PHY204 Lecture 3 [rln3]

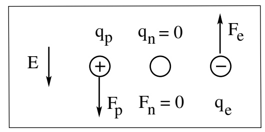
Particle in Uniform Electric or Gravitational Field

particle	charge	mass
electron	$q_e = -e$	$m_e = 9.109 \times 10^{-31}\text{kg}$
proton	$q_p = +e$	$m_p = 1.673 \times 10^{-27}\text{kg}$
neutron	$q_n = 0$	$m_n = 1.675 \times 10^{-27}\text{kg}$

Elementary charge:
$e = 1.602 \times 10^{-19}\text{C}$.

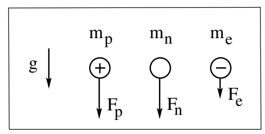
Electric field

- equation of motion: $\vec{F} = m\vec{a}$
- force law: $\vec{F} = q\vec{E}$
- acceleration: $\vec{a} = (q/m)\vec{E}$



Gravitational field

- equation of motion: $\vec{F} = m\vec{a}$
- force law: $\vec{F} = m\vec{g}$
- acceleration: $\vec{a} = \vec{g}$



tsl24

Motion of charged particles in a uniform electric field is the primary theme of this lecture. An electric field $\vec{E}$ is uniform when it has the same magnitude and direction at all points within a certain region of space. We postpone the discussion about how a uniform field can be established to the next lecture.

Consider the electron, the proton, and the neutron, which are the constituent particles of atomic matter (see the table on the slide). All three particles have mass, only two have charge.

Massive particles experience a force $\vec{F} = m\vec{g}$ in a gravitational field $\vec{g}$ and charged particles a force $\vec{F} = q\vec{E}$ in an electric field $\vec{E}$.

Newton's second law, $\vec{F} = m\vec{a}$, predicts that any net force $\vec{F}$ acting on a particle of mass m causes an acceleration $\vec{a}$ in the same direction.

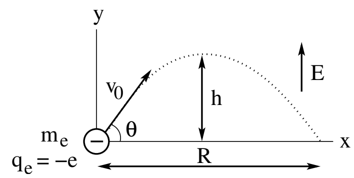
In the case of a gravitational force $F = m\vec{g}$, the resulting acceleration is $\vec{a} = \vec{g}$, independent of the mass and always in the direction of the gravitational field.

In the case of an electric force $F = q\vec{E}$, the resulting acceleration is $\vec{a} = (q/m)\vec{E}$, which depends on both mass and charge of the particle. The proton (with positive charge) experiences a force and an acceleration in the direction of the electric field. For the electron (with negative charge), force and acceleration are both directed opposite to the electric field. The neutron (with zero charge) experiences no electric force and, therefore, no acceleration.

Projectile Motion in Electric Field

- electrostatic force: $F_x = 0 \quad F_y = -eE$
- equation of motion: $\vec{F} = m_e\vec{a}$
- acceleration: $a_x = 0 \qquad a_y = -\frac{e}{m_e}E \equiv -a$
- velocity: $v_x(t) = v_0\cos\theta \qquad v_y(t) = v_0\sin\theta - at$
- position: $x(t) = v_0[\cos\theta]t \qquad y(t) = v_0[\sin\theta]t - \frac{1}{2}at^2$

- height: $h = \frac{v_0^2}{2a}\sin^2\theta$
- range: $R = \frac{v_0^2}{a}\sin(2\theta)$



tsl26

Motion in a uniform electric field is motion subject to a constant force, thus motion with constant acceleration. Such motion is always confined to a plane, which we can take to be the xy-plane. That plane is defined by the direction of the initial velocity $\vec{v}_0$ and the direction of the force $\vec{F}$. We thus write,

$$\vec{v}_0 = v_{0x}\,\hat{\mathbf{i}} + v_{0y}\,\hat{\mathbf{j}}, \quad \vec{F} = F_x\,\hat{\mathbf{i}} + F_y\,\hat{\mathbf{j}}.$$

The components of acceleration $\vec{a}$ as inferred from Newton's second law are

$$a_x = \frac{F_x}{m}, \quad a_y = \frac{F_y}{m}.$$

Velocity $\vec{v}(t)$ and position $\vec{r}(t)$ as functions of time are determined by kinematic relations via integration:

$$v_x(t) = v_{0x} + a_x t, \quad v_y(t) = v_{0y} + a_y t,$$

$$x(t) = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2, \quad y(t) = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2,$$

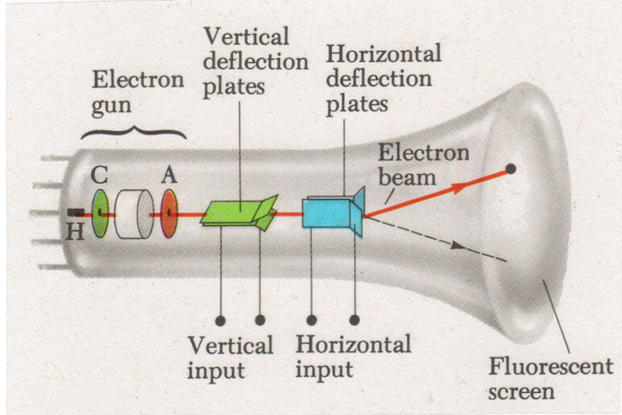
where the components of the initial velocity, v_{0x}, v_{0y}, and the components of the initial position, x_0, y_0, are integration constants.

The first five items on the slide represent a specific application of these equations. The parabolic shape of the path (dotted line) follows if we solve $x(t)$ for time t and substitute the result into the expression of $y(t)$.

The expressions for height h and range R of the projectile path are given on the slide and are derived as follows.

If the projectile reaches maximum height at time t_1, then it returns to the x-axis at time $2t_1$. We use $v_y(t_1) = v_0[\sin\theta] - at_1 = 0$, from which we extract t_1. Then we use $h = y(t_1)$ and $R = x(2t_1)$.

Cathode Ray Tube



tsl419

The slide shows a vacuum tube. The part labeled C, named cathode, is a metal disk and the part labeled A, named anode, is another metal disk with a hole at its center. The cathode is being negatively charge and the anode positively.

Heating up the cathode helps electrons escape from its surface. They are attracted by the anode and are accelerated toward it. A fraction of those electrons shoot right through the hole and form a beam directed toward the right.

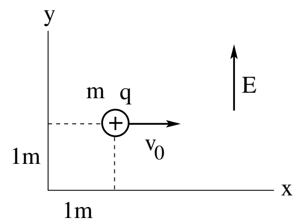
The pairs of horizontal and vertical metallic plates are oppositely charged in a time-controlled way, producing electric forces in vertical and horizontal directions, respectively.

For example, if the top horizontal plate is charged positively and the bottom plate negatively, an electron (with negative charge) passing between them will be attracted to the former and repelled by the latter. It thus experiences an upward force.

With this setup, a skilled technician can guide the electron beam, in the manner of a marionettist, to any point on the fluorescent screen in quick succession. Wherever the beam hits the screen, it leaves a bright spot for a time significantly longer than it takes the electronics to scan the entire screen. We are only a few steps away from a primitive TV image.

Particle Projected Perpendicular to Uniform Electric Field

A charged particle ($m = 3\text{kg}$, $q = 1\mu\text{C}$) is launched at $t_0 = 0$ with initial speed $v_0 = 2\text{m/s}$ in an electric field of magnitude $E = 6 \times 10^6 \text{N/C}$ as shown.



(a) Find the position of the particle at $t_1 = 3\text{s}$.

(b) By what angle does the velocity vector turn between $t_0 = 0$ and $t_1 = 3\text{s}$?

tsl27

We recognize that this exercise is about motion with constant acceleration. The acceleration is constant due to the constant (electric) force on a charged particle, which, in turn, is due to the uniform electric field.

We know that such motion is confined to the plane spanned by the vectors $\vec{v}_0$ and $\vec{E}$. The graph specifies an xy coordinate system in that plane. We can now use the solutions worked out on page 2,

$$v_x(t) = v_{0x} + a_x t, \quad v_y(t) = v_{0y} + a_y t,$$

$$x(t) = x_0 + v_{0x} t + \tfrac{1}{2} a_x t^2, \quad y(t) = y_0 + v_{0y} t + \tfrac{1}{2} a_y t^2,$$

and adapt them to the present situation (for time $t_1 = 3\text{s}$):

Acceleration:

$$a_x = 0, \quad a_y = \frac{F}{m} = \frac{q}{m} E = 2\text{m/s}^2,$$

Velocity:

$$v_{1x} = v_0 = 2\text{m/s}, \quad v_{1y} = a_y t_1 = (2\text{m/s}^2)(3\text{s}) = 6\text{m/s}.$$

Position:

$$x_1 = x_0 + v_0 t_1 = 1\text{m} + (2\text{m/s})(3\text{s}) = 7\text{m}.$$

$$y_1 = y_0 + \tfrac{1}{2} a_y t_1^2 = 1\text{m} + \tfrac{1}{2}(2\text{m/s}^2)(3\text{s})^2 = 10\text{m}.$$

Initial angle: $\theta_0 = 0$ (relative to x-axis).

Final angle:

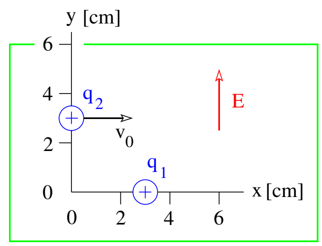
$$\tan\theta_1 = \frac{v_{1y}}{v_{1x}} = \frac{6\text{m/s}}{2\text{m/s}} = 3 \quad \Rightarrow \quad \theta_1 = 72^\circ.$$

Particles Accelerated by Uniform Electric Field

A uniform electric field $E = 0.75 \times 10^3$N/C exists in the box.

(a) A charged particle of mass $m_1 = 1.9 \times 10^{-9}$kg is released from rest at $x = 3$cm, $y = 0$. It exits the box at $x = 3$cm, $y = 6$cm after a time $t_1 = 5.7 \times 10^{-5}$s. Find the charge q_1.

(b) A second charged particle of mass $m_2 = 2.7 \times 10^{-14}$kg is projected from position $x = 0$, $y = 3$cm with initial speed $v_0 = 3.2 \times 10^4$m/s. It exits the box at $x = 3.9$cm, $y = 6$cm. Find the charge q_2.



tsl28

Here we have two variations on the theme of the previous page.

(a): Particle 1 moves parallel to the y-axis, accelerated by the electric field. The chain of reasoning that brings us from what is given to what is being asked is as follows:

$$t_1 = 5.7\times10^{-5}\text{s}, \quad m_1 = 1.9\times10^{-9}\text{kg}, \quad \Delta y_1 = 0.06\text{m}, \quad E = 0.75\times10^3\text{N/C}.$$

$$\Delta y_1 = \frac{1}{2}a_1 t_1^2 \quad \Rightarrow \; a_1 = \frac{2\Delta y_1}{t_1^2} = 3.69 \times 10^7\text{m/s}^2.$$

$$\Rightarrow \; F_1 = m_1 a_1 = 7.02 \times 10^{-2}\text{N} = q_1 E \quad \Rightarrow \; q_1 = \frac{F_1}{E} = 93.6\mu\text{C}.$$

(b) Particle 2 is launched horizontally to the right and accelerated upward. Here our reasoning a few more steps.

$$v_0 = 3.2 \times 10^4\text{m/s}, \quad m_2 = 2.7 \times 10^{-14}\text{kg}, \quad \Delta x_2 = 0.039\text{m}, \quad \Delta y_2 = 0.03\text{m}.$$

$$\Delta x_2 = v_0 t_2 \quad \Rightarrow \; t_2 = \frac{\Delta x_2}{v_0} = 1.22 \times 10^{-6}\text{s}.$$

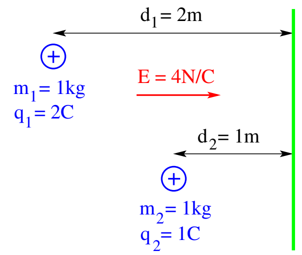
$$\Delta y_2 = \frac{1}{2}a_2 t_2^2 \quad \Rightarrow \; a_2 = \frac{2\Delta y_2}{t_2^2} = 4.04 \times 10^{10}\text{m/s}^2.$$

$$F_2 = m_2 a_2 = 1.09 \times 10^{-3}\text{N} = q_2 E \quad \Rightarrow \; q_2 = \frac{F_2}{E} = 1.45\mu\text{C}.$$

Is the Faster also the Quicker?

Charged particles 1 and 2 are released from rest in a uniform electric field.

(a) Which particle moves faster when it hits the wall?
(b) Which particle reaches the wall more quickly?



tsl372

To solve this problem we recall one relation from dynamics and two relations from kinematics:

$$a = \frac{q}{m}E, \quad v = at, \quad d = \frac{1}{2}at^2.$$

The cause is the electric force qE and the effect an acceleration $a = F/m$; the acceleration is constant, the velocity is the integral of acceleration, and the distance the integral of velocity.

For part (a) we eliminate t from the last two relations:

$$v = \sqrt{2ad}.$$

Part (a):

$$a_1 = \frac{2\text{C}}{1\text{kg}} 4\text{N/C} = 8\text{m/s}^2, \quad a_2 = \frac{1\text{C}}{1\text{kg}} 4\text{N/C} = 4\text{m/s}^2,$$

$$v_1 = \sqrt{2(8\text{m/s}^2)(2\text{m})} = 5.66\text{m/s}, \quad v_1 = \sqrt{2(4\text{m/s}^2)(1\text{m})} = 2.83\text{m/s}.$$

Part (b):

$$t_1 = \frac{v_2}{a_2} = 0.707\text{s}, \quad t_2 = \frac{v_1}{a_1} = 0.707\text{s}.$$

In answer to the title question: The faster is not quicker in this instance.

Intermediate Exam I: Problem #3 (Spring '06)

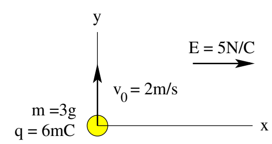
Consider a region of uniform electric field as shown. A charged particle is projected at time $t = 0$ with initial velocity as shown. Ignore gravity.

(a) Find the components a_x and a_y of the acceleration at time $t = 0$.

(b) Find the components v_x and v_y of the velocity at time $t = 0$.

(c) Find the components v_x and v_y of the velocity at time $t = 1.2$s.

(d) Find the components x and y of the position at time $t = 1.2$s.



tsl350

This exercise was previously used as an exam problem. Evidently, it is about motion with constant acceleration driven by an electric force. Solving the exercise requires that we recall the kinematic relations from page 2 and adapt them to the present situation.

The individual steps are pretty straightforward:

(a) $a_x = \frac{q}{m}E = \frac{6 \times 10^{-3}\text{C}}{3 \times 10^{-3}\text{kg}}(5\text{N/C}) = 10\text{m/s}^2, \quad a_y = 0.$

(b) $v_x = 0, \quad v_y = v_0 = 2\text{m/s}.$

(c) $v_x = a_x t = (10\text{m/s}^2)(1.2\text{s}) = 12\text{m/s}, \quad v_y = v_0 = 2\text{m/s}.$

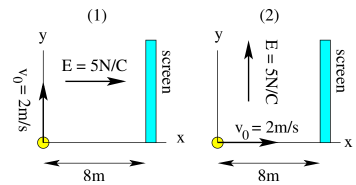
(d) $x = \frac{1}{2}a_x t^2 = 0.5(10\text{m/s}^2)(1.2\text{s})^2 = 7.2\text{m},$

$y = v_y t = (2\text{m/s})(1.2\text{s}) = 2.4\text{m}.$

Unit Exam I: Problem #3 (Spring '07)

Consider two regions of uniform electric field as shown. Charged particles of mass $m = 2$kg and charge $q = 1$C are projected at time $t = 0$ with initial velocities as shown. Both particles will hit the screen eventually. Ignore gravity.

(a) At what time t_1 does the particle in region (1) hit the screen?
(b) At what height y_1 does the particle in region (1) hit the screen?
(c) At what time t_2 does the particle in region (2) hit the screen?
(d) At what height y_2 does the particle in region (2) hit the screen?



tsl361

Here is another problem from a previous exam. We can solve it in like manner.

(a) $x_1 = \frac{1}{2}at_1^2 \quad \text{with} \quad a = \frac{q}{m}E = 2.5\text{m/s}^2,$

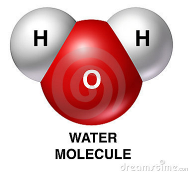
$x_1 = 8\text{m} \quad \Rightarrow \; t_1 = 2.53\text{s}.$

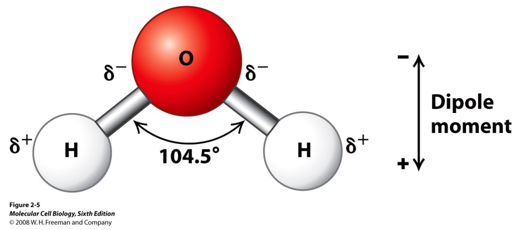
(b) $y_1 = v_0 t_1 = 5.06\text{m}.$

(c) $x_2 = v_0 t_2 \quad \Rightarrow \; t_2 = \frac{8\text{m}}{2\text{m/s}} = 4\text{s}.$

(d) $y_2 = \frac{1}{2}at_2^2 = 20\text{m}.$

Water Molecule





tsl510

Here and on the next two pages we briefly introduce a different topic.

We know that atoms are electrically neutral unless they are ionized. Some elements give up an electron easily, other elements accommodate an extra electron readily, yet other elements resist both operations.

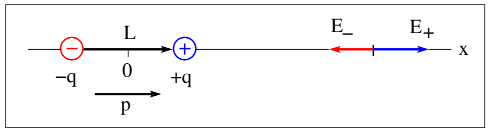
In molecules, which have multiple nuclei with shared electrons, the electronic charge distribution is considerably more complex than in atoms. Quite often, the center of negative charge is somewhat displaced from the center of positive charge. A close-up observation perceives such a molecule as a pair of opposite charges slightly displaced: an electric dipole.

The H_2O molecule is a prominent example. When considering its shape, think of a Mickey Mouse head. The region of the two smaller hydrogen atoms contains an excess of positive charge, whereas the region of the oxygen atom has more negative charge.

This has the consequence that the hydrogen atoms of one molecule are attracted to the oxygen atom of neighboring atoms and vice versa. Water in frozen form is stabilized by so-called hydrogen bonds. In ice, each ear of one mouse head touches the neck of a neighboring mouse head. Each neck touches two ears but not of the same mouse head.

In liquid water, ears and necks form a loose, dynamic network of hydrogen bonds that are being formed and broken continually.

Electric Dipole Field



$$E = \frac{kq}{(x-L/2)^2} - \frac{kq}{(x+L/2)^2} = kq\left[\frac{(x+L/2)^2-(x-L/2)^2}{(x-L/2)^2(x+L/2)^2}\right] = \frac{2kqLx}{(x^2-L^2/4)^2}$$
$$\simeq \frac{2kqL}{x^3} = \frac{2kp}{x^3} \qquad (\text{for } x \gg L)$$

Electric dipole moment: $\vec{p} = q\vec{L}$

- Note the more rapid decay of the electric field with distance from an electric dipole ($\sim r^{-3}$) than from an electric point charge ($\sim r^{-2}$).
- The dipolar field is not radial.

tsl23

Any electric dipole can be modeled, quite generally, as two point charges $+q$ and $-q$, separated a distance L as shown.

The electric dipole moment is quantitatively specified as the vector, $\vec{p} = q\vec{L}$, pointing from the negative toward the positive charge.

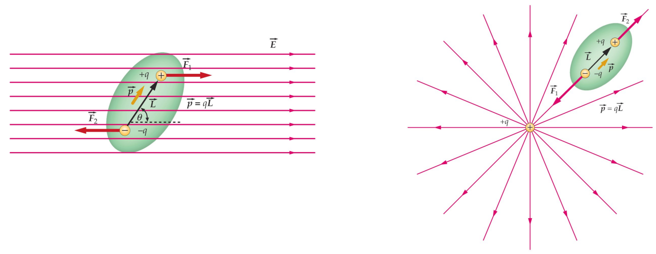
If the dipole is positioned on the x-axis and aligned as shown, the electric field to its right is pointing away from it, because the positive charge is somewhat close than the negative charge. On the left of the dipole, the field point toward it (not shown), because the negative charge is somewhat closer than the positive charge.

Coulomb's law tells us that the (radial) electric field of a single point charge decays in strength like $\sim r^{-2}$ with distance. The two lines of equations on the slide tell us that in the case of a dipole the strength of the electric field decays faster, namely like $\sim r^{-3}$ with distance.

This decay law is only shown here for points on the x-axis, but it is generally true. The direction is not radial, in general. We will take a closer look at the field lines of an electric dipole later.

Force and Torque on Electric Dipole

- The net force on an electric dipole in a *uniform* electric field vanishes.
- However, this dipole experiences a torque $\vec{\tau} = \vec{p} \times \vec{L}$ that tends to align the vector $\vec{p}$ with the vector $\vec{E}$.
- Now consider an electric dipole that is already aligned (locally) with a *nonuniform* electric field. This dipole experiences a net force that is always in the direction where the field has the steepest increase.



tsl328

What happens if an electric dipole, e.g. a water molecule is placed into a uniform electric field? The charges $+q$ and $-q$ are pulled in opposite directions with equal strength of force. The net force is zero, implying that no acceleration ensues.

However, the dipole experiences a torque that aims to align the vector $\vec{p}$ with the vector $\vec{E}$ as illustrated in the graph on the left. The phenomenon of aligning electric dipoles is named electric polarization.

In molecules that have no intrinsic electric dipole moment, a dipole moment is being induced when the molecule is place into an electric field. The nuclei are being pulled one way and the electrons in the opposite direction, producing a slight effective displacement.

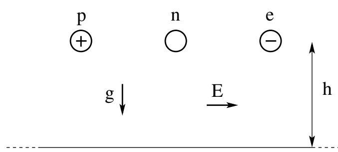
If an electric dipole is placed into an nonuniform electric field, such as illustrated in the graph on the right, it does experience a net force. The forces on the charges $+q$ and $-q$ are no longer equal in magnitude. Once the dipole is properly aligned by the acting torque, the dipole is always attracted toward the region of strong electric field.

The overall conclusion is that electric dipoles produce cohesive forces, i.e. forces of an attractive nature between molecules.

We shall return to electric dipoles later and also introduce magnetic dipoles.

Particle in Uniform Electric and Gravitational Field (1)

A proton, a neutron, and an electron are dropped from rest in a vertical gravitational field $\vec{g}$ and in a horizontal electric field $\vec{E}$ as shown. Both fields are uniform.



(a) Which particle travels the shortest distance?
(b) Which particle travels the longest distance?
(c) Which particle travels the shortest time?
(d) Which particle reaches the highest speed?

tsl25

This is the quiz for lecture 3.

We return to motion with constant acceleration. Two of the particles experience a constant electric force directed horizontally. All three particles experience a constant gravitational force directed vertically down.

All three particles thus undergo constant acceleration in different directions and with different magnitudes. All three hit the floor eventually.