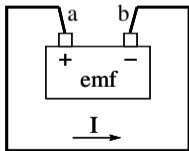




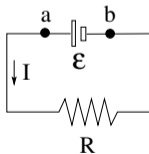
Consider a wire with resistance $R = \rho \ell / A$ connected to a battery.

- **Resistor rule:** In the direction of I across a resistor with resistance R , the electric potential drops: $\Delta V = -IR$.
- **EMF rule:** From the $(-)$ terminal to the $(+)$ terminal in an ideal source of emf, the potential rises: $\Delta V = \mathcal{E}$.
- **Loop rule:** The algebraic sum of the changes in potential encountered in a complete traversal of any loop in a circuit must be zero: $\sum \Delta V_i = 0$.

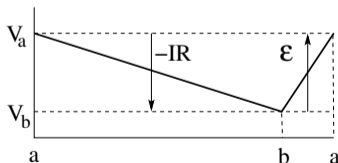
physical system



circuit diagram



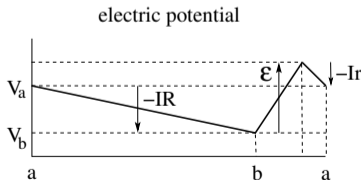
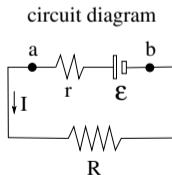
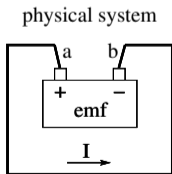
electric potential



Battery with Internal Resistance



- Real batteries have an internal resistance r .
- The terminal voltage $V_{ba} \equiv V_a - V_b$ is smaller than the emf \mathcal{E} written on the label if a current flows through the battery.
- Usage of the battery increases its internal resistance.
- Current from loop rule: $\mathcal{E} - Ir - IR = 0 \Rightarrow I = \frac{\mathcal{E}}{R + r}$
- Current from terminal voltage: $V_{ba} = \mathcal{E} - Ir = IR \Rightarrow I = \frac{V_{ba}}{R}$

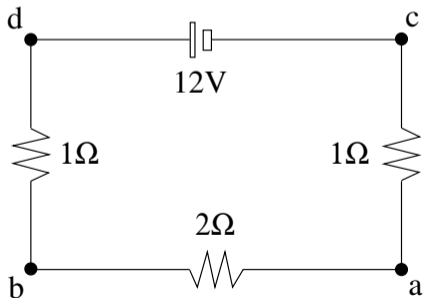


Resistor Circuit (4)



Consider the resistor circuit shown.

- (a) Find the direction of the positive current (cw/ccw).
- (b) Find the magnitude of the current.
- (c) Find the voltage $V_{ab} = V_b - V_a$.
- (d) Find the voltage $V_{cd} = V_d - V_c$.

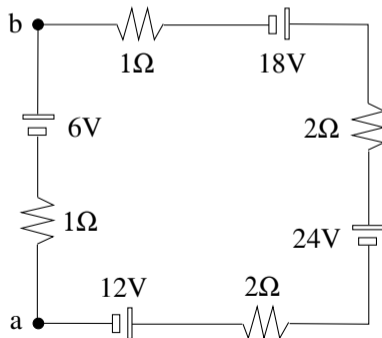


Resistor Circuit (6)



Consider the resistor circuit shown.

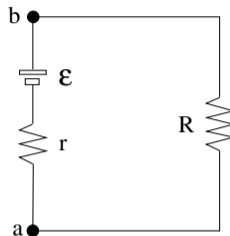
- (a) Choose a current direction and use the loop rule to determine the current.
- (b) Name the direction of positive current (cw/ccw).
- (c) Find $V_{ab} \equiv V_b - V_a$ along two different paths.





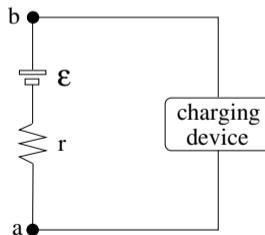
Battery in use

- Terminal voltage: $V_{ab} = \mathcal{E} - Ir = IR$
- Power output of battery: $P = V_{ab}I = \mathcal{E}I - I^2r$
 - Power generated in battery: $\mathcal{E}I$
 - Power dissipated in battery: I^2r
- Power transferred to load: $P = I^2R$



Battery being charged:

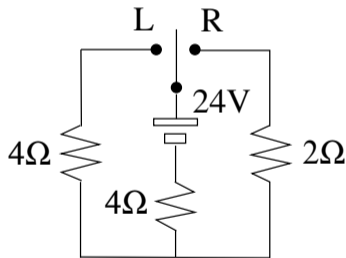
- Terminal voltage: $V_{ab} = \mathcal{E} + Ir$
- Power supplied by charging device: $P = V_{ab}I$
- Power input into battery: $P = \mathcal{E}I + I^2r$
 - Power stored in battery: $\mathcal{E}I$
 - Power dissipated in battery: I^2r



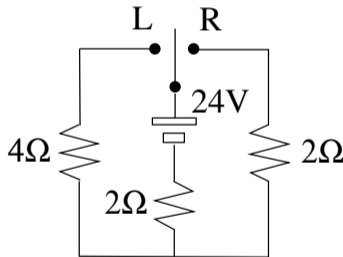


Consider two 24V batteries with internal resistances (a) $r = 4\Omega$, (b) $r = 2\Omega$.

- Which setting of the switch (L/R) produces the larger power dissipation in the resistor on the side?



(a)



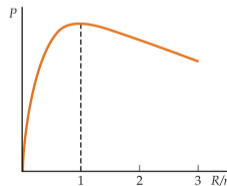
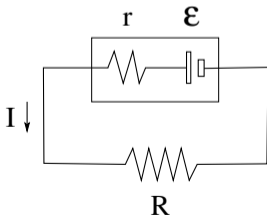
(b)



A battery providing an emf \mathcal{E} with internal resistance r is connected to an external resistor of resistance R as shown.

For what value of R does the battery deliver the maximum power to the external resistor?

- Electric current: $\mathcal{E} - Ir - IR = 0 \Rightarrow I = \frac{\mathcal{E}}{R + r}$
- Power delivered to external resistor: $P = I^2 R = \frac{\mathcal{E}^2 R}{(R + r)^2} = \frac{\mathcal{E}^2}{r} \frac{R/r}{(R/r + 1)^2}$
- Condition for maximum power: $\frac{dP}{dR} = 0 \Rightarrow R = r$

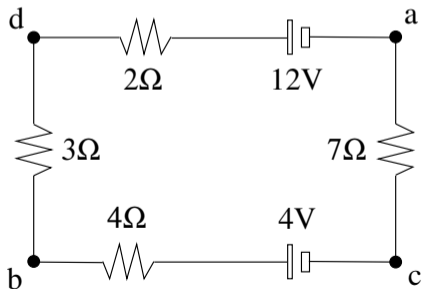


Resistor Circuit (5)



Consider the resistor circuit shown.

- (a) Choose a current direction and use the loop rule to determine the current.
- (b) Name the direction of positive current (cw/ccw).
- (c) Find the potential difference $V_{ab} = V_b - V_a$.
- (d) Find the voltage $V_{cd} = V_d - V_c$.



Symbols Used in Circuit Diagrams



resistor



ammeter (connect in series)



capacitor



voltmeter (connect in parallel)



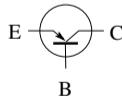
inductor



diode



emf source

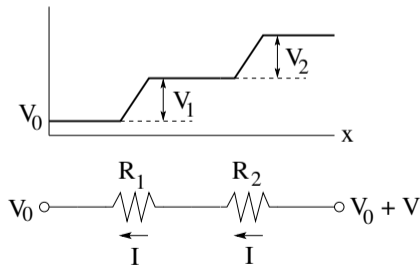


transistor



Find the equivalent resistance of two resistors connected in series.

- Current through resistors: $I_1 = I_2 = I$
- Voltage across resistors: $V_1 + V_2 = V$
- Equivalent resistance: $R \equiv \frac{V}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2}$
- $\Rightarrow R = R_1 + R_2$

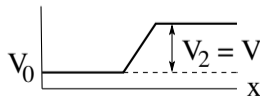
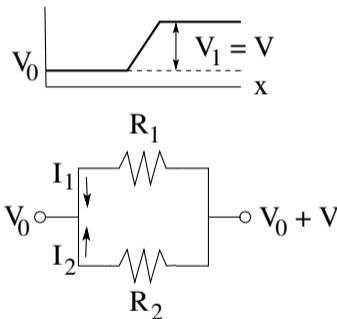


Resistors Connected in Parallel



Find the equivalent resistance of two resistors connected in parallel.

- Current through resistors: $I_1 + I_2 = I$
- Voltage across resistors: $V_1 = V_2 = V$
- Equivalent resistance: $\frac{1}{R} \equiv \frac{I}{V} = \frac{I_1}{V_1} + \frac{I_2}{V_2}$
- $\Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

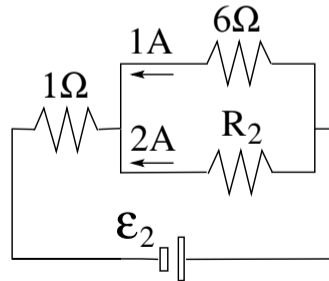
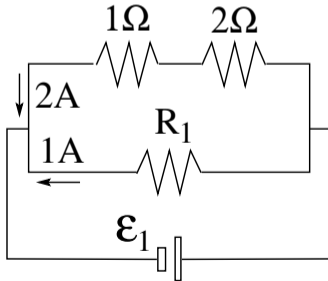


Resistor Circuit (1)



Consider the two resistor circuits shown.

- (a) Find the resistance R_1 .
- (b) Find the emf \mathcal{E}_1 .
- (c) Find the resistance R_2 .
- (d) Find the emf \mathcal{E}_2 .

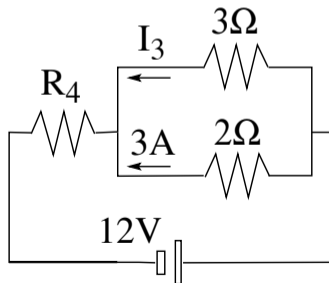
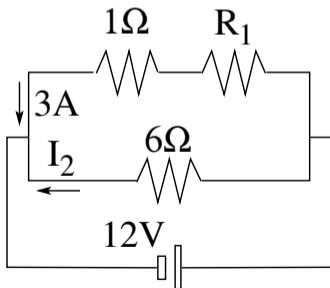


Resistor Circuit (2)



Consider the two resistor circuits shown.

- (a) Find the resistance R_1 .
- (b) Find the current I_2 .
- (c) Find the current I_3 .
- (d) Find the resistance R_4 .

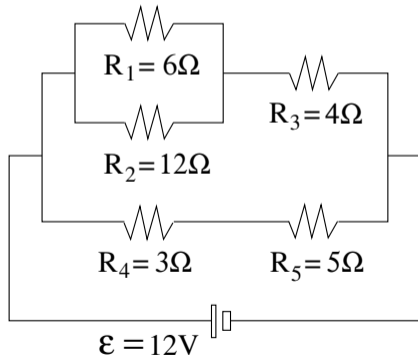


Resistor Circuit (8)



Consider the circuit of resistors shown.

- Find the equivalent resistance R_{eq} .
- Find the currents I_1, \dots, I_5 through each resistor and the voltages V_1, \dots, V_5 across each resistor.
- Find the total power P dissipated in the circuit.





Loop Rule

- When any closed-circuit loop is traversed, the algebraic sum of the changes in electric potential must be zero.

Junction Rule

- At any junction in a circuit, the sum of the incoming currents must equal the sum of the outgoing currents.

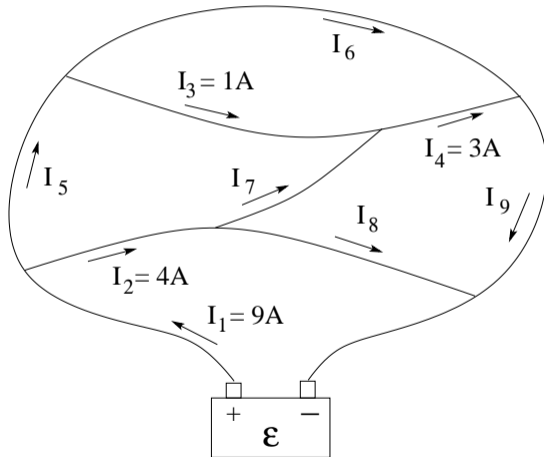
Strategy

- Use the junction rule to name all independent currents.
- Use the loop rule to determine the independent currents.

Applying the Junction Rule



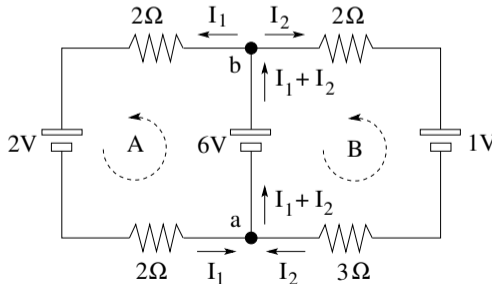
In the circuit of steady currents, use the junction rule to find the unknown currents I_5, \dots, I_9 .





Consider the circuit shown below.

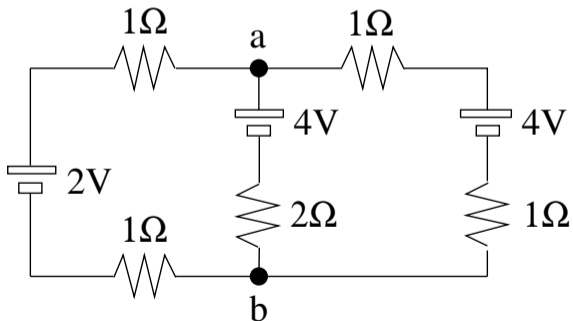
- Junction a : I_1, I_2 (in); $I_1 + I_2$ (out)
- Junction b : $I_1 + I_2$ (in); I_1, I_2 (out)
- Two independent currents require the use of two loops.
- Loop A (ccw): $6V - (2\Omega)I_1 - 2V - (2\Omega)I_1 = 0$
- Loop B (ccw): $(3\Omega)I_2 + 1V + (2\Omega)I_2 - 6V = 0$
- Solution: $I_1 = 1A, \quad I_2 = 1A$





Consider the electric circuit shown.

- Identify all independent currents via junction rule.
- Determine the independent currents via loop rule.
- Find the Potential difference $V_{ab} = V_b - V_a$.

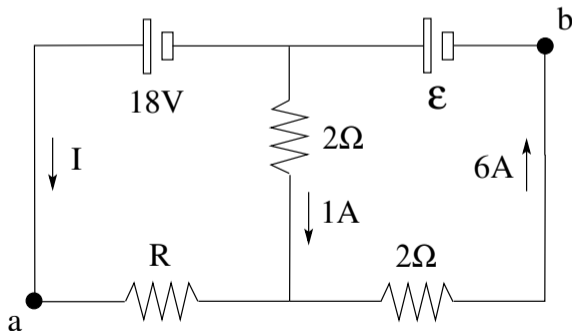


Resistor Circuit (9)



Use Kirchhoff's rules to find

- (a) the current I ,
- (b) the resistance R ,
- (c) the emf \mathcal{E} ,
- (d) the voltage $V_{ab} \equiv V_b - V_a$.

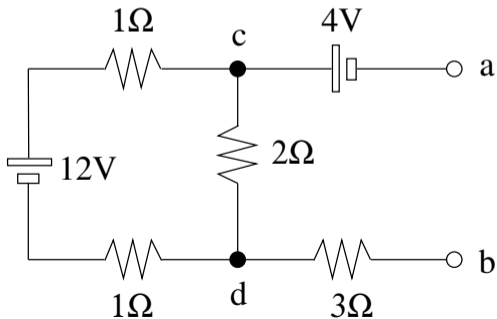


Resistor Circuit (10)



Consider the electric circuit shown.

- (a) Find the current through the 12V battery.
- (b) Find the current through the 2Ω resistor.
- (c) Find the total power dissipated.
- (d) Find the voltage $V_{cd} \equiv V_d - V_c$.
- (e) Find the voltage $V_{ab} \equiv V_b - V_a$.

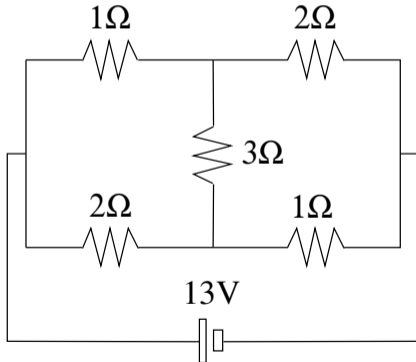


Resistor Circuit (12)



Consider the electric circuit shown.

- Find the equivalent resistance R_{eq} of the circuit.
- Find the total power P dissipated in the circuit.



Resistor Circuit (3)



Consider the resistor and capacitor circuits shown.

- (a) Find the equivalent resistance R_{eq} .
- (b) Find the power P_2, P_3, P_4 dissipated in each resistor.
- (c) Find the equivalent capacitance C_{eq} .
- (d) Find the energy U_2, U_3, U_4 stored in each capacitor.

