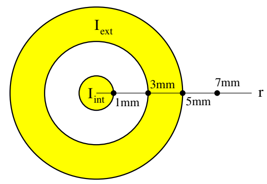
PHY204 Lecture 26 [rln26]

Unit Exam III: Problem #3 (Spring '12)

The coaxial cable shown in cross section has surfaces at radii 1mm, 3mm, and 5mm. Equal currents flow through both conductors: $I_{int} = I_{ext} = 0.03\text{A}$ ⊙ (out). Find direction (↑,↓) and magnitude (B_1, B_3, B_5, B_7) of the magnetic field at the four radii indicated (•).



Solution:

$$2\pi(1\text{mm})B_1 = \mu_0(0.03\text{A}) \quad \Rightarrow \quad B_1 = 6\mu\text{T} \quad \uparrow$$
$$2\pi(3\text{mm})B_3 = \mu_0(0.03\text{A}) \quad \Rightarrow \quad B_3 = 2\mu\text{T} \quad \uparrow$$
$$2\pi(5\text{mm})B_5 = \mu_0(0.06\text{A}) \quad \Rightarrow \quad B_5 = 2.4\mu\text{T} \quad \uparrow$$
$$2\pi(7\text{mm})B_7 = \mu_0(0.06\text{A}) \quad \Rightarrow \quad B_7 = 1.71\mu\text{T} \quad \uparrow$$

tsl437

In this lecture we work out further applications of Ampère's law and the law of Biot and Savart. All applications have to do with sources of magnetic field in the form of steady currents. Steady currents produce time-independent magnetic fields.

The slide on this page shows a coaxial cable in cross section. It consists of an inner conductor and a surrounding outer conductor separated by an insulator.

This is tailor-made for applying Ampère's law. We use loops of radius, 1mm, 3mm, 5mm, and 7mm. Only the currents inside the loop matter.

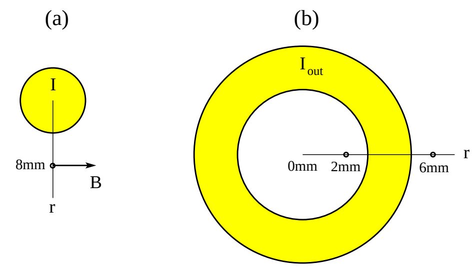
Since both currents are flowing out of the page. It is expedient to integrate counterclockwise. Then both currents count as positive. The solution on the slide shows in detail how to do it.

Since the right-hand side of Ampère's law is positive for each loop, the left-hand side must be positive as well. Since we are integrating counterclockwise and the result must be positive, it follows that the magnetic field is pointing in the counterclockwise tangential direction to the (circular field line). At the four point marked by bullets, the field is pointing up.

Unit Exam III: Problem #2 (Spring '08)

(a) Consider a solid wire of radius $R = 3\text{mm}$.
Find magnitude I and direction (in/out) that produces a magnetic field $B = 7\mu\text{T}$ at radius $r = 8\text{mm}$.

(b) Consider a hollow cable with inner radius $R_{int} = 3\text{mm}$ and outer radius $R_{ext} = 5\text{mm}$.
A current $I_{out} = 0.9\text{A}$ is directed out of the plane.
Find direction (up/down) and magnitude B_2, B_6 of the magnetic field at radius $r_2 = 2\text{mm}$ and $r_6 = 6\text{mm}$, respectively.



Solution:

(a) $7\mu\text{T} = \dfrac{\mu_0 I}{2\pi(8\text{mm})} \quad \Rightarrow \; I = 0.28\text{A}$ (out).

(b) $B_2 = 0, \qquad B_6 = \dfrac{\mu_0(0.9\text{A})}{2\pi(6\text{mm})} = 30\mu\text{T}$ (up).

tsl382

This slide features two more applications of Ampère's law.

In part (a) we know the magnetic field at a certain distance from the center of a long, straight wire seen in cross section and calculate magnitude and direction of the current flowing through the wire. Note that that the radius of the wire does not matter as long as the field point is outside, which it is. The denominator in the expression on the right-hand side is the circumference of the loop.

The field lines curl around the wire counterclockwise. The right-hand rule then tells us that the positive current direction is $\odot$.

In part (b) we have a hollow cable, again shown in cross section. The current is given and we calculate magnitude and direction of the magnetic field at two points. For each point we construct a circular loop which is concentric with the conductor. The inner loop has no current inside: $I_C = 0$. Hence the field is zero. All the current is inside the outer loop. Hence we have $I_C = I_{out}$ for the outer loop. The denominator is again the circumference of the loop.

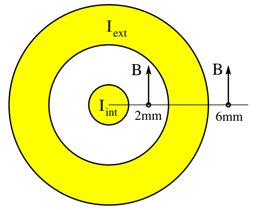
The right-hand rule tells us that the field at that radius is tangential to the loop in directed counterclockwise. This means that that at the point marked it is directed $\uparrow$.

Unit Exam III: Problem #2 (Spring '11)

The coaxial cable shown has surfaces at radii 1mm, 3mm, and 5mm. The magnetic field is the same at radii 2mm and 6mm, namely $B = 7\mu$T in the direction shown.

(a) Find magnitude (in SI units) and direction (in/out) of the current $I_{\rm int}$ flowing through the inner conductor.

(b) Find magnitude (in SI units) and direction (in/out) of the current $I_{\rm ext}$ flowing through the outer conductor.



Solution:

(a) $(7\mu\mathrm{T})(2\pi)(0.002\mathrm{m}) = \mu_0 I_{\rm int} \quad \Rightarrow \; I_{\rm int} = 0.07\mathrm{A}$ (out)

(b) $(7\mu\mathrm{T})(2\pi)(0.006\mathrm{m}) = \mu_0 (I_{\rm int} + I_{\rm ext}) \quad \Rightarrow \; I_{\rm int} + I_{\rm ext} = 0.21\mathrm{A}$ (out)

$\Rightarrow \; I_{\rm ext} = 0.14\mathrm{A}$ (out)

tsl416

Here we have another application of Ampère's law involving a coaxial cable.

Our goal is to calculate the currents in the inner and outer conductors. The only information given are direction an magnitude of the magnetic field at two points.

When we decide to integrate counterclockwise around the loop, then the loop integrals at both radii come out positive. This implies that the positive current direction is out ($\odot$).

It is smart to first deal with an Amperian loop at radius 2mm because it only contains one of the two unknown currents, namely I_{int}. The calculation is worked out on the slide.

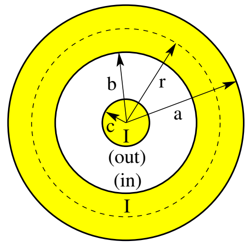
Now we consider the outer loop, which contains both currents. The relevant current now is $I_{int} + I_{ext}$ but only I_{ext} is still unknown. The calculation yields $I_{int} + I_{ext}$, from which we infer I_{ext}.

Ampère's Law: Coaxial Cable

Consider a long coaxial cable, consisting of two cylindrical conductors separated by an insulator as shown in a cross-sectional view.

There is a current I flowing out of the plane in the inner conductor and a current of equal magnitude I flowing into the plane in the outer conductor.

Calculate the magnetic field B as a function of the radial coordinate r.



tsl341

Stray magnetic fields are a potential source of trouble in electronic equipment. Coaxial cables are a design that avoids stray magnetic fields if the currents in the two conductors are equal in magnitude and opposite in direction, which they frequently are in practical applications.

In the slide you see the cross section of a coaxial cable with an inner conductor of radius c and an outer conductor of inner radius b and outer radius a. At a particular instant in time, the current I is directed $\odot$ on the inner conductor and $\otimes$ on the outer conductor.

We assume that the current density is uniform across both conductors. Symmetry dictates that the magnetic field is tangential to circular field lines and constant in magnitude along each such circle to make the loop integrals simple.

What is the magnitude B of the magnetic field as a function of radial distance r from the center of the cable? We employ Ampère's law and declare $\odot$ to be the positive direction for the current, implying that we integrate along circular Amperian loops in counterclockwise direction.

We begin with determining the magnetic field at a radius $r < c$, i.e. at a location inside the inner conductor. For that purpose we must choose an Amperian loop in the shape of a circle with radius $r < c$. Ampère's law then looks as follows:

$$2\pi r B = \mu_0 I \frac{\pi r^2}{\pi c^2} \quad \Rightarrow \quad B = \frac{\mu_0 I}{2\pi} \frac{r}{c^2} \quad : \; r < c.$$

The loop integral on the left-hand side reduces to the product of the circumference and the magnetic field to be determined. The right-hand side is the product of the permeability constant and the current flowing through the loop. That current is a fraction of I equal to the ratio of the cross sectional area inside the loop and the total cross section of the inner conductor.

Next we consider a point at radius $c < r < b$ between the two conductors. Here the current that flows through the loop is I.

$$2\pi r B = \mu_0 I \quad \Rightarrow \; B = \frac{\mu_0 I}{2\pi}\frac{1}{r} \quad : \; c < r < b.$$

For a point inside the outer conductor i.e. at radius $b < r < a$, the entire inner current (counted positively) and a fraction of the outer conductor (counted negatively) flow through the loop (dashed circle):

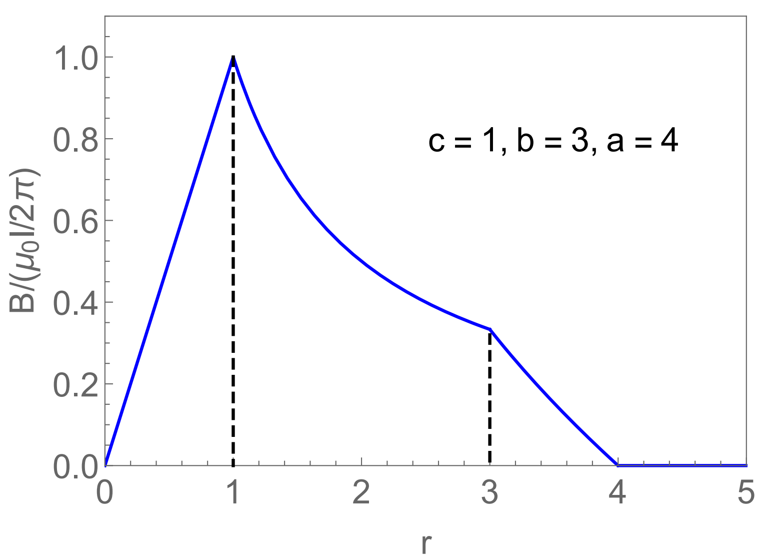
$$2\pi r B = \mu_0 I - \mu_0 I \frac{\pi r^2 - \pi b^2}{\pi a^2 - \pi b^2} \quad \Rightarrow \; B = \frac{\mu_0 I}{2\pi}\frac{1}{r}\frac{a^2 - r^2}{a^2 - b^2} \quad : \; b < r < a.$$

Getting from the first equation to the second equation takes a couple of intermediate steps in this case.

Finally, for a point outside the cable, at radius $r > a$, we have

$$2\pi r B = \mu_0 I - \mu_0 I = 0 \quad \Rightarrow \; B = 0 \quad : \; r > a.$$

The net current through the loop is zero, implying zero magnetic field. In the graph below we show the magnetic field strength B in units of $\mu_0 I/2\pi$ versus the radial distance r. This convention means that the actual magnetic field (in units of Tesla) is the value read off the graph multiplied by $\mu_0 I/2\pi$.



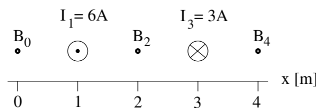
Intermediate Exam III: Problem #2 (Spring '07)

Consider two very long, straight wires with currents $I_1 = 6\text{A}$ at $x = 1\text{m}$ and $I_3 = 3\text{A}$ at $x = 3\text{m}$ in the directions shown. Find magnitude and direction (up/down) of the magnetic field

(a) B_0 at $x = 0$,

(b) B_2 at $x = 2\text{m}$,

(c) B_4 at $x = 4\text{m}$.



Solution:

(a) $B_0 = -\frac{\mu_0(6\text{A})}{2\pi(1\text{m})} + \frac{\mu_0(3\text{A})}{2\pi(3\text{m})} = -1.2\mu\text{T} + 0.2\mu\text{T} = -1.0\mu\text{T}$ (down),

(b) $B_2 = \frac{\mu_0(6\text{A})}{2\pi(1\text{m})} + \frac{\mu_0(3\text{A})}{2\pi(1\text{m})} = 1.2\mu\text{T} + 0.6\mu\text{T} = 1.8\mu\text{T}$ (up),

(c) $B_4 = \frac{\mu_0(6\text{A})}{2\pi(3\text{m})} - \frac{\mu_0(3\text{A})}{2\pi(1\text{m})} = 0.4\mu\text{T} - 0.6\mu\text{T} = -0.2\mu\text{T}$ (down).

tsl366

We continue with an application of calculating the magnetic field in the vicinity of two currents in long, parallel wires with currents in opposite direction. We know that the magnetic field a distance r from the center of long, straight wire is $B = \mu_0 I/2\pi r$, directed tangential to circular field lines directed as determined by the right-hand rule (see lecture 24).

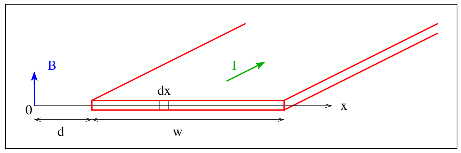
For current I_1, which is directed $\odot$, the field is directed $\downarrow$ to its left and $\uparrow$ to its right. The opposite is the case for current I_2, because it is directed $\otimes$. In the presence of two currents there are two magnetic fields at each point, one from each current. The resultant field to be calculated is the superposition of both field.

In the solution shown on the slide, fields pointing $\uparrow$ are counted positively and field pointing $\downarrow$ negatively.

Consider a very long ribbon of width w carrying a current I in the direction shown.

The current density is assumed to be uniform.

Find the magnetic field B generated a distance d from the ribbon as shown.



Divide the ribbon into thin strips of width dx.
Treat each strip as a wire with current $dI = Idx/w$.
Sum up the field contributions from parallel wires.

$$dB = \frac{\mu_0}{2\pi}\frac{dI}{x} = \frac{\mu_0 I}{2\pi w}\frac{dx}{x}$$

$$B = \frac{\mu_0 I}{2\pi w}\int_d^{d+w}\frac{dx}{x} = \frac{\mu_0 I}{2\pi w}\ln\left(1+\frac{w}{d}\right)$$

tsl519

In this application we wish to calculate the magnetic field generated by a long current-carrying ribbon. The field point is some distance to the side.

The strategy we adopt for this calculation is to divide the ribbon into infinitesimal segments of width dx. Each such segment can then be treated as a long, straight wire carrying the fraction $dI = Idx/w$ of the total current.

The slide shows what the magnetic-field contribution dB of such a current segment a distance x from the field point is. What remains to be done is to sum up the contributions from all current segments that make up the ribbon.

On the previous page we had a similar situation but with just two wires. Here we are dealing with infinitely many wire segments of infinitesimal width. The sum turns into an integral, which is evaluated on the slide.

It is noteworthy that we can recover the result for a single wire by shrinking the width w of the ribbon until it is much smaller than the distance d to the field point. If we expand the logarithmic function in powers of w/d we have

$$\ln\left(1+\frac{w}{d}\right) = \frac{w}{d} + \mathrm{O}\left(\frac{w}{d}\right)^2,$$

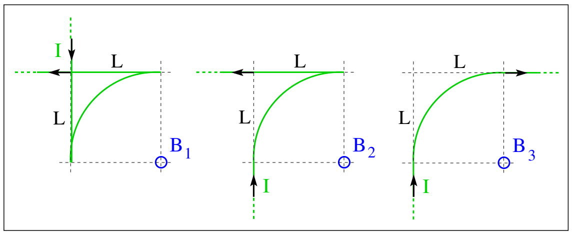
where the last expression is symbolic for terms of higher order, beginning with a quadratic term. If w/d is tiny, e.g. 10^{-2}, then $(w/d)^2$ is much tinier, namely 10^{-4}, and $(w/d)^3$ even tinier, 10^{-6}. When we substitute the leading power, w/d, for the logarithmic function in the result on the slide we obtain the familiar result for a single wire: $B = \mu_0 I/2\pi d$.

Magnetic Field Application (3)

Two semi-infinite straight wires are connected to a segment of circular wire in three different ways. A current I flows in the direction indicated.

(a) Find the direction ($\odot, \otimes$) of the magnetic fields $\vec{B}_1, \vec{B}_1, \vec{B}_3$.

(b) Rank the magnetic fields according to strength.



tsl219

This slide shows three configurations of currents flowing through consecutive segments of wire, two of which are semi-infinite straight lines. Between the two straight segments the wire bends into a quarter circle. At the field points indicated, vertical, horizontal, and circular segments produces a magnetic fields of the following magnitude:

$$B_V = B_H = \frac{\mu_0 I}{4\pi L}, \quad B_C = \frac{\mu_0 I}{8L}.$$

Where do these expressions come from? The first expression, which applies to both vertical and horizontal semi-infinite lines, is simply half the field generated by an infinite line at a distance L from it. The second expression is one fourth of the field generated at the center of a full circle of radius L.

In each configuration the magnetic field at the point marked, thus consists of the sum of a term $\pm B_V$ from the vertical straight segment, a term $\pm B_C$ from the bent segment, and a term $\pm B_H$ from the horizontal straight segment. The right-hand rule determines whether it is + or − in each case.

The solution for the convention that out (in) is positive (negative) reads,

$$B_1 = B_V - B_C + B_H, \quad B_2 = -B_V - B_C + B_H, \quad B_3 = -B_V - B_C - B_H.$$

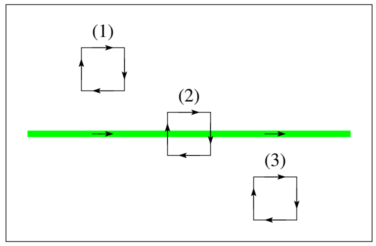
The resultant field is positive in the first case and negative in the other two cases. Hence the direction of B_1 is $\odot$ and the direction of B_2 and B_3 is $\otimes$.

Given that $B_V = B_H$ are weaker than B_C due to the fact that $4\pi > 8$, it follows that the strongest field is B_3 and the weakest field B_1.

Magnetic Field Application (8)

Three squares with equal clockwise currents are placed in the magnetic field of a straight wire with a current flowing to the right.

- Find the direction (↑, ↓, zero) of the magnetic force acting on each square.



tsl225

The task on this slide is somewhat similar to the task on quiz 20. There we had four current-carrying squares positioned partially or fully in a region of uniform magnetic field. Here we have three current-carrying squares in a region of nonuniform magnetic field, namely the field generated by the current from left to right inside the green wire oriented horizontally.

What matters is (i) that the magnetic field generated by the current in the green horizontal conductor is directed $\odot$ above it and $\otimes$ below it and (ii) that the strength of that magnetic field decreases with distance from the green conductor.

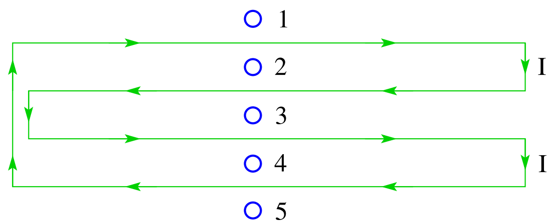
The consequence of (i) is that the magnetic force on each side of the current squares is directed toward the inside above the green line and toward the outside below the green line. Confirm this using the right-hand rule.

The consequence of (ii) is that the forces on the vertical sides of each square are equal in magnitude and opposite in direction ($\leftarrow$ or $\rightarrow$), which makes the net horizontal force on each square is zero. Hence the net force must be vertical ($\uparrow$ or $\downarrow$) and come from the horizontal sides of the squares.

In square (1) the bottom side experiences a force up and the top side a force down. The former is stronger than the latter and the net force is directed $\uparrow$. In square (3) the bottom side experiences a weaker force down and the top side a stronger force up, thus producing a net force directed $\uparrow$ again. In square (2) both horizontal sides experience a force down because here not only the current direction switches but the magnetic-field direction switches too. Hence the net force is directed $\downarrow$.

An electric current I flows through the wire as indicated by arrows.

(a) Find the direction ($\odot$, $\otimes$) of the magnetic field generated by the current at the points $1, \ldots, 5$.
(b) At which points do we observe the strongest and weakest magnetic fields?



tsl220

When analyzing this situation, we assume that what primarily matters are the horizontal portions of the current. The vertical portions are more distant from the field points and can be neglected for the purpose of answering the questions posed.

Next we note that all distances between field points and horizontal currents are odd multiples of the distance R between any field point and the nearest current.

For a simple albeit approximate quantitative analysis we assume that the horizontal currents are very long. This allows us to introduce a reference magnetic field, $B_0 = \mu_0 I/2\pi R$, which is the magnetic field generated at any field point by one of its nearest-neighbor horizontal currents.

Magnetic fields generated by more distant horizontal currents are fractions of B_0 because the distance is a multiple of R.

The four magnetic-field contributions at each field point are directed perpendicular to the page.

If we declare magnetic fields out ($\odot$) to be counted positively and fields in ($\otimes$) negatively, we thus obtain the following results:

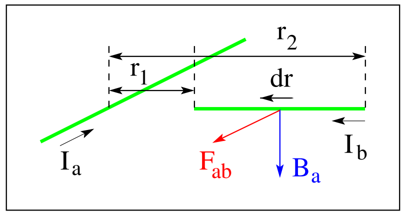
$$B_1 = B_5 = B_0\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}\right) \simeq 0.72B_0 \quad \odot$$

$$B_2 = B_4 = B_0\left(-1 - 1 + \frac{1}{3} - \frac{1}{5}\right) \simeq -1.87B_0 \quad \otimes$$

$$B_3 = B_0\left(1 + 1 - \frac{1}{3} - \frac{1}{3}\right) \simeq 1.33B_0 \quad \odot$$

Force Between Perpendicular Lines of Electric Current

- Electric currents: I_a, I_b
- Magnetic field generated by line a: $B_a = \frac{\mu_0}{2\pi}\frac{I_a}{r}$
- Magnetic force on segment dr of line b: $dF_{ab} = I_b B_a dr$
- Magnetic force on line b: $F_{ab} = \frac{\mu_0}{2\pi} I_a I_b \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0}{2\pi} I_a I_b \ln\frac{r_2}{r_1}$



tsl233

This application is related to what we discussed at the beginning of lecture 24, namely forces between straight current-carrying wires. We already know that two currents that run along parallel wires in the same direction produce an attractive magnetic force between the two wires.

What if the wires are perpendicular to each other? This slide is designed to help us analyze any such case. It calculates direction and magnitude of a straight segment of current I_b in the magnetic field of a long wire with current I_a.

At the position of the segment, the magnetic field $\vec{B}_a$ generated by current I_a is directed down. Therefore, the force $\vec{F}_{ab}$ exerted on the segment is directed parallel to the long wire as shown. We use $d\vec{F}_{ab} = I_b d\vec{r} \times \vec{B}_a$ to determine that direction.

The force acting between endpoints $\vec{r}_1$ and $\vec{r}_2$ is not uniform . The magnitude of total force F_{ab} is worked out on the slide.

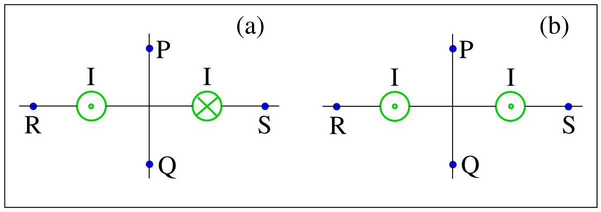
Now let us think of the segment shown as part of the current I_b in a long wire. Then we pick another segment (not shown) positioned symmetrically on the opposite side of current I_a. The force exerted on that segment is the same in magnitude but opposite in direction because B_a has opposite direction at that location.

How would the long wire with current I_b respond when it is oriented perpendicular to the wire carrying I_a? The part to right of I_a will be pulled to the front and the part to the left of I_a will be pulled to the back. This amounts to a torque, which aims to align the two currents flowing in the same direction.

Magnetic Field Application (10)

Consider two currents of equal magnitude in straight wires flowing perpendicular to the plane.

- In configurations (a) and (b), find the direction $(\rightarrow, \leftarrow, \uparrow, \downarrow)$ of the magnetic field generated by the two currents at points P, Q, R, S



tsl227

This is the quiz for lecture 26.

At all four points marked in configuration (a) and all four points marked in configuration (b) each of the two currents in the long, straight wires generate a magnetic field. When you add the two fields thus obtained, the resulting field has one of the four directions indicated on the slide. None of the fields vanish.