protons and electrons
in Earth's magnetic field.



tsl407

What happens if a charged particle is launched in a uniform magnetic field $\vec{B}$ with an initial velocity $\vec{v}_0$ that is, unlike on page 2, not perpendicular to the field? Let us say we have,

$$\vec{B} = B_z\,\hat{\mathbf{k}}, \quad \vec{v}_0 = v_{x0}\,\hat{\mathbf{i}} + v_{y0}\,\hat{\mathbf{j}} + v_{z0}\,\hat{\mathbf{k}}.$$

In that case the particle still moves at constant speed but its path now is a spiral with axis in z-direction and radius,

$$r = \frac{mv_\perp}{qB}, \quad v_\perp = \sqrt{v_{x0}^2 + v_{y0}^2}.$$

The particle advances in z-direction with constant $v_z = v_{z0}$.

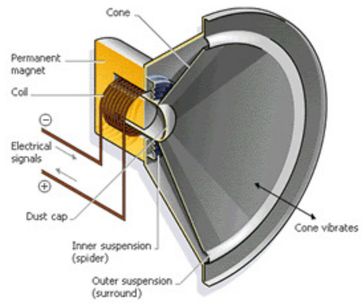
If the magnetic field is non-uniform in the way shown on the left, the bulge represents a region of weaker field. As the particle circles around in a vertical plane, the magnetic force now also has a component parallel to the horizontal axis, pointing into region of weaker field (toward the bulge).

The particle now spirals back and forth inside the bulge. We effectively have a magnetic bottle for charged particles.

Such magnetic traps, named van Allen belts, do exist in the Earth's magnetic field. Protons and electrons, ejected by the sun as solar wind, get trapped in different regions due to their different masses.

Loudspeaker

Conversion of electric signal into mechanical vibration.



tsl410

Sound carrying music or spoken information travels through air as a superposition of longitudinal pressure and density waves. In a microphone (electrical transducer) sound causes a mechanical vibration which is converted into an electrical signal. When the electrical signal is amplified, the sound can be recreated in a loudspeaker (mechanical transducer) as shown schematically.

The amplified electrical signal is a current on which the vibrations picked up by the microphone are encoded as a pattern. The current flows through a coil attached to a cone and positioned in a magnetic field such that the magnetic force is either toward the front or the back of the cone.

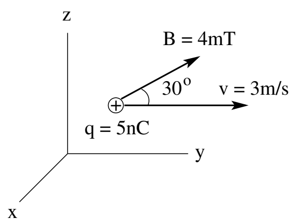
The magnetic force thus reproduces, on the cone, the vibrational pattern picked up by the microphone.

The cone, in turn, emits sound waves traveling through the air, carrying the original music or spoken information.

Intermediate Exam II: Problem #4 (Spring '05)

Consider a charged particle moving in a uniform magnetic field as shown. The velocity is in y-direction and the magnetic field in the yz-plane at $30°$ from the y-direction.

(a) Find the direction of the magnetic force acting on the particle.
(b) Find the magnitude of the magnetic force acting on the particle.



Solution:

(a) Use the right-hand rule: positive x-direction (front, out of page).
(b) $F = qvB\sin 30° = (5\times10^{-9}\text{C})(3\text{m/s})(4\times10^{-3}\text{T})(0.5) = 3\times10^{-11}\text{N}$.

tsl339

We conclude this lecture with two simple quantitative applications of magnetic force.

We recognize at once that the problem posed on the slide here is an application of the magnetic force on a charged particle: $\vec{F} = q\vec{v}\times\vec{B}$.

For part (a) we employ the right-hand rule as explained in the previous lecture: index to the right and middle finger to the upper right makes the thumb point out of the plane.

For part (b) we recall how to calculate the magnitude of a vector that is the result of the cross product between two vectors as also explained in the previous lecture.

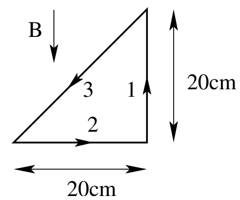
There are plenty of similar problems among the exam 2 slides.

Intermediate Exam II: Problem #4 (Spring '06)

A current loop in the form of a right triangle is placed in a uniform magnetic field of magnitude $B = 30\text{mT}$ as shown. The current in the loop is $I = 0.4\text{A}$ in the direction indicated.

(a) Find magnitude and direction of the force $\vec{F}_1$ on side 1 of the triangle.

(b) Find magnitude and direction of the force $\vec{F}_2$ on side 2 of the triangle.



Solution:

(a) $\vec{F}_1 = I\vec{L} \times \vec{B} = 0$ (angle between $\vec{L}$ and $\vec{B}$ is $180°$).

(b) $F_2 = ILB = (0.4\text{A})(0.2\text{m})(30 \times 10^{-3}\text{T}) = 2.4 \times 10^{-3}\text{N}$.
Direction of $\vec{F}_2$: $\otimes$ (into plane).

tsl354

This an application of magnetic force on segments of currents: $\vec{F} = I\vec{L} \times \vec{B}$, which is readily recognizable. Again, there are many problems among the exam 2 slides.

The solution of both parts (a) and (b) are straightforward and worked out on the slide.

Suppose we add a part (c) asking ourselves to determine the force on side 3.

The right-hand rule tells us that the direction of force $\vec{F}_3$ is into the plane, i.e. opposite in direction to force $\vec{F}_2$.

For the magnitude of $\vec{F}_3$ we must take into account that side 3 is longer than side two by a factor $\sqrt{2}$ and that the angle between $\vec{L}$ and $\vec{B}$ is $45°$. Hence w can write,

$$F_3 = (0.4\text{A})(\sqrt{2})(0.2\text{m})(30 \times 10^{-3}\text{T}) \underbrace{\sin(45°)}_{1/\sqrt{2}} = 2.4 \times 10^{-3}\text{N}.$$

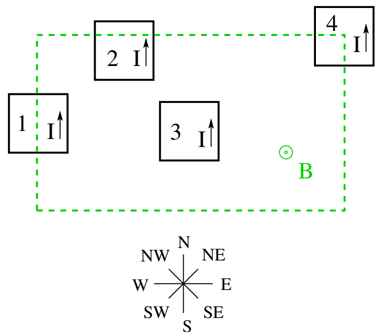
The magnitude of $\vec{F}_3$ is the same as that of $\vec{F}_2$.

It turns out that the net force on a current loop in a uniform magnetic field is always zero. That does not mean, as we shall see in the following lecture, that a free current loop does not respond with motion when positioned in a constant magnetic field. Zero net force does not mean zero torque.

Magnetic Force Application (3)

The dashed rectangle marks a region of uniform magnetic field $\vec{B}$ pointing out of the plane.

- Find the direction of the magnetic force acting on each loop with a ccw current I.



tsl190

This is the quiz for lecture 20.

It is an application of the vector equation $\vec{F} = I\vec{L} \times \vec{B}$.

Possible answers are either no force or a force directed toward one of the eight compass points indicated.