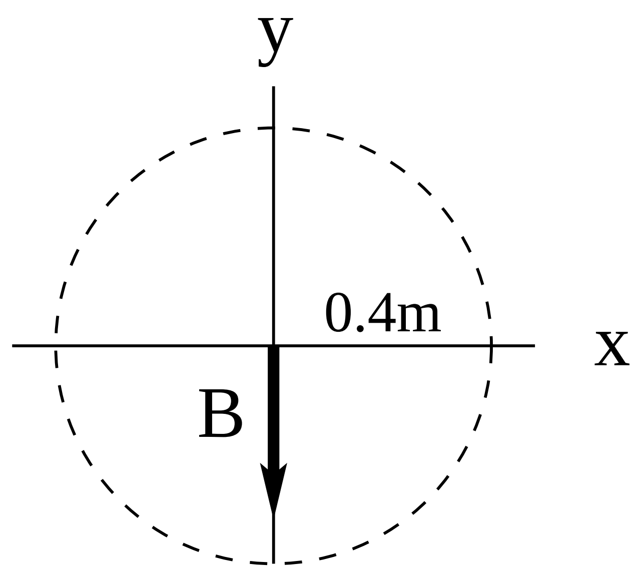
Intermediate Exam III: Problem #1 (Spring '05)

An infinitely long straight current of magnitude $I = 6$A is directed into the plane ($\otimes$) and located a distance $d = 0.4$m from the coordinate origin (somewhere on the dashed circle). The magnetic field $\vec{B}$ generated by this current is in the negative y-direction as shown.

(a) Find the magnitude B of the magnetic field.

(b) Mark the location of the position of the current $\otimes$ on the dashed circle.



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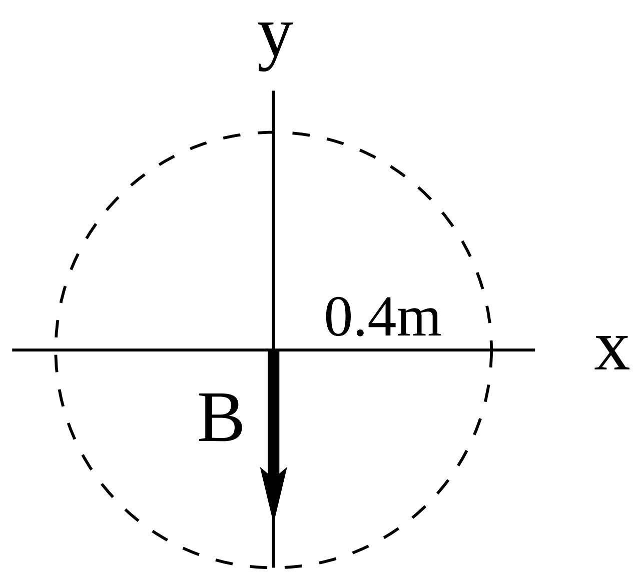


Solution:

(a) $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{d} = 3\mu\mathrm{T}.$

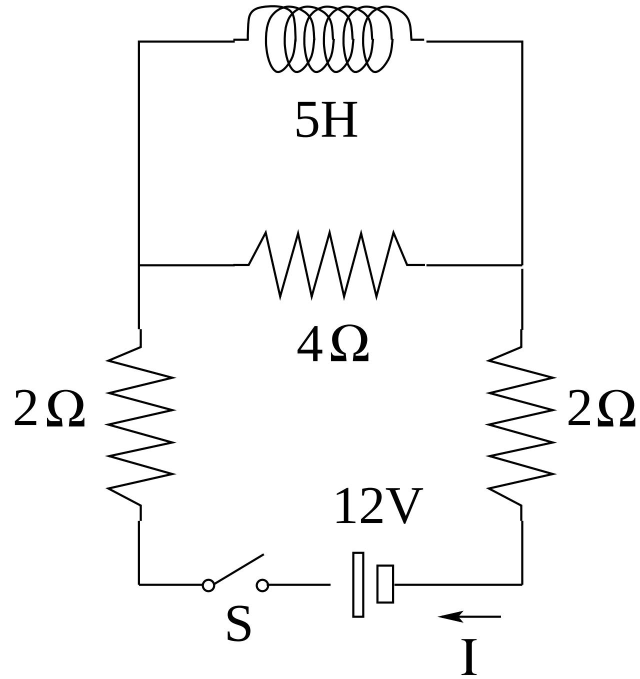
(b) Position of current $\otimes$ is at $y = 0,\ x = -0.4$m.

Intermediate Exam III: Problem #2 (Spring ’05)

In the circuit shown we close the switch S at time $t = 0$. Find the current I through the battery and the voltage V_L across the inductor

(a) immediately after the switch has been closed,

(b) a very long time later.



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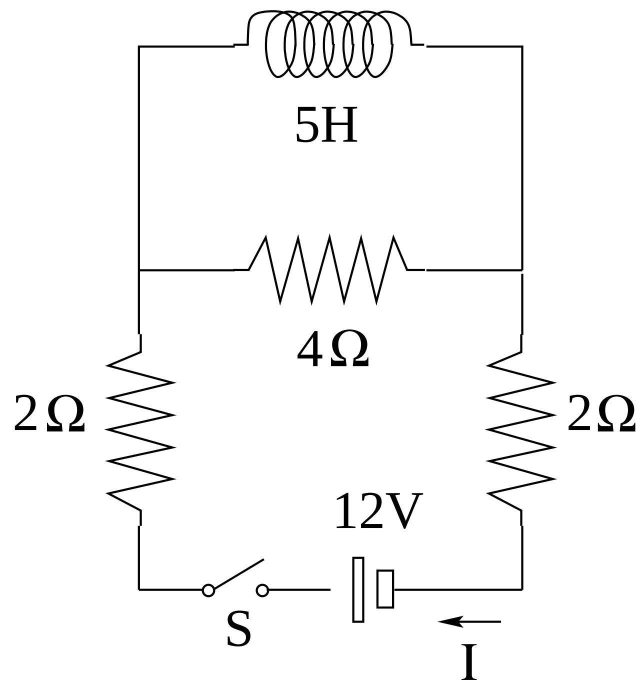


Solution:

(a) $I = \dfrac{12\text{V}}{2\Omega + 4\Omega + 2\Omega} = 1.5\text{A}, \qquad V_L = (4\Omega)(1.5\text{A}) = 6\text{V}.$

(b) $I = \dfrac{12\text{V}}{2\Omega + 2\Omega} = 3\text{A}, \qquad V_L = 0.$

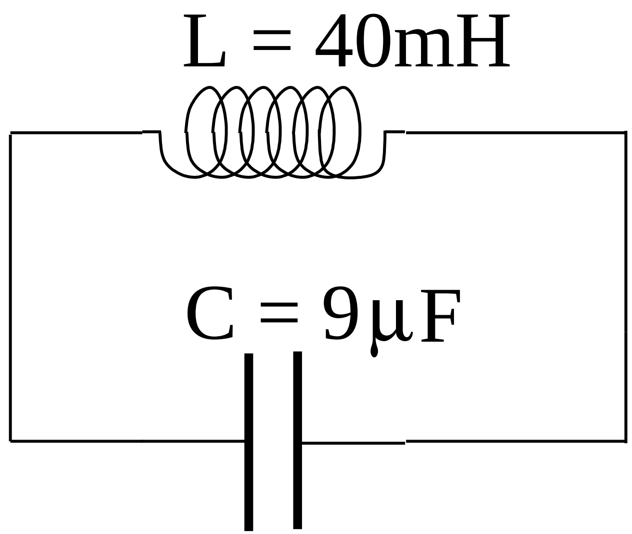
Intermediate Exam III: Problem #3 (Spring '05)

At time $t = 0$ the capacitor is charged to $Q_{max} = 3\mu$C and the current is instantaneously zero.

(a) How much energy is stored in the capacitor at time $t = 0$?

(b) At what time t_1 does the current reach its maximum value?

(c) How much energy is stored in the inductor at time t_1?



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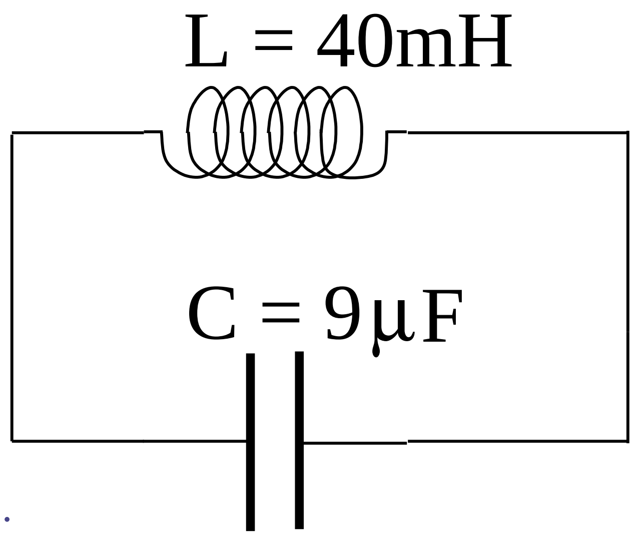


Solution:

(a) $U_C = \frac{Q_{max}^2}{2C} = 0.5\mu\text{J}.$

(b) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC} = 3.77\text{ms}, \qquad t_1 = \frac{T}{4} = 0.942\text{ms}.$

(c) $U_L = U_C = 0.5\mu\text{J}$ (energy conservation.)

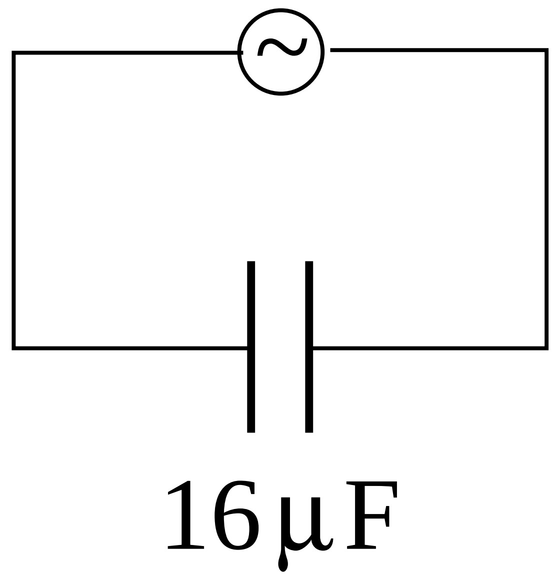
Intermediate Exam III: Problem #4 (Spring ’05)

Consider the circuit shown. The ac voltage supplied is $\mathcal{E} = \mathcal{E}_{max}\cos(\omega t)$ with $\mathcal{E}_{max} = 170$V and $\omega = 377$rad/s.

(a) What is the maximum value I_{max} of the current?

(b) What is the emf $\mathcal{E}(t)$ at $t = 0.01$s?

(c) What is the current $I(t)$ at $t = 0.01$s?



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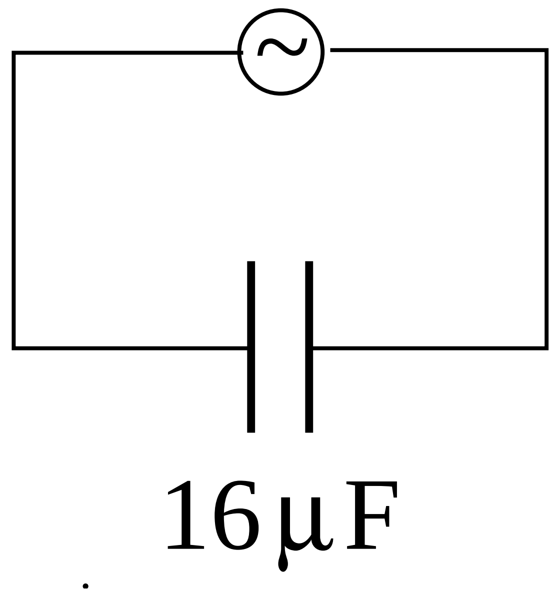


Solution:

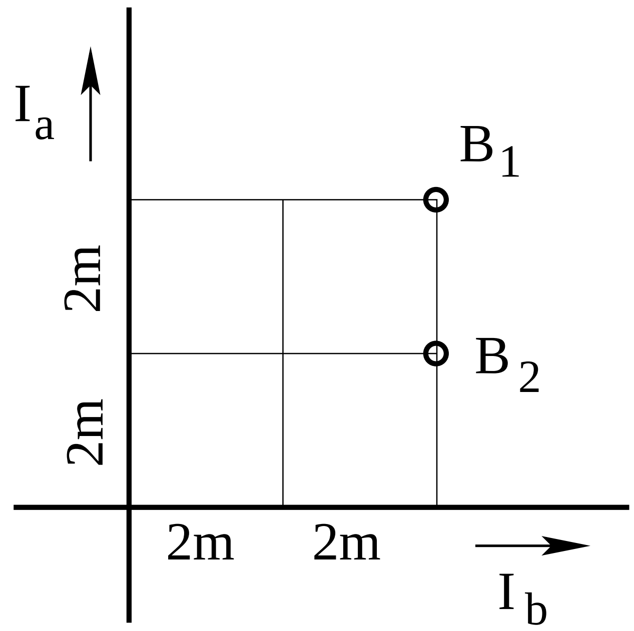
(a) $I_{max} = \dfrac{\mathcal{E}_{max}}{X_C} = \mathcal{E}_{max}\omega C = 1.03\text{A}.$

(b) $\mathcal{E} = (170\text{V})\cos(3.77\text{rad}) = (170\text{V})(-0.809) = -138\text{V}.$

(c) $I = \mathcal{E}_{max}\omega C\cos(3.77\text{rad} + \pi/2) = (1.03\text{A})(0.588) = 0.605\text{A}.$

Intermediate Exam III: Problem #1 (Spring ’06)

Consider two infinitely long, straight wires with currents of equal magnitude $I_1 = I_2 = 5$A in the directions shown.
Find the direction (in/out) and the magnitude of the magnetic fields $\mathbf{B}_1$ and $\mathbf{B}_2$ at the points marked in the graph.



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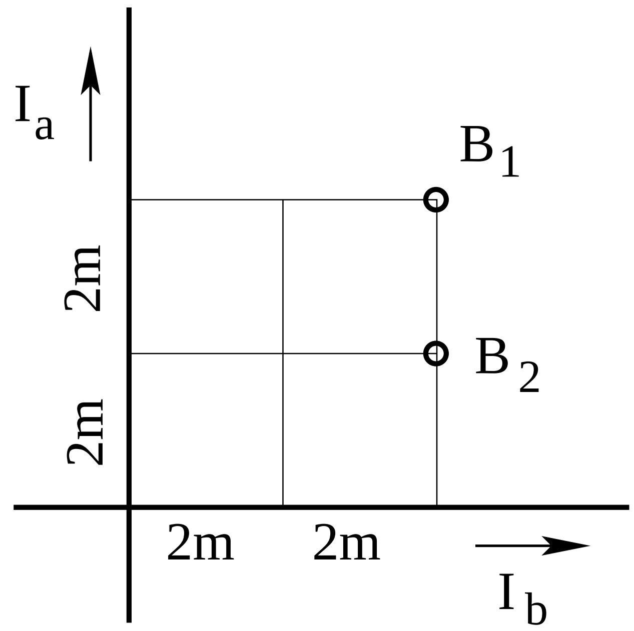
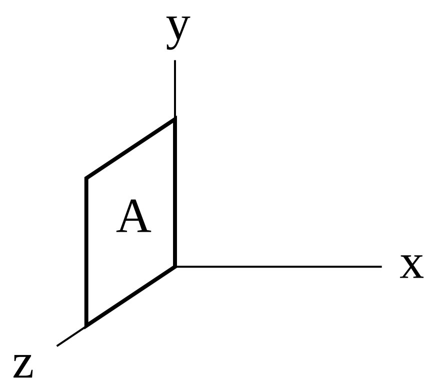


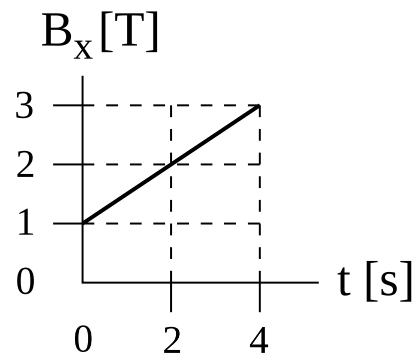
Solution:

- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{5\text{A}}{4\text{m}} - \frac{5\text{A}}{4\text{m}}\right) = 0$ (no direction).
- $B_2 = \frac{\mu_0}{2\pi}\left(\frac{5\text{A}}{2\text{m}} - \frac{5\text{A}}{4\text{m}}\right) = 0.25\mu\text{T}$ (out of plane).

Intermediate Exam III: Problem #2 (Spring '06)

A conducting loop in the shape of a square with area $A = 4\mathrm{m}^2$ and resistance $R = 5\Omega$ is placed in the yz-plane as shown. A time-dependent magnetic field $\mathbf{B} = B_x\hat{\mathbf{i}}$ is present. The dependence of B_x on time is shown graphically.
(a) Find the magnetic flux Φ_B through the loop at time $t = 0$.
(b) Find magnitude and direction (cw/ccw) of the induced current I at time $t = 2$s.





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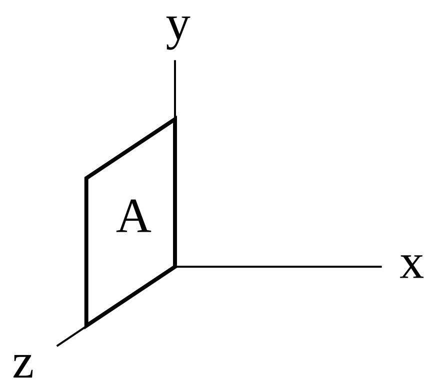
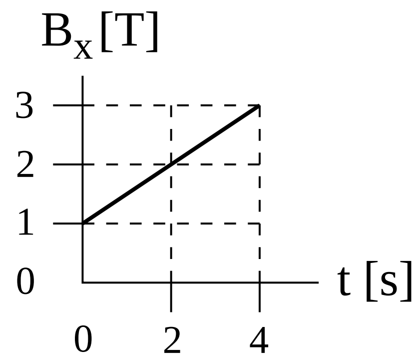




Choice of area vector: $\odot/\otimes$ $\Rightarrow$ positive direction = ccw/cw.

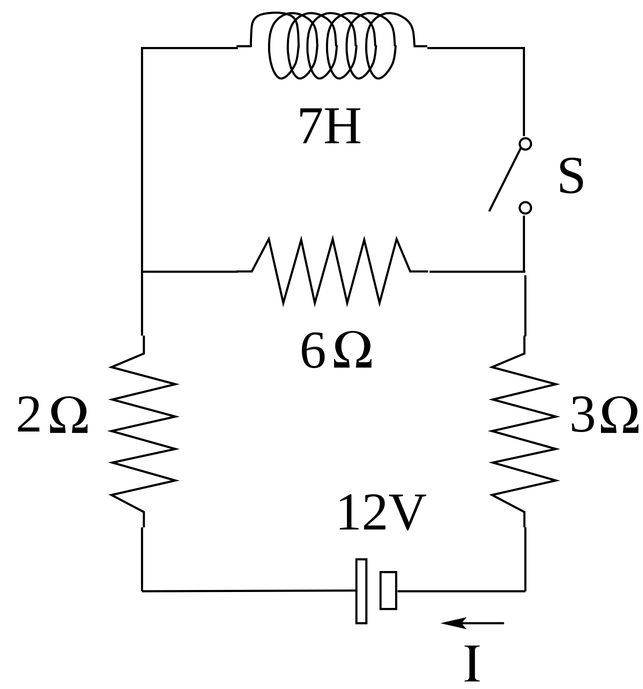
(a) $\Phi_B = \pm(1\text{T})(4\text{m}^2) = \pm 4\text{Tm}^2.$

(b) $\dfrac{d\Phi_B}{dt} = \pm(0.5\text{T/s})(4\text{m}^2) = \pm 2\text{V} \qquad \Rightarrow \; \mathcal{E} = -\dfrac{d\Phi_B}{dt} = \mp 2\text{V}.$

$\Rightarrow \; I = \dfrac{\mathcal{E}}{R} = \mp\dfrac{2\text{V}}{5\Omega} = \mp 0.4\text{A}$ (cw).

Intermediate Exam III: Problem #3 (Spring '06)

In the circuit shown the switch S is initially open.
Find the current I through the battery
(a) while the switch is open,
(b) immediately after the switch has been closed,
(c) a very long time later.



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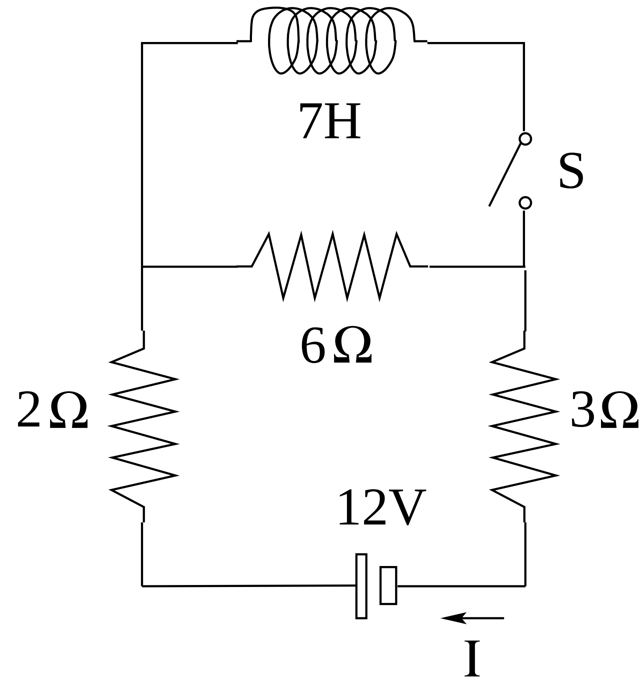


(a) $I = \dfrac{12\text{V}}{2\Omega + 3\Omega + 6\Omega} = 1.09\text{A}.$

(b) $I = \dfrac{12\text{V}}{2\Omega + 3\Omega + 6\Omega} = 1.09\text{A}.$

(c) $I = \dfrac{12\text{V}}{2\Omega + 3\Omega} = 2.4\text{A}.$

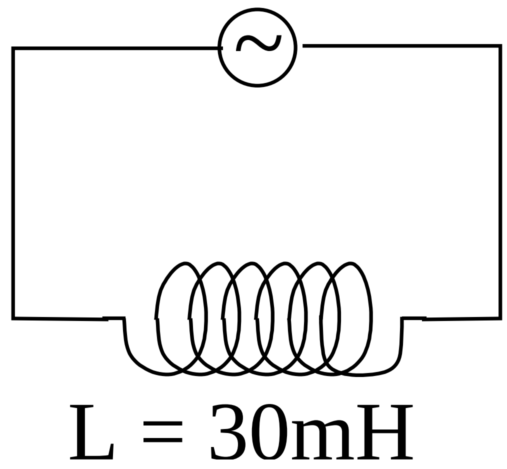
Intermediate Exam III: Problem #4 (Spring '06)

Consider the circuit shown. The ac voltage supplied is $\mathcal{E} = \mathcal{E}_{max}\cos(\omega t)$ with $\mathcal{E}_{max} = 170$V and $\omega = 377$rad/s.

(a) What is the maximum value I_{max} of the current?

(b) What is the emf $\mathcal{E}$ at $t = 0.02$s?

(c) What is the current I at $t = 0.02$s?



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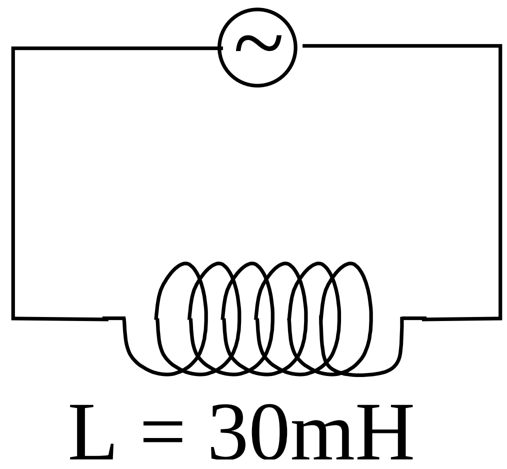


(a) $I_{max} = \frac{\mathcal{E}_{max}}{X_L} = \frac{\mathcal{E}_{max}}{\omega L} = \frac{170\text{V}}{11.3\Omega} = 15.0\text{A}.$

(b) $\mathcal{E} = \mathcal{E}_{max}\cos(7.54\text{rad}) = (170\text{V})(0.309) = 52.5\text{V}.$

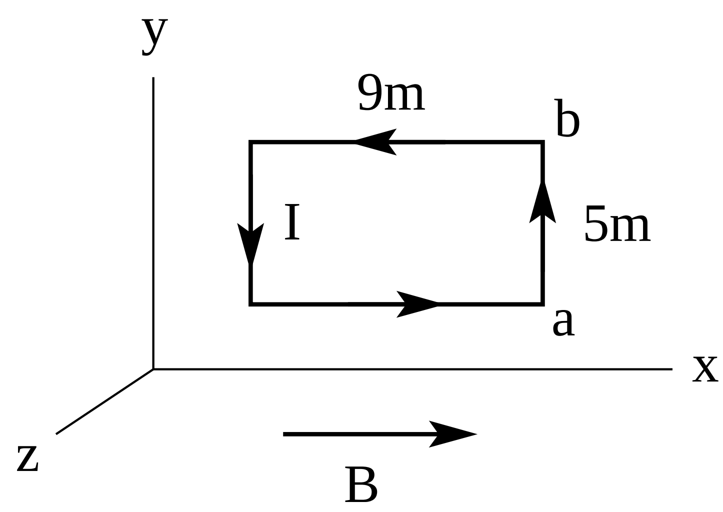
(c) $I = I_{max}\cos(7.54\text{rad} - \pi/2) = (15.0\text{A})(0.951) = 14.3\text{A}.$

Consider a rectangular conducting loop in the xy-plane with a counterclockwise current $I = 7$A in a uniform magnetic field $\vec{B} = 3\text{T}\hat{i}$.

(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.

(b) Find the force $\vec{F}$ (magnitude and direction) acting on the side ab of the rectangle.

(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



Intermediate Exam III: Problem #1 (Spring ’07)

Consider a rectangular conducting loop in the xy-plane with a counterclockwise current $I = 7$A in a uniform magnetic field $\vec{B} = 3\text{T}\hat{i}$.

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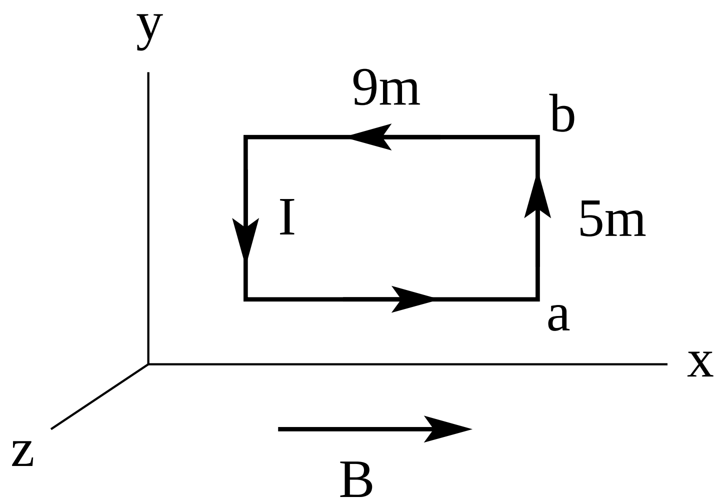


Solution:

(a) $\vec{\mu} = (7\text{A})(45\text{m}^2)\hat{k} = 315\text{Am}^2\hat{k}.$

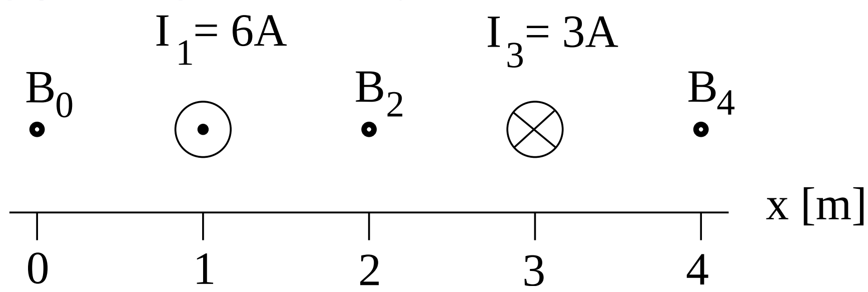
(b) $\vec{F} = I\vec{L} \times \vec{B} = (7\text{A})(5\text{m}\hat{j}) \times (3\text{T}\hat{i}) = -105\text{N}\hat{k}.$

(c) $\vec{\tau} = \vec{\mu} \times \vec{B} = (315\text{Am}^2\hat{k}) \times (3\text{T}\hat{i}) = 945\text{Nm}\hat{j}$

Intermediate Exam III: Problem #2 (Spring '07)

Consider two very long, straight wires with currents $I_1 = 6\text{A}$ at $x = 1\text{m}$ and $I_3 = 3\text{A}$ at $x = 3\text{m}$ in the directions shown. Find magnitude and direction (up/down) of the magnetic field

(a) B_0 at $x = 0$,
(b) B_2 at $x = 2\text{m}$,
(c) B_4 at $x = 4\text{m}$.



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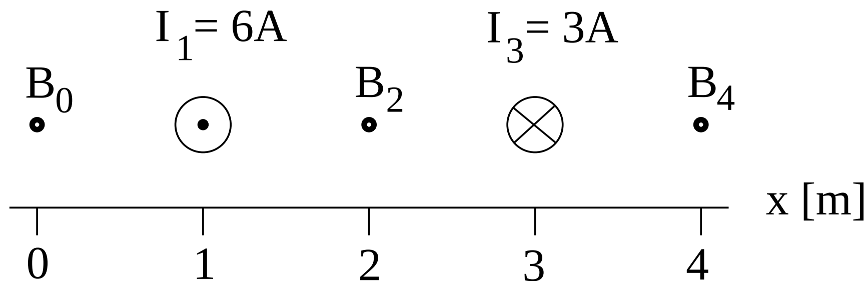


Solution:

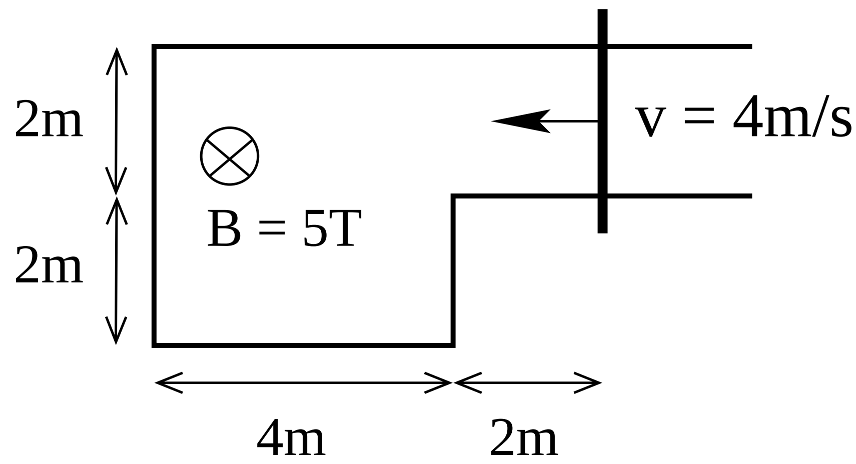
(a) $B_0 = -\frac{\mu_0(6\text{A})}{2\pi(1\text{m})} + \frac{\mu_0(3\text{A})}{2\pi(3\text{m})} = -1.2\mu\text{T} + 0.2\mu\text{T} = -1.0\mu\text{T}$ (down),

(b) $B_2 = \frac{\mu_0(6\text{A})}{2\pi(1\text{m})} + \frac{\mu_0(3\text{A})}{2\pi(1\text{m})} = 1.2\mu\text{T} + 0.6\mu\text{T} = 1.8\mu\text{T}$ (up),

(c) $B_4 = \frac{\mu_0(6\text{A})}{2\pi(3\text{m})} - \frac{\mu_0(3\text{A})}{2\pi(1\text{m})} = 0.4\mu\text{T} - 0.6\mu\text{T} = -0.2\mu\text{T}$ (down).

Intermediate Exam III: Problem #3 (Spring ’07)

A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown.
(a) Find the magnetic flux Φ_B through the frame at the instant shown.
(b) Find the induced emf $\mathcal{E}$ at the instant shown.
(c) Find the direction (cw/ccw) of the induced current.



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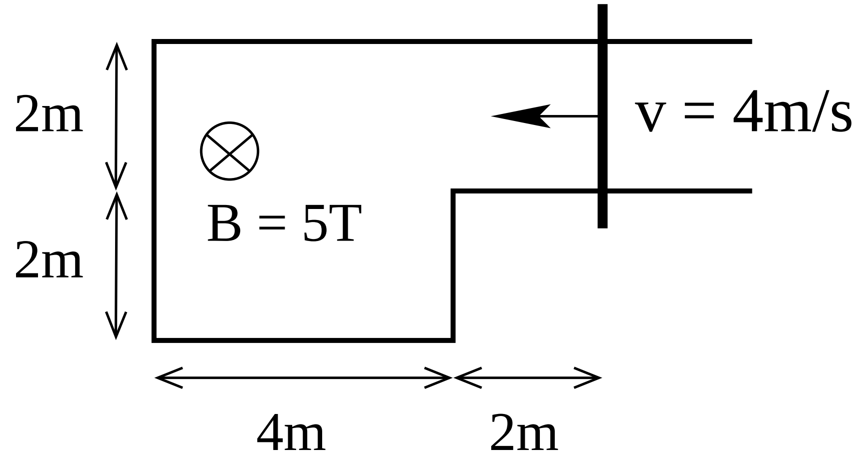


Solution:

(a) $\Phi_B = \vec{A} \cdot \vec{B} = \pm(20\text{m}^2)(5\text{T}) = \pm 100\text{Wb}.$

(b) $\mathcal{E} = -\frac{d\Phi_B}{dt} = \pm(5\text{T})(2\text{m})(4\text{m/s}) = \pm 40\text{V}.$

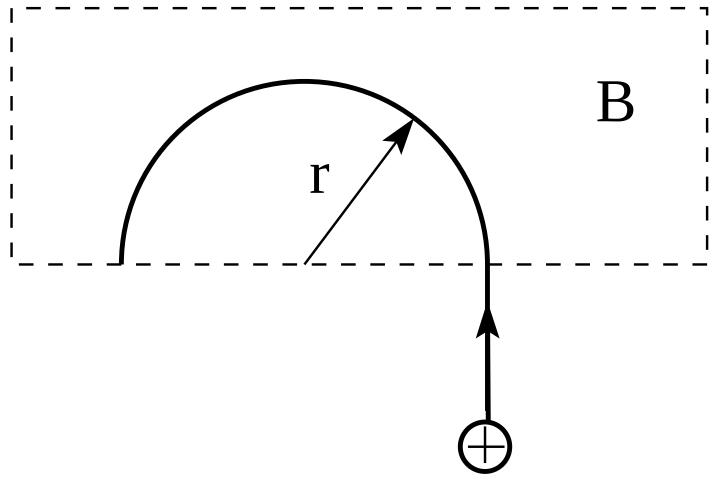
(c) clockwise.

A proton ($m = 1.67 \times 10^{-27}$kg, $q = 1.60 \times 10^{-19}$C) with velocity $v = 3.7 \times 10^4$m/s enters a region of magnetic field B directed perpendicular to the plane of the sheet. The field bends the path of the proton into a semicircle of radius $r = 19$cm as shown.

(a) Find the force necessary to keep the proton moving on the circle

(b) Find the direction ($\odot$ or $\otimes$) and the magnitude of the magnetic field B that provides this force.

(c) Find the time t it takes the proton to complete the semicircular motion.



Unit Exam III: Problem #4 (Spring '07)

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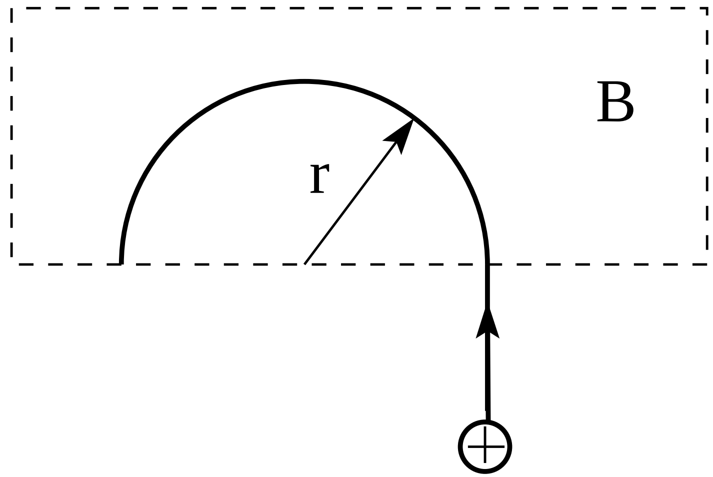


Solution:

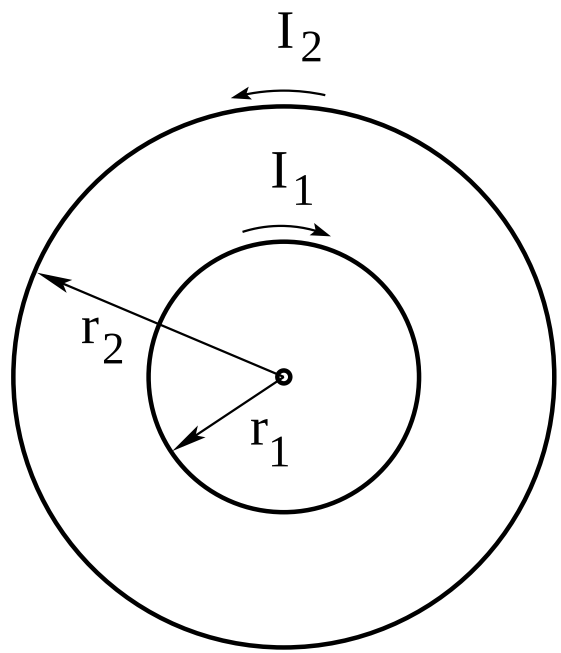
(a) $F = \frac{mv^2}{r} = 1.20 \times 10^{-17}\text{N}.$

(b) $F = qvB \quad \Rightarrow \; B = \frac{F}{qv} = 2.03 \times 10^{-3}\text{T}. \qquad \otimes$

(c) $vt = \pi r \quad \Rightarrow \; t = \frac{\pi r}{v} = 1.61 \times 10^{-5}\text{s}.$

Unit Exam III: Problem #1 (Spring '08)

Consider two circular currents $I_1 = 3$A at radius $r_1 = 2$m and $I_2 = 5$A at radius $r_2 = 4$m in the directions shown.
(a) Find magnitude B and direction $(\odot, \otimes)$ of the resultant magnetic field at the center.
(b) Find magnitude μ and direction $(\odot, \otimes)$ of the magnetic dipole moment generated by the two currents.



Unit Exam III: Problem #1 (Spring '08)

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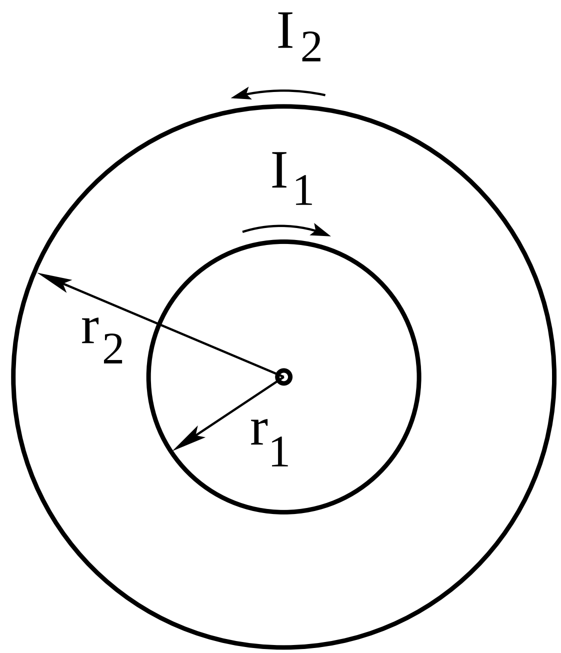


Solution:

(a) $B = \frac{\mu_0(3\text{A})}{2(2\text{m})} - \frac{\mu_0(5\text{A})}{2(4\text{m})} = (9.42 - 7.85) \times 10^{-7}\text{T}$

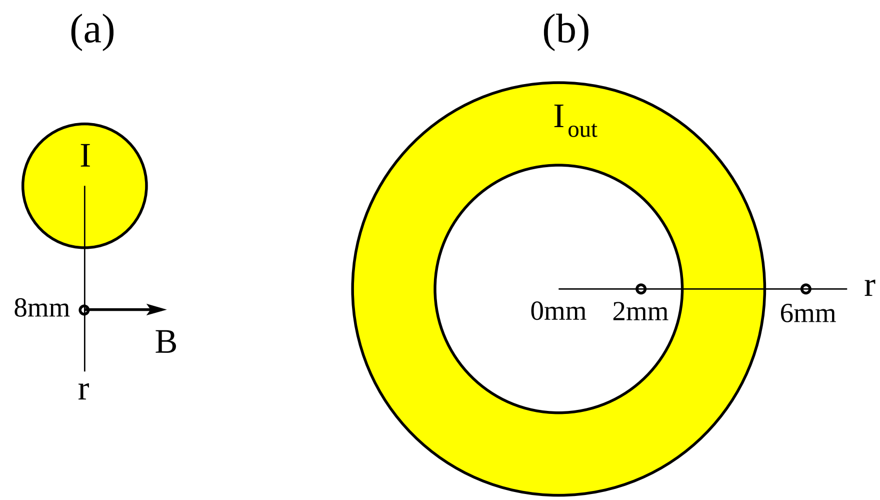
$\Rightarrow \ B = 1.57 \times 10^{-7}\text{T} \quad \otimes$

(b) $\mu = \pi(4\text{m})^2(5\text{A}) - \pi(2\text{m})^2(3\text{A}) = (251 - 38)\text{Am}^2$

$\Rightarrow \ \mu = 213\text{Am}^2 \quad \odot$

Unit Exam III: Problem #2 (Spring '08)

(a) Consider a solid wire of radius $R = 3$mm.
Find magnitude I and direction (in/out) that produces a magnetic field $B = 7\mu\mathrm{T}$ at radius $r = 8$mm.
(b) Consider a hollow cable with inner radius $R_{int} = 3$mm and outer radius $R_{ext} = 5$mm.
A current $I_{out} = 0.9$A is directed out of the plane.
Find direction (up/down) and magnitude B_2, B_6 of the magnetic field at radius $r_2 = 2$mm and $r_6 = 6$mm, respectively.



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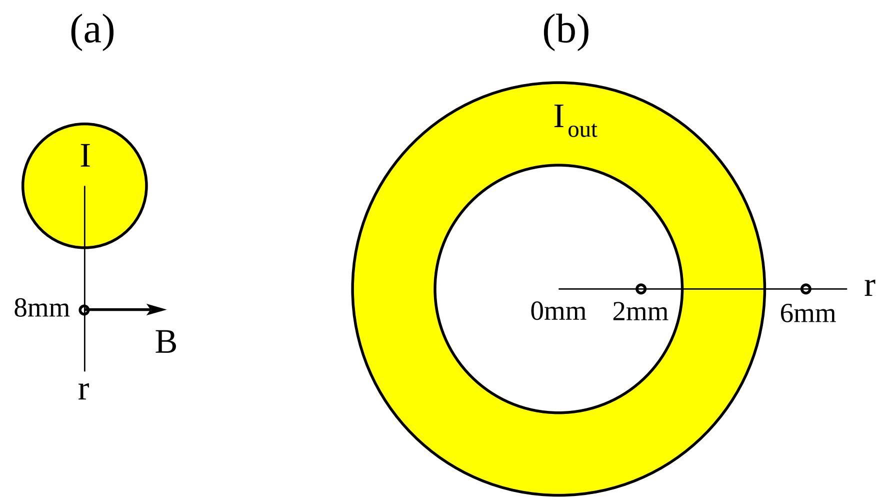
(b) Consider a hollow cable with inner radius $R_{int} = 3$mm and outer radius $R_{ext} = 5$mm.
A current $I_{out} = 0.9$A is directed out of the plane.
Find direction (up/down) and magnitude B_2, B_6 of the magnetic field at radius $r_2 = 2$mm and $r_6 = 6$mm, respectively.



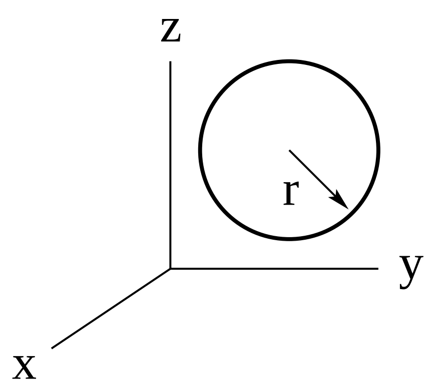
Solution:

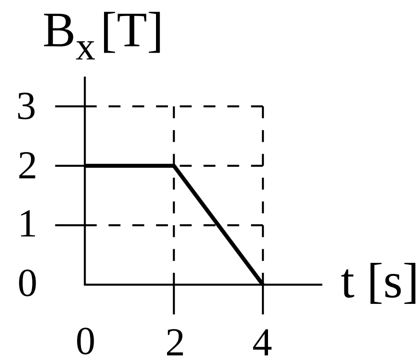
(a) $7\mu\mathrm{T} = \dfrac{\mu_0 I}{2\pi(8\mathrm{mm})} \quad \Rightarrow I = 0.28\mathrm{A}$ (out).

(b) $B_2 = 0, \qquad B_6 = \dfrac{\mu_0(0.9\mathrm{A})}{2\pi(6\mathrm{mm})} = 30\mu\mathrm{T}$ (up).

A circular wire of radius $r = 2.5$m and resistance $R = 4.8\Omega$ is placed in the yz-plane as shown.
A time-dependent magnetic field $\mathbf{B} = B_x\hat{\mathbf{i}}$ is present.
The dependence of B_x on time is shown graphically.

(a) Find the magnitude $|\Phi_B^{(1)}|$ and $|\Phi_B^{(3)}|$ of the magnetic flux through the cicle at times $t = 1$s and $t = 3$s, respectively.
(b) Find magnitude I_1, I_3 and direction (cw/ccw) of the induced current at times $t = 1$s and $t = 3$s, respectively.





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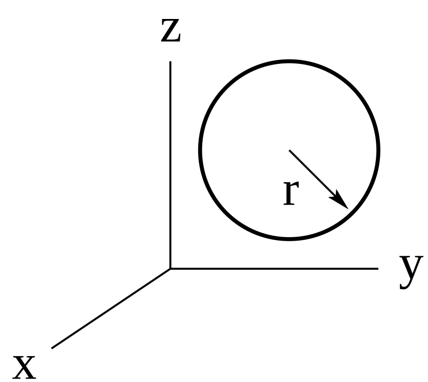
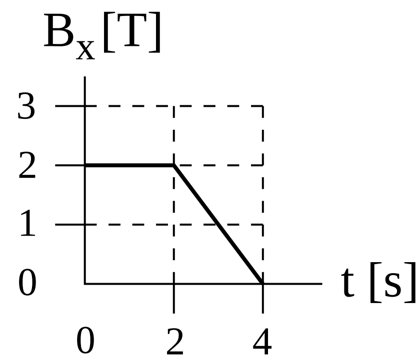




Solution:

(a) $|\Phi_B^{(1)}| = \pi(2.5\text{m})^2(2\text{T}) = 39.3\,\text{Wb},$

$|\Phi_B^{(3)}| = \pi(2.5\text{m})^2(1\text{T}) = 19.6\,\text{Wb}.$

(b) $\left|\frac{d\Phi_B^{(1)}}{dt}\right| = 0 \quad \Rightarrow \quad I_1 = 0,$

$\left|\frac{d\Phi_B^{(3)}}{dt}\right| = |\pi(2.5\text{m})^2(-1\text{T/s})| = 19.6\text{V} \qquad \Rightarrow \quad I_3 = \frac{19.6\text{V}}{4.8\Omega} = 4.1\text{A}$ (ccw).

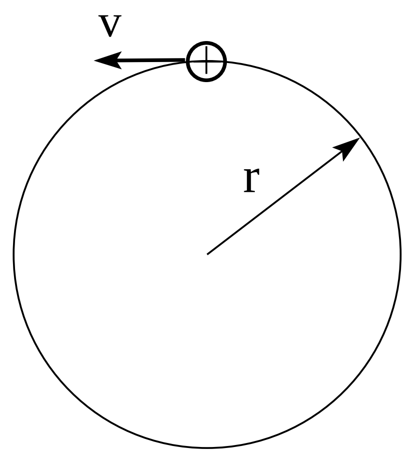
Unit Exam III: Problem #3 (Spring '08)

A proton ($m = 1.67 \times 10^{-27}$kg, $q = 1.60 \times 10^{-19}$C) with velocity $v = 3.7 \times 10^4$m/s moves on a circle of radius $r = 0.49$m in a counterclockwise direction.

(a) Find the centripetal force F needed to keep the proton on the circle.

(b) Find direction ($\odot$ or $\otimes$) and magnitude of the field $\mathbf{B}$ that provides the centripetal force F.

(c) Find the electric current I produced by the rotating proton.



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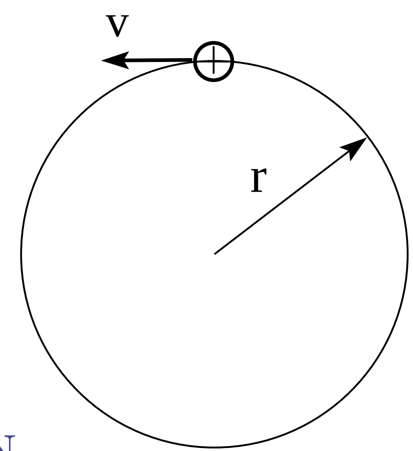


Solution:

(a) $F = \dfrac{mv^2}{r} = \dfrac{(1.67 \times 10^{-27}\mathrm{kg})(3.7 \times 10^{4}\mathrm{m/s})^2}{0.49\mathrm{m}} = 4.67 \times 10^{-18}\mathrm{N}.$

(b) $F = qvB \quad \Rightarrow B = \dfrac{F}{qv} = \dfrac{4.67 \times 10^{-18}\mathrm{N}}{(1.60 \times 10^{-19}\mathrm{C})(3.7 \times 10^{4}\mathrm{m/s})} = 0.788\mathrm{mT} \quad \otimes$ (in).

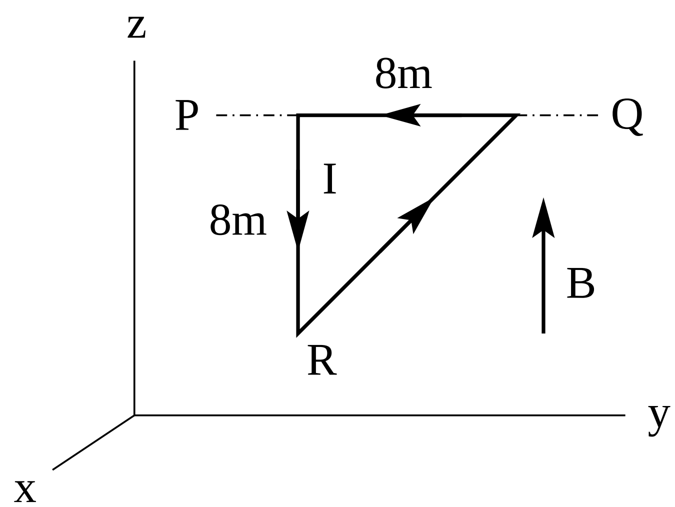
(c) $I = \dfrac{q}{\tau}, \quad \tau = \dfrac{2\pi r}{v} \quad \Rightarrow I = \dfrac{qv}{2\pi r} = 1.92 \times 10^{-15}\mathrm{A}.$

A triangular conducting loop in the yz-plane with a counterclockwise current $I = 3$A is free to rotate about the axis PQ. A uniform magnetic field $\vec{B} = 0.5\mathrm{T}\hat{k}$ is present. (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the triangle.

(b) Find the magnetic torque $\vec{\tau}$ (magnitude and direction) acting on the triangle.

(c) Find the magnetic force $\vec{F}_H$ (magnitude and direction) acting on the long side (hypotenuse) of the triangle.

(d) Find the force $\vec{F}_R$ (magnitude and direction) that must be applied to the corner R to keep the triangle from rotating.



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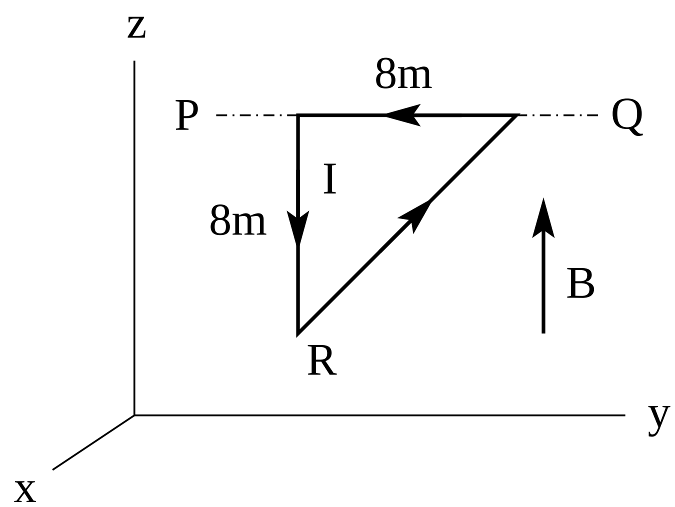


Solution:

(a) $\vec{\mu} = (3\mathrm{A})(32\mathrm{m}^2)\hat{i} = 96\mathrm{Am}^2\hat{i}$.

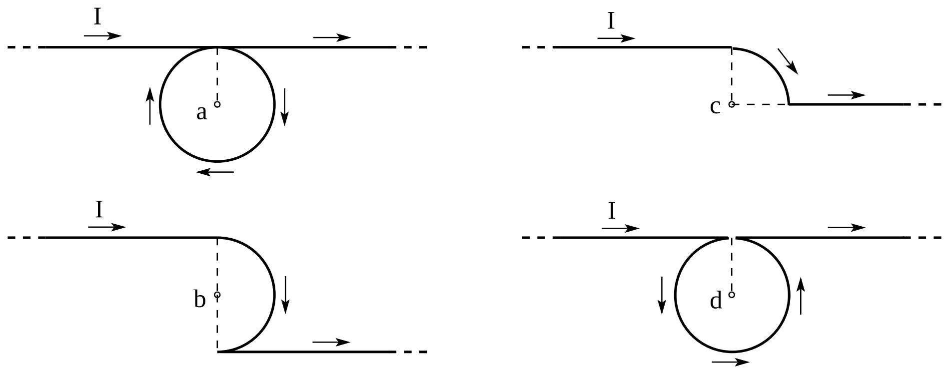
(b) $\vec{\tau} = \vec{\mu} \times \vec{B} = (96\mathrm{Am}^2\hat{i}) \times (0.5\mathrm{T}\hat{k}) = -48\mathrm{Nm}\hat{j}$.

(c) $F_H = (3\mathrm{A})(8\sqrt{2}\mathrm{m})(0.5\mathrm{T})(\sin 45^\circ) = 12\mathrm{N} \quad \odot$.

(d) $(-8\mathrm{m}\hat{k}) \times \vec{F}_R = -\vec{\tau} = 48\mathrm{Nm}\hat{j} \quad \Rightarrow \quad \vec{F}_R = -6\mathrm{N}\hat{i}$.

Unit Exam III: Problem #2 (Spring ’09)

Two semi-infinite straight wires are connected to a curved wire in the form of a full circle, quarter circle, or half circle of radius $R = 1$m in four different configurations. A current $I = 1$A flows in the directions shown. Find magnitude B_a, B_b, B_c, B_d and direction $(\odot/\otimes)$ of the magnetic field thus generated at the points a, b, c, d.



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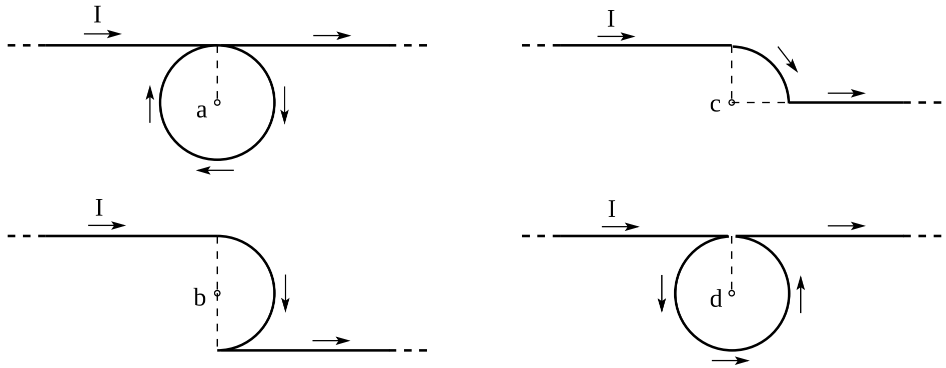


Solution:

$$B_a = \left|\frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R}\right| = |100\text{nT} + 628\text{nT} + 100\text{nT}| = 828\text{nT} \quad \otimes$$

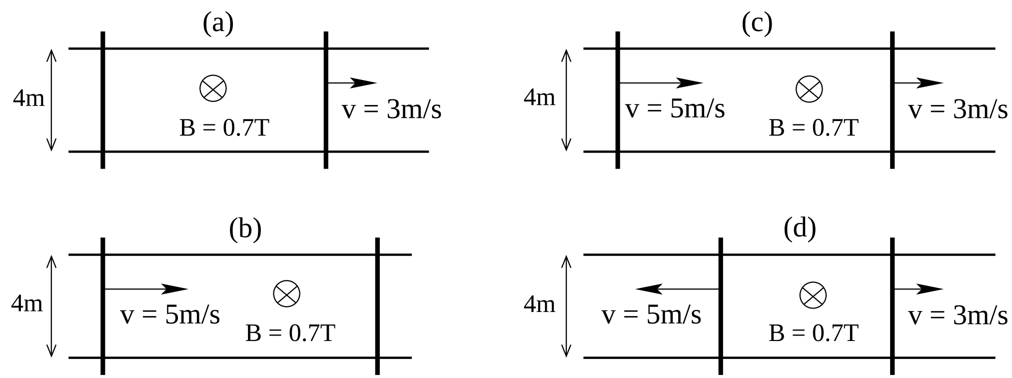
$$B_b = \left|\frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4R} - \frac{\mu_0 I}{4\pi R}\right| = |100\text{nT} + 314\text{nT} - 100\text{nT}| = 314\text{nT} \quad \otimes$$

$$B_c = \left|\frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{8R} + 0\right| = |100\text{nT} + 157\text{nT}| = 257\text{nT} \quad \otimes$$

$$B_d = \left|\frac{\mu_0 I}{4\pi R} - \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R}\right| = |100\text{nT} - 628\text{nT} + 100\text{nT}| = 428\text{nT} \quad \odot$$

Unit Exam III: Problem #3 (Spring ’09)

A pair of rails are connected by two mobile rods. A uniform magnetic field B directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is $R = 0.2\Omega$ in each case. Find magnitude I and direction (cw/ccw) of the induced current in each case.



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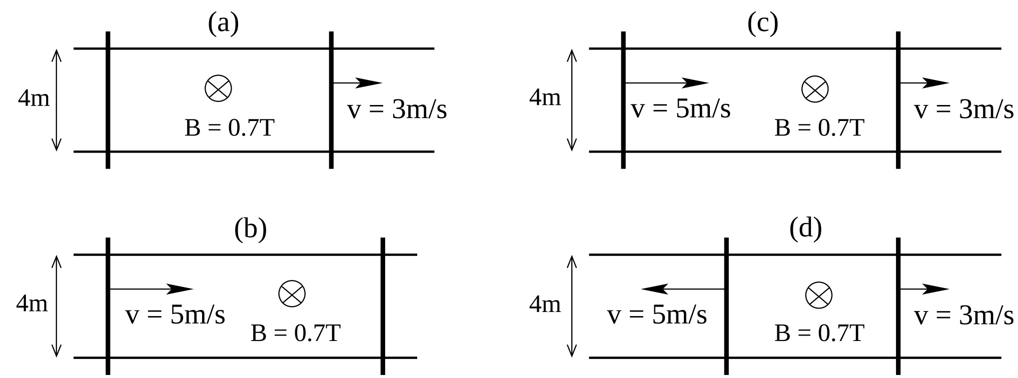


Solution:

(a) $|\mathcal{E}| = (3\text{m/s})(0.7\text{T})(4\text{m}) = 8.4\text{V}, \qquad I = \dfrac{8.4\text{V}}{0.2\Omega} = 42\text{A}$ ccw

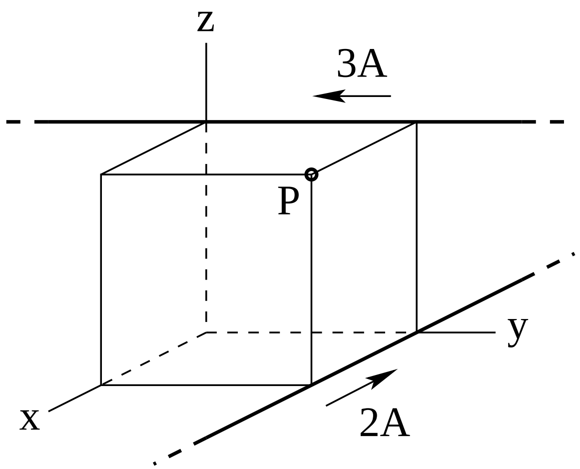
(b) $|\mathcal{E}| = (5\text{m/s})(0.7\text{T})(4\text{m}) = 14\text{V}, \qquad I = \dfrac{14\text{V}}{0.2\Omega} = 70\text{A}$ cw

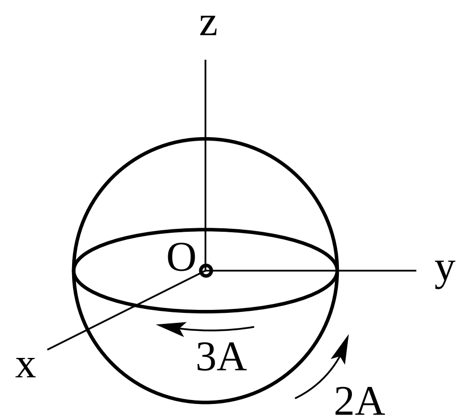
(c) $|\mathcal{E}| = (5\text{m/s} - 3\text{m/s})(0.7\text{T})(4\text{m}) = 5.6\text{V}, \qquad I = \dfrac{5.6\text{V}}{0.2\Omega} = 28\text{A}$ cw

(d) $|\mathcal{E}| = (5\text{m/s} + 3\text{m/s})(0.7\text{T})(4\text{m}) = 22.4\text{V}, \qquad I = \dfrac{22.4\text{V}}{0.2\Omega} = 112\text{A}$ ccw

(a) Two very long straight wires carry currents as shown. A cube with edges of length 8cm serves as scaffold. Find the magnetic field at point P in the form $\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$ with B_x, B_y, B_z in SI units.

(b) Two circular currents of radius 5cm, one in the xy-lane and the other in the yz-plane, carry currents as shown. Both circles are centered at point O. Find the magnetic field at point O in the form $\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$ with B_x, B_y, B_z in SI units.





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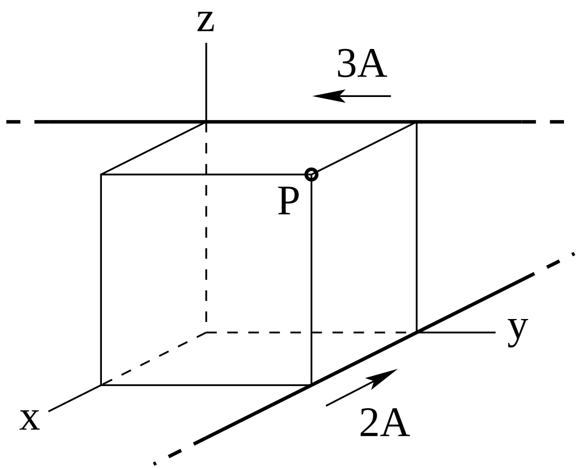
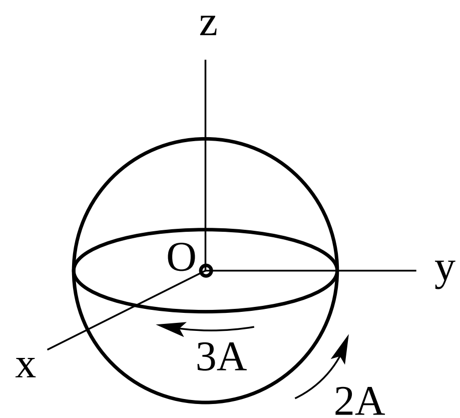




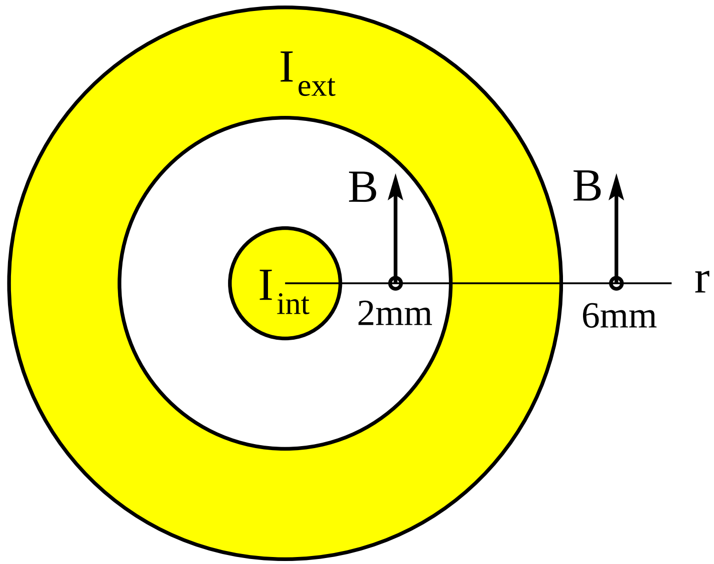
Solution:

(a) $B_x = 0, \quad B_y = \dfrac{\mu_0(2\text{A})}{2\pi(0.08\text{m})} = 5\mu\text{T}, \quad B_z = \dfrac{\mu_0(3\text{A})}{2\pi(0.08\text{m})} = 7.5\mu\text{T}.$

(b) $B_x = \dfrac{\mu_0(2\text{A})}{2(0.05\text{m})} = 25.1\mu\text{T}, \quad B_y = 0, \quad B_z = -\dfrac{\mu_0(3\text{A})}{2(0.05\text{m})} = -37.7\mu\text{T}.$

Unit Exam III: Problem #2 (Spring ’11)

The coaxial cable shown has surfaces at radii 1mm, 3mm, and 5mm. The magnetic field is the same at radii 2mm and 6mm, namely $B = 7\mu$T in the direction shown.
(a) Find magnitude (in SI units) and direction (in/out) of the current $I_{\rm int}$ flowing through the inner conductor.
(b) Find magnitude (in SI units) and direction (in/out) of the current $I_{\rm ext}$ flowing through the outer conductor.



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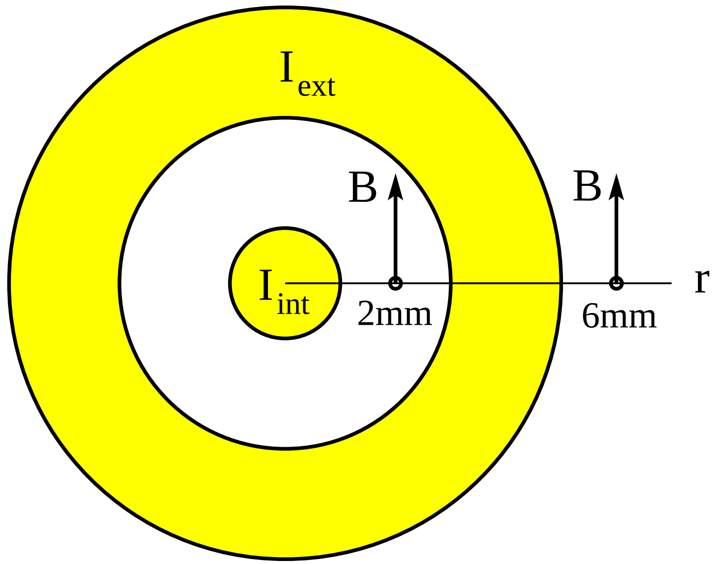
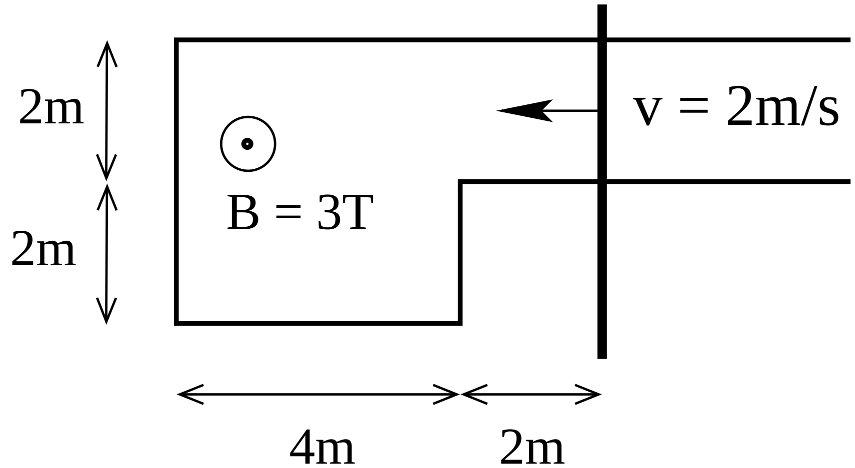


Solution:

(a) $(7\mu\text{T})(2\pi)(0.002\text{m}) = \mu_0 I_{\rm int} \quad \Rightarrow \; I_{\rm int} = 0.07\text{A}$ (out)

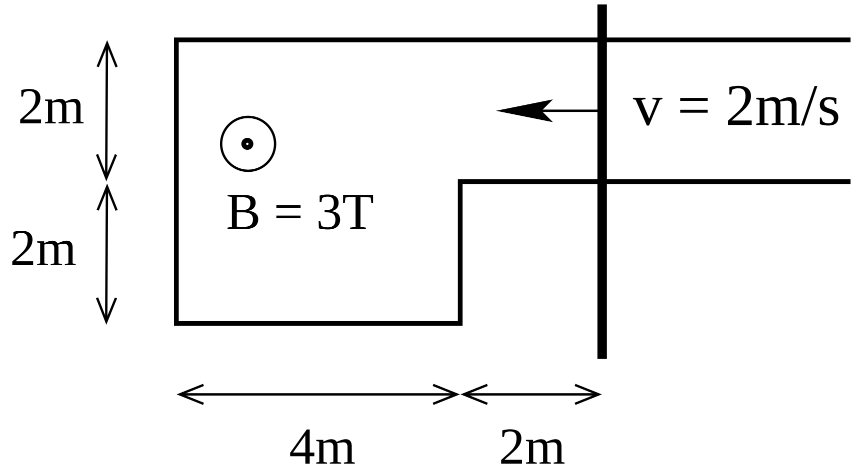
(b) $(7\mu\text{T})(2\pi)(0.006\text{m}) = \mu_0 (I_{\rm int} + I_{\rm ext}) \quad \Rightarrow \; I_{\rm int} + I_{\rm ext} = 0.21\text{A}$ (out)
$\Rightarrow \; I_{\rm ext} = 0.14\text{A}$ (out)

A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown. The rod moves at constant velocity.
(a) Find the magnetic flux Φ_B through the frame and the induced emf $\mathcal{E}$ around the frame at the instant shown.
(b) Find the magnetic flux Φ_B through the frame and the induced emf $\mathcal{E}$ around the frame two seconds later.
Write magnitudes only (in SI units), no directions.



Unit Exam III: Problem #3 (Spring '11)

A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown. The rod moves at constant velocity.
(a) Find the magnetic flux Φ_B through the frame and the induced emf $\mathcal{E}$ around the frame at the instant shown.
(b) Find the magnetic flux Φ_B through the frame and the induced emf $\mathcal{E}$ around the frame two seconds later.
Write magnitudes only (in SI units), no directions.



Solution:

(a) $\Phi_B = (20\text{m}^2)(3\text{T}) = 60\text{Wb}, \quad \mathcal{E} = (2\text{m/s})(3\text{T})(2\text{m}) = 12\text{V}.$

(b) $\Phi_B = (8\text{m}^2)(3\text{T}) = 24\text{Wb}, \quad \mathcal{E} = (2\text{m/s})(3\text{T})(4\text{m}) = 24\text{V}.$

Unit Exam III: Problem #1 (Spring '12)

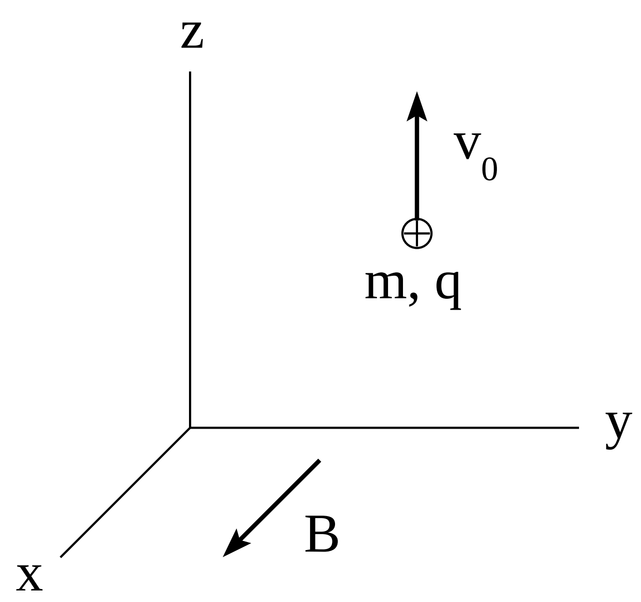
In a region of uniform magnetic field $\mathbf{B} = 5\mathrm{mT}\hat{\mathbf{i}}$, a proton ($m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C}$) is launched with velocity $\mathbf{v}_0 = 4000\mathrm{m/s}\hat{\mathbf{k}}$.

(a) Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.

(b) Calculate the radius r of the circular path.

(c) Calculate the time T it takes the proton to go around that circle once.

(d) Sketch the circular path of the proton in the graph.



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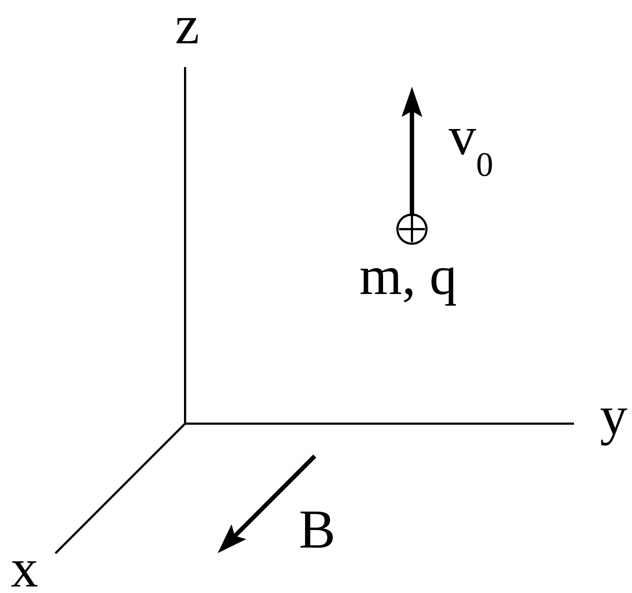


Solution:

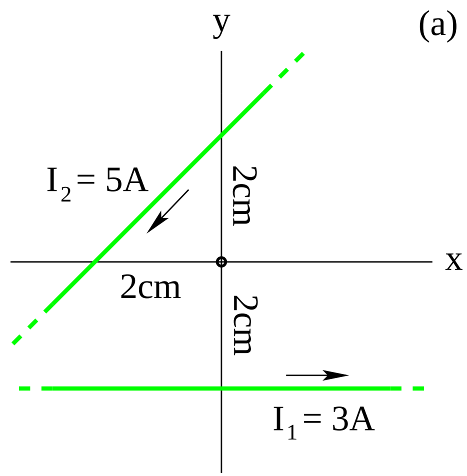
(a) $F = qv_0B = 3.2 \times 10^{-18}\mathrm{N}.$

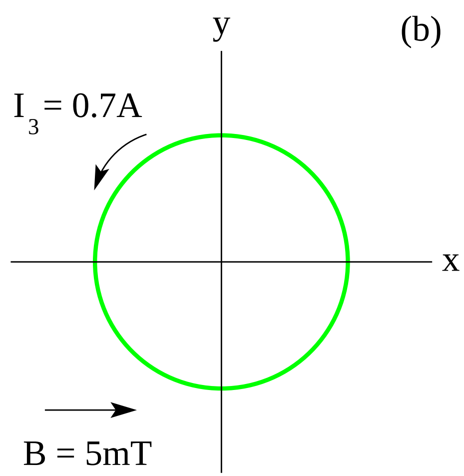
(b) $\dfrac{mv_0^2}{r} = qv_0B \quad \Rightarrow r = \dfrac{mv_0}{qB} = 8.35\mathrm{mm}.$

(c) $T = \dfrac{2\pi r}{v_0} = \dfrac{2\pi m}{qB} = 13.1\mu\mathrm{s}.$

(d) Center of circle to the right of proton’s initial position (cw motion).

(a) Two very long straight wires positioned in the xy-plane carry electric currents I_1, I_2 as shown. Calculate magnitude (B_1, B_2) and direction $(\odot, \otimes)$ of the magnetic field produced by each current at the origin of the coordinate system.
(b) A conducting loop of radius $r = 3$cm placed in the xy-plane carries a current $I_3 = 0.7$A in the direction shown. Find direction and magnitude of the torque $\vec{\tau}$ acting on the loop if it is placed in a magnetic field $\mathbf{B} = 5\text{mT}\hat{\mathbf{i}}$.





Unit Exam III: Problem #2 (Spring ’12)

(a) Two very long straight wires positioned in the xy-plane carry electric currents I_1, I_2 as shown. Calculate magnitude (B_1, B_2) and direction $(\odot, \otimes)$ of the magnetic field produced by each current at the origin of the coordinate system.
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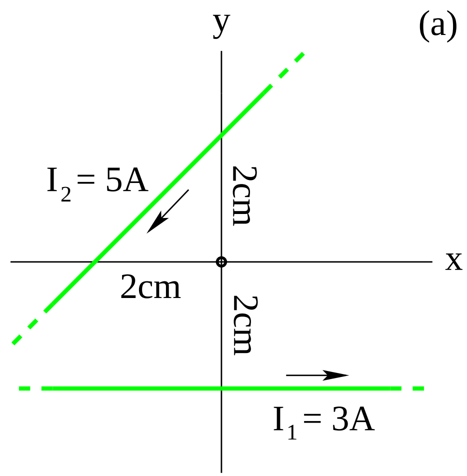
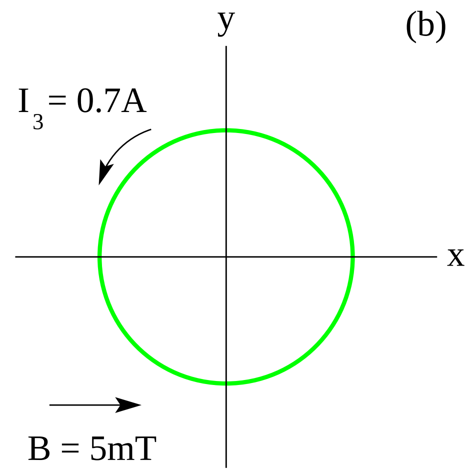




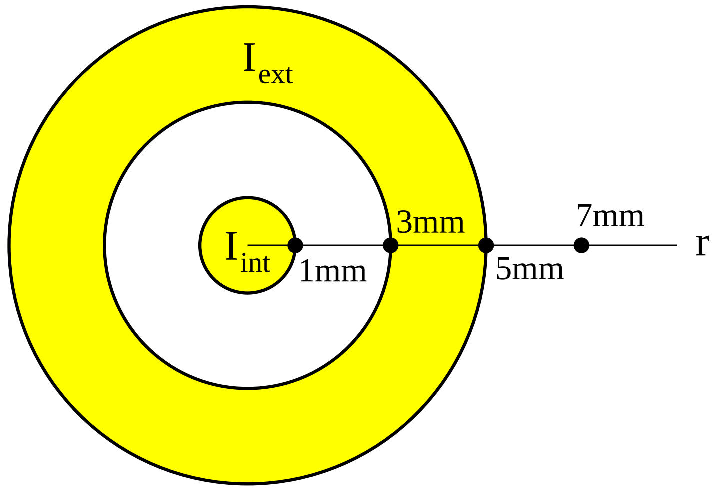
Solution:

(a) $B_1 = \dfrac{\mu_0(3\text{A})}{2\pi(2\text{cm})} = 30\mu\text{T}.\quad \odot \qquad B_2 = \dfrac{\mu_0(5\text{A})}{2\pi(1.41\text{cm})} = 70.9\mu\text{T}.\quad \odot$

(b) $\vec{\mu} = \pi(3\text{cm})^2(0.7\text{A})\hat{\mathbf{k}} = 1.98 \times 10^{-3}\text{Am}^2\hat{\mathbf{k}} \quad \Rightarrow \quad \vec{\tau} = \vec{\mu} \times \mathbf{B} = 9.90 \times 10^{-6}\text{Nm}\hat{\mathbf{j}}.$

Unit Exam III: Problem #3 (Spring ’12)

The coaxial cable shown in cross section has surfaces at radii 1mm, 3mm, and 5mm. Equal currents flow through both conductors: $I_{int} = I_{ext} = 0.03\text{A}$ $\odot$ (out). Find direction $(\uparrow, \downarrow)$ and magnitude (B_1, B_3, B_5, B_7) of the magnetic field at the four radii indicated ($\bullet$).



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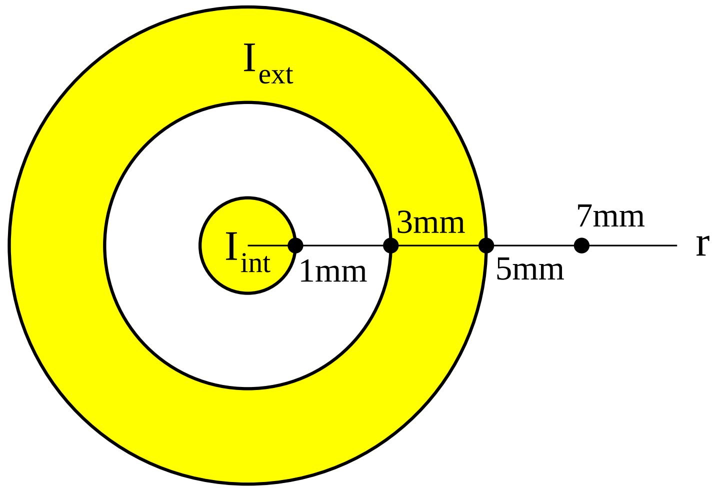


Solution:

$$2\pi(1\text{mm})B_1 = \mu_0(0.03\text{A}) \quad \Rightarrow \quad B_1 = 6\mu\text{T} \quad \uparrow$$

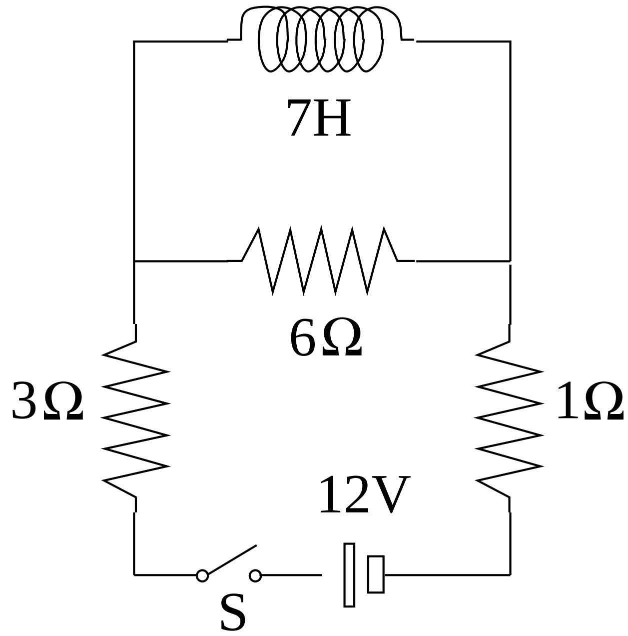
$$2\pi(3\text{mm})B_1 = \mu_0(0.03\text{A}) \quad \Rightarrow \quad B_1 = 2\mu\text{T} \quad \uparrow$$

$$2\pi(5\text{mm})B_1 = \mu_0(0.06\text{A}) \quad \Rightarrow \quad B_1 = 2.4\mu\text{T} \quad \uparrow$$

$$2\pi(7\text{mm})B_1 = \mu_0(0.06\text{A}) \quad \Rightarrow \quad B_1 = 1.71\mu\text{T} \quad \uparrow$$

Unit Exam IV: Problem #1 (Spring ’12)

In the circuit shown we close the switch S at time $t = 0$. Find the current I_L through the inductor and the voltage V_6 across the 6Ω-resistor
(a) immediately after the switch has been closed,
(b) a very long time later.



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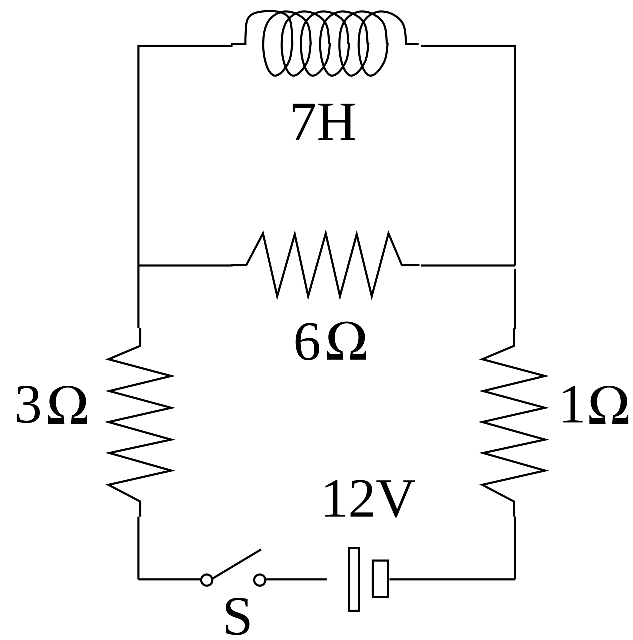


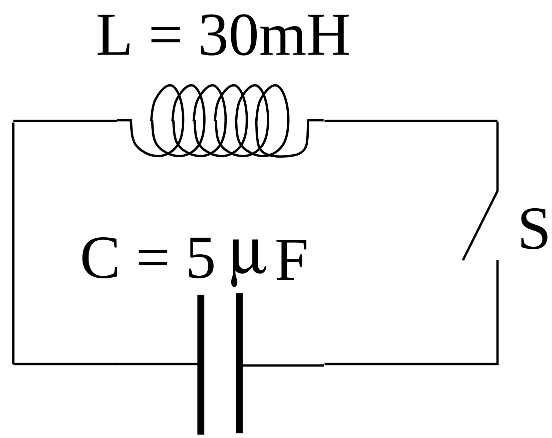
Solution:

(a) $I_L = 0, \quad I_6 = \dfrac{12\text{V}}{10\Omega} = 1.2\text{A}, \quad V_6 = (6\Omega)(1.2\text{A}) = 7.2\text{V}.$

(b) $I_L = \dfrac{12\text{V}}{4\Omega} = 3\text{A}, \quad V_6 = 0.$

Unit Exam IV: Problem #2 (Spring ’12)

At time $t = 0$ the capacitor is charged to $Q_{max} = 4\mu$C and the switch is being closed. The charge on the capacitor begins to decrease and the current through the inductor begins to increase.
(a) At what time t_1 is the capacitor discharged for the first time?
(b) At what time t_2 has the current through the inductor returned to zero for the first time?
(c) What is the maximum energy stored in the capacitor at any time?
(d) What is the maximum energy stored in the inductor at any time?



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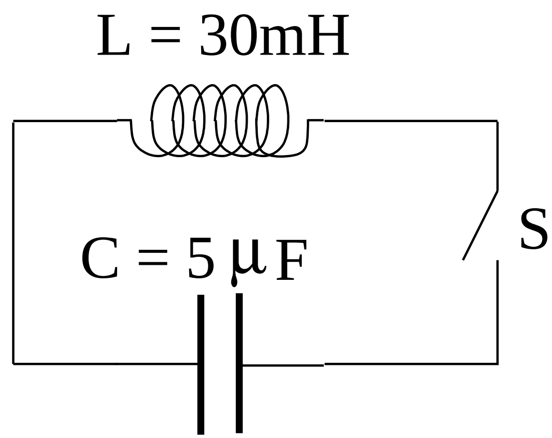


Solution:

(a) $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{LC} = 2.43\text{ms}, \qquad t_1 = \dfrac{T}{4} = 0.608\text{ms}.$

(b) $t_2 = \dfrac{T}{2} = 1.22\text{ms}.$

(c) $U_C^{max} = \dfrac{Q_{max}^2}{2C} = 1.6\mu\text{J}.$

(d) $U_L^{max} = U_C^{max} = 1.6\mu\text{J}$ (energy conservation.)

Unit Exam IV: Problem #3 (Spring ’12)

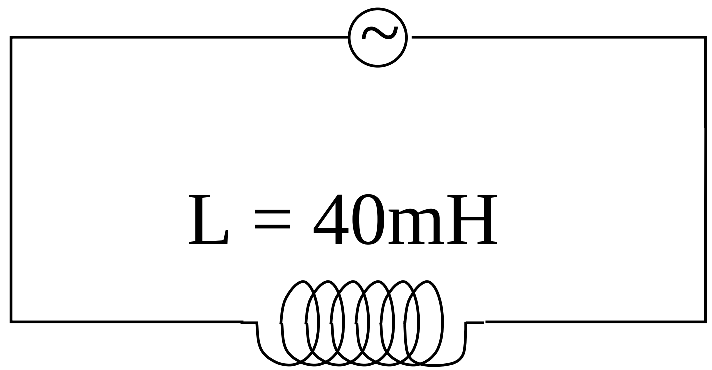
The ac voltage supplied in the circuit shown is $\mathcal{E} = \mathcal{E}_{max}\cos(\omega t)$ with $\mathcal{E}_{max} = 170$V and $\omega = 377$rad/s.

(a) What is the maximum value I_{max} of the current?

(b) What is the emf $\mathcal{E}(t)$ at $t = 5$ms?

(c) What is the current $I(t)$ at $t = 5$ms?

(d) What is the power transfer $P(t)$ between ac source and device at $t = 5$ms?



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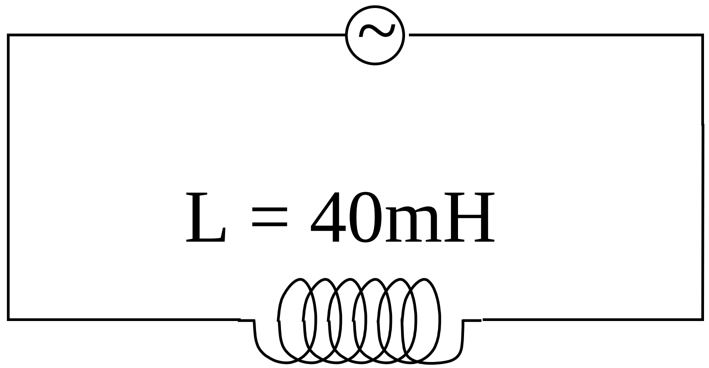


Solution:

(a) $I_{max} = \dfrac{\mathcal{E}_{max}}{\omega L} = \dfrac{170\text{V}}{(377\text{rad/s})(40\text{mH})} = 11.3\text{A}.$

(b) $\mathcal{E} = (170\text{V})\cos(1.885\text{rad}) = (170\text{V})(-0.309) = -52.5\text{V}.$

(c) $I = (11.3\text{A})\cos(1.885\text{rad} - \pi/2) = (11.3\text{A})\cos(0.314) = (11.3\text{A})(0.951) = 10.7\text{A}.$

(d) $P = \mathcal{E}I = (-52.5\text{V})(10.7\text{A}) = -562\text{W}.$

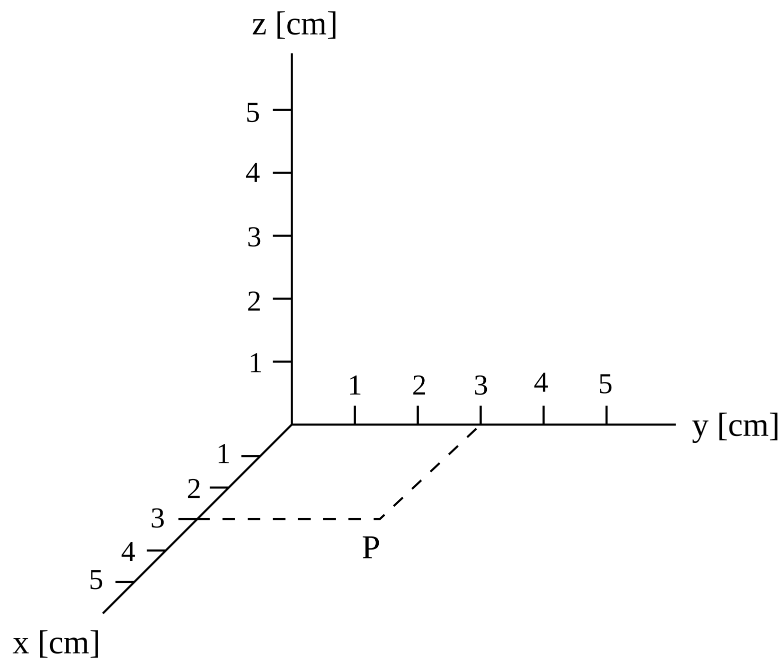
Unit Exam III: Problem #1 (Spring ’13)

In a region of uniform magnetic field $\mathbf{B}$ a proton $(m = 1.67 \times 10^{-27}\text{kg},\ q = 1.60 \times 10^{-19}\text{C})$ experiences a force $\mathbf{F} = 9.0 \times 10^{-19}\text{N}\,\hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 3000\text{m/s}\,\hat{\mathbf{j}}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).

(b) Calculate the radius r of the circular path.

(c) Locate the center C of the circular path in the coordinate system on the page.



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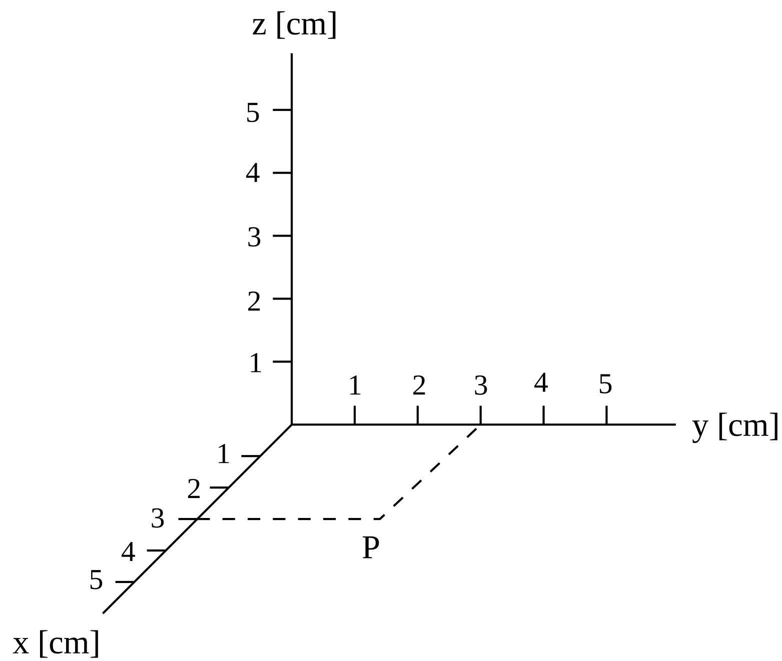


Solution:

(a) $B = \dfrac{F}{qv_0} = 1.88 \times 10^{-3}\text{T}, \quad \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{k}}$

$\Rightarrow\ \mathbf{B} = 1.88 \times 10^{-3}\text{T}\,\hat{\mathbf{k}}.$

(b) $F = \dfrac{mv_0^2}{r} = qv_0B$

$\Rightarrow\ r = \dfrac{mv_0^2}{F} = \dfrac{mv_0}{qB} = 1.67\text{cm}.$

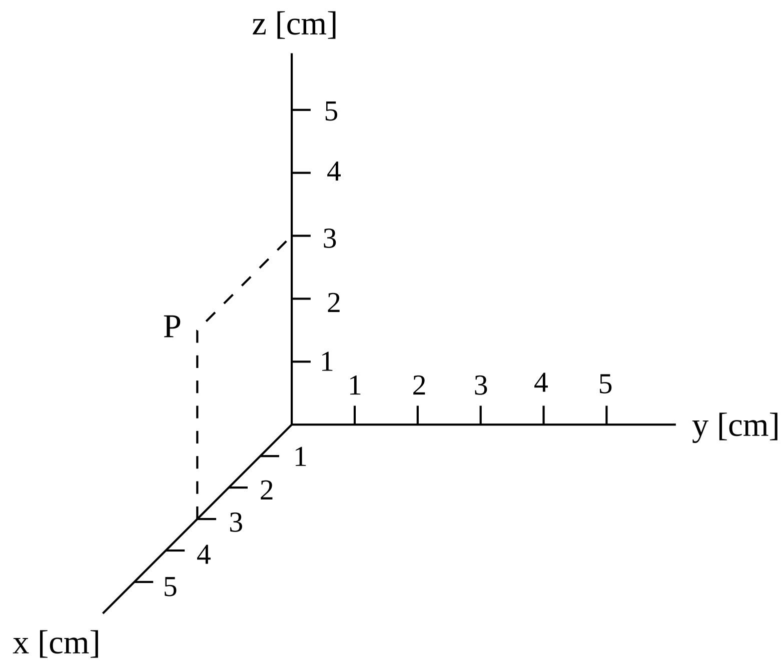
(c) $C = 4.67\text{cm}\,\hat{\mathbf{i}} + 3.00\text{cm}\,\hat{\mathbf{j}}.$

In a region of uniform magnetic field $\mathbf{B}$ a proton $(m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C})$ experiences a force $\mathbf{F} = 8.0 \times 10^{-19}\mathrm{N}\,\hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 2000\mathrm{m/s}\,\hat{\mathbf{k}}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).

(b) Calculate the radius r of the circular path.

(c) Locate the center C of the circular path in the coordinate system on the page.



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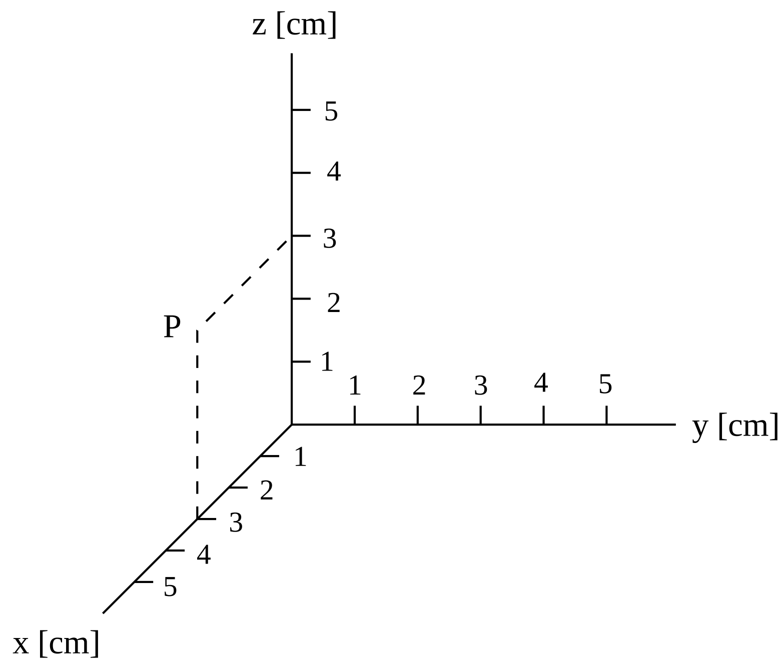


Solution:

(a) $B = \dfrac{F}{qv_0} = 2.50 \times 10^{-3}\text{T}, \quad \hat{\mathbf{i}} = \hat{\mathbf{k}} \times (-\hat{\mathbf{j}})$

$\Rightarrow \mathbf{B} = -2.50 \times 10^{-3}\text{T}\,\hat{\mathbf{j}}.$

(b) $F = \dfrac{mv_0^2}{r} = qv_0B$

$\Rightarrow r = \dfrac{mv_0^2}{F} = \dfrac{mv_0}{qB} = 0.835\text{cm}.$

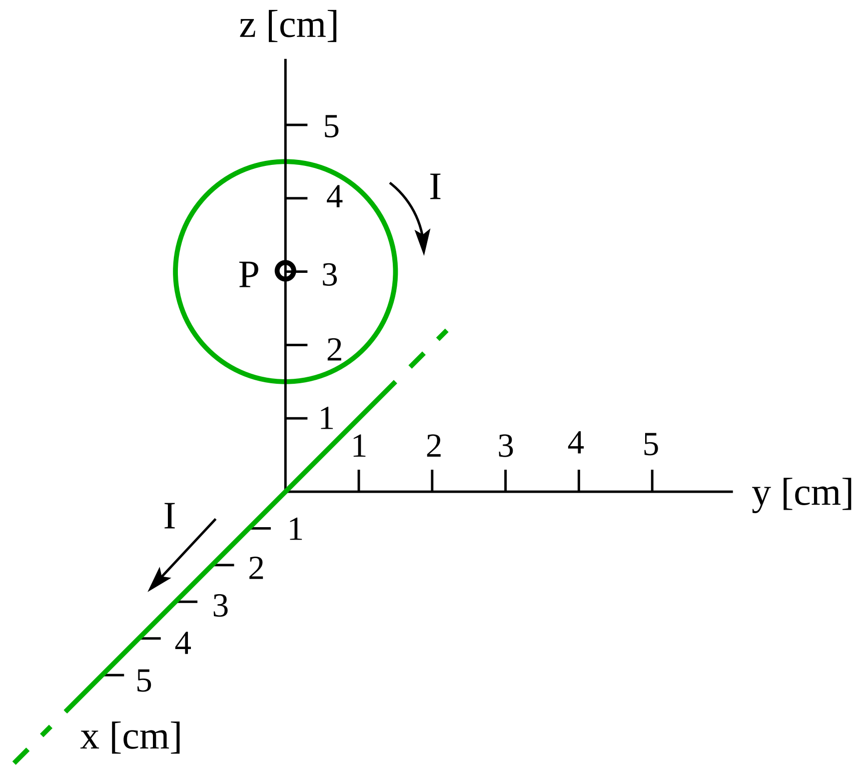
(c) $C = 3.84\text{cm}\,\hat{\mathbf{i}} + 3.00\text{cm}\,\hat{\mathbf{k}}.$

A very long, straight wire is positioned along the x-axis and a circular wire of 1.5cm radius in the yz plane with its center P on the z-axis as shown. Both wires carry a current $I = 0.6$A in the directions shown.

(a) Find the magnetic field $\mathbf{B}_c$ (magnitude and direction) generated at point P by the current in the circular wire.

(b) Find the magnetic field $\mathbf{B}_s$ (magnitude and direction) generated at point P by the current in the straight wire.

(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the circular current.



Unit Exam III: Problem #2 (Spring ’13)

A very long, straight wire is positioned along the x-axis and a circular wire of 1.5cm radius in the yz plane with its center P on the z-axis as shown. Both wires carry a current $I = 0.6$A in the directions shown.
(a) Find the magnetic field $\mathbf{B}_c$ (magnitude and direction) generated at point P by the current in the circular wire.
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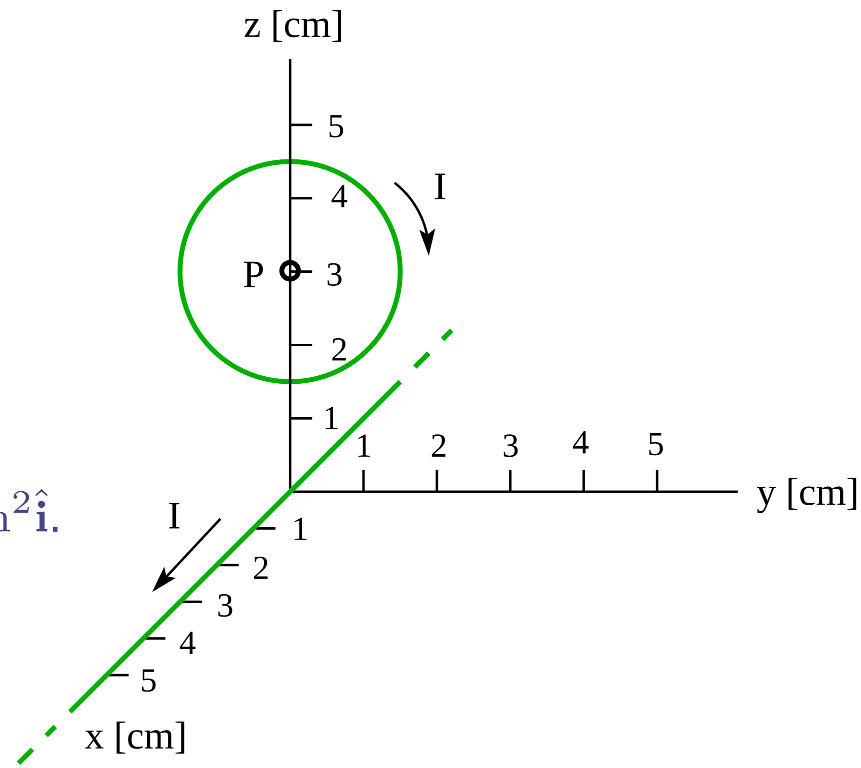

Solution:

(a) $\mathbf{B}_c = \dfrac{\mu_0(0.6\text{A})}{2(0.015\text{m})}(-\hat{\mathbf{i}}) = -2.51 \times 10^{-5}\text{T}\,\hat{\mathbf{i}}.$

(b) $\mathbf{B}_s = \dfrac{\mu_0(0.6\text{A})}{2\pi(0.03\text{m})}(-\hat{\mathbf{j}}) = -4.00 \times 10^{-6}\text{T}\,\hat{\mathbf{j}}.$

(c) $\vec{\mu} = \pi(0.015\text{mm})^2(0.6\text{A})(-\hat{\mathbf{i}}) = -4.24 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}.$

Unit Exam III: Problem #2 (Spring ’13)

A very long straight wire is positioned along the x-axis and a circular wire of 2.0cm radius in the yz plane with its center P on the y-axis as shown. Both wires carry a current $I = 0.5$A in the directions shown.

(a) Find the magnetic field $\mathbf{B}_c$ (magnitude and direction) generated at point P by the current in the circular wire.

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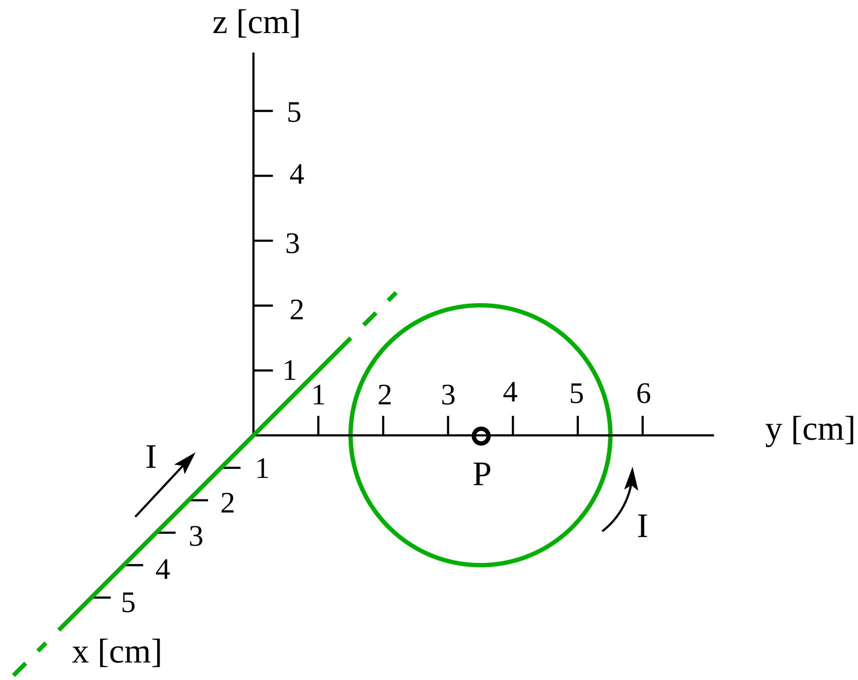

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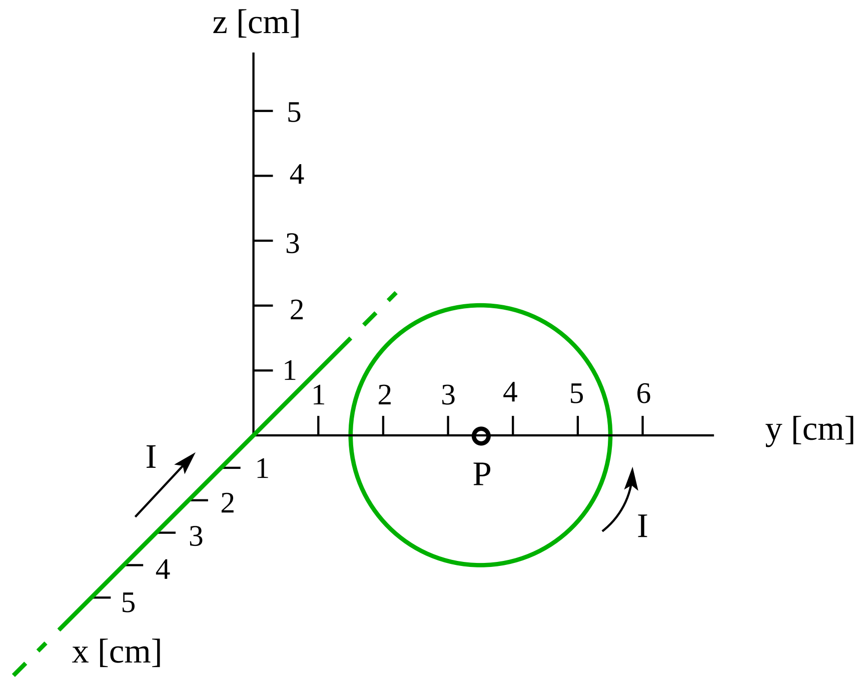


Solution:

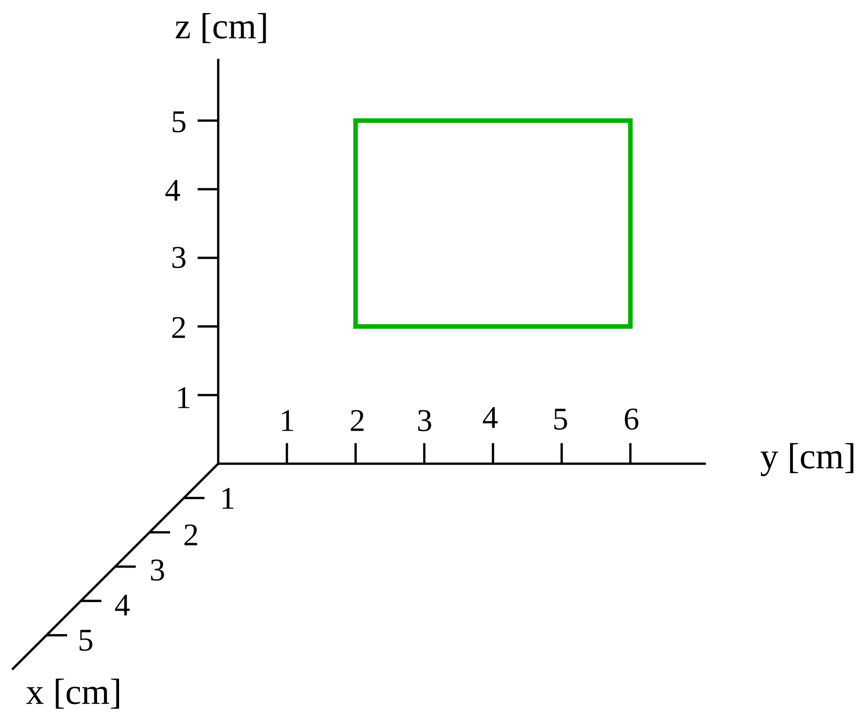
(a) $\mathbf{B}_c = \dfrac{\mu_0(0.5\text{A})}{2(0.02\text{m})}\,\hat{\mathbf{i}} = 1.57 \times 10^{-5}\text{T}\,\hat{\mathbf{i}}.$

(b) $\mathbf{B}_s = \dfrac{\mu_0(0.5\text{A})}{2\pi(0.035\text{m})}(-\hat{\mathbf{k}}) = -2.86 \times 10^{-6}\text{T}\,\hat{\mathbf{k}}.$

(c) $\vec{\mu} = \pi(0.02\text{m})^2(0.5\text{A})\,\hat{\mathbf{i}} = 6.28 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}.$

Unit Exam III: Problem #3 (Spring ’13)

Consider a wire with a resistance per unit length of 1Ω/cm bent into a rectangular loop and placed into the yz-plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (2\hat{\mathbf{i}} + 1\hat{\mathbf{j}} + 3\hat{\mathbf{k}})t$T/s, where t is the time in seconds.
(a) Find the magnetic flux Φ_B through the rectangle at time $t = 2$s.
(b) Find magnitude and direction (cw/ccw) of the induced EMF $\mathcal{E}$ around the rectangle at time $t = 2$s.
(c) Infer the induced current I from the induced EMF.



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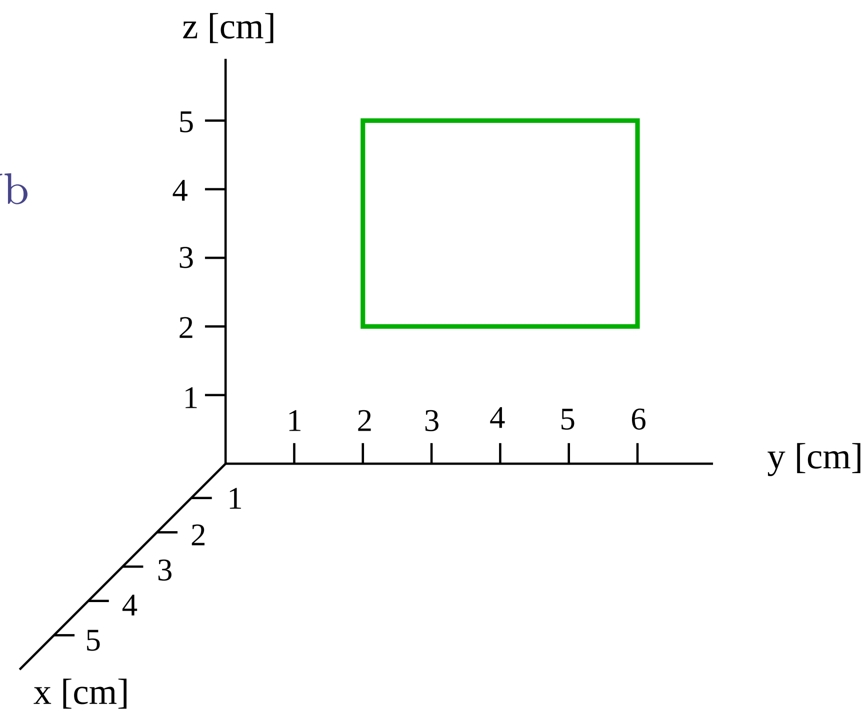
(c) Infer the induced current I from the induced EMF.

Solution:

(a) $\Phi_B = \pm(4\text{cm})(3\text{cm})(2\text{T/s})(2\text{s}) = \pm 4.8 \times 10^{-3}\text{Wb}$

(b) $\mathcal{E} = \mp(4\text{cm})(3\text{cm})(2\text{T/s}) = \mp 2.4\text{mV}$ (cw)

(c) $I = \dfrac{2.4\text{mV}}{(1\Omega/\text{cm})(14\text{cm})} = 0.171\text{mA}$



Unit Exam III: Problem #3 (Spring ’13)

Consider a wire with a resistance per unit length of 1Ω/cm bent into a rectangular loop and placed into the yz-plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (3\hat{\mathbf{i}} + 1\hat{\mathbf{j}} + 2\hat{\mathbf{k}})t$T/s, where t is the time in seconds.

(a) Find the magnetic flux Φ_B through the rectangle at time $t = 2$s.

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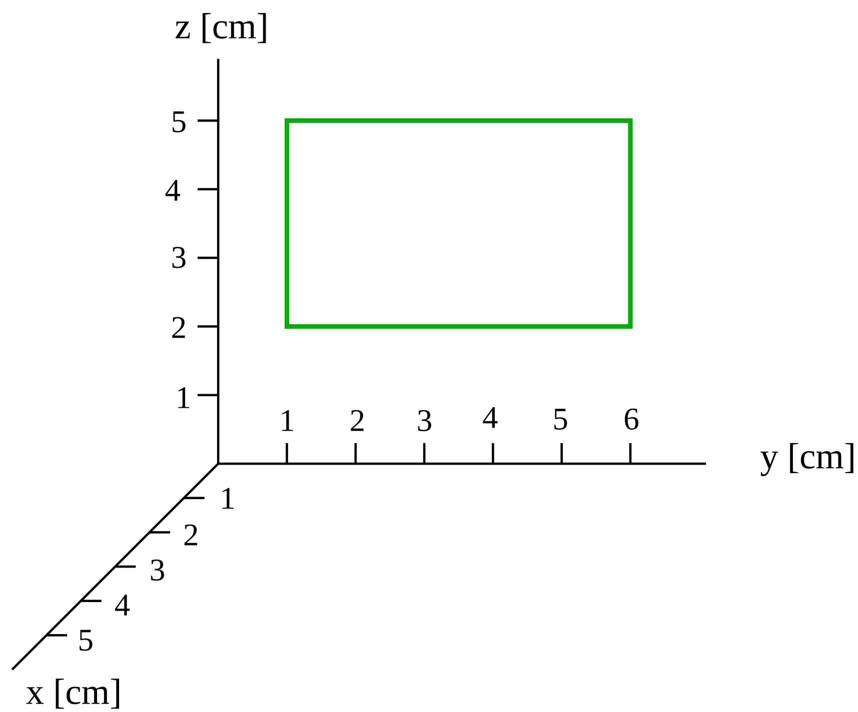

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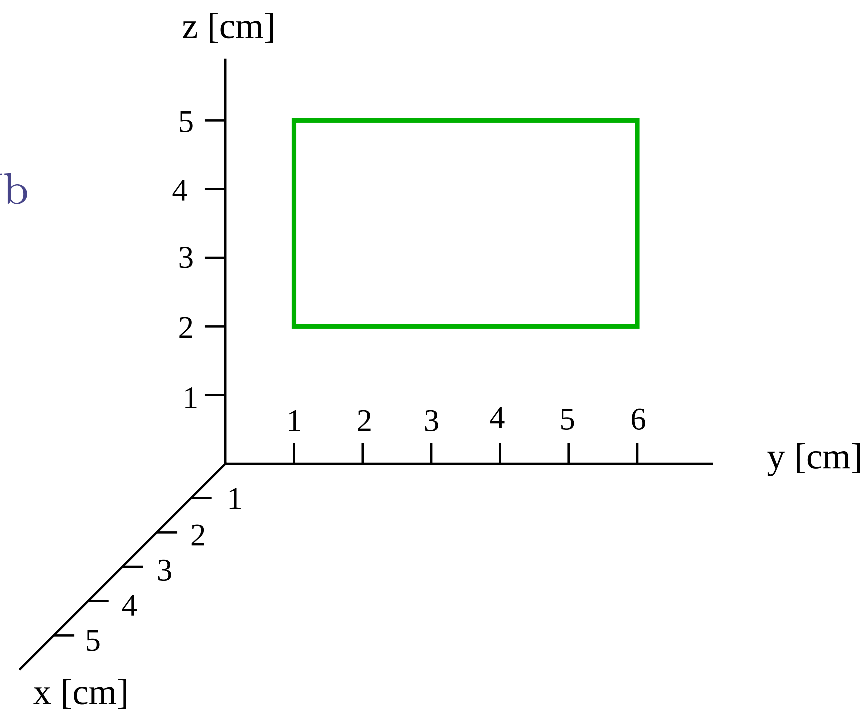
(c) Infer the induced current I from the induced EMF.

Solution:

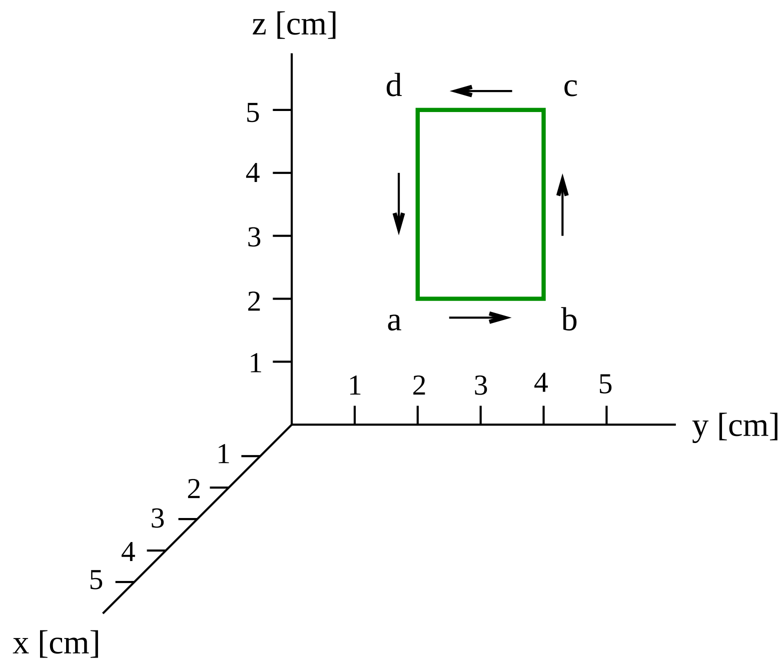
(a) $\Phi_B = \pm(5\text{cm})(3\text{cm})(3\text{T/s})(2\text{s}) = \pm 9.0 \times 10^{-3}\text{Wb}$

(b) $\mathcal{E} = \mp(5\text{cm})(3\text{cm})(3\text{T/s}) = \mp 4.5\text{mV}$ (cw)

(c) $I = \dfrac{4.5\text{mV}}{(1\Omega/\text{cm})(16\text{cm})} = 0.281\text{mA}$



A counterclockwise current $I = 1.7$A [$I = 1.3$A] is flowing through the conducting rectangular frame shown in a region of magnetic field $\mathbf{B} = 6$mT$\hat{\mathbf{j}}$ [$\mathbf{B} = 6$mT$\hat{\mathbf{k}}$].
(a) Find the force $\mathbf{F}_{bc}$ [$\mathbf{F}_{ab}$] (magnitude and direction) acting on side bc [ab] of the rectangle.
(b) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the current loop.



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(a) Find the force $\mathbf{F}_{bc}$ [$\mathbf{F}_{ab}$] (magnitude and direction) acting on side bc [ab] of the rectangle.
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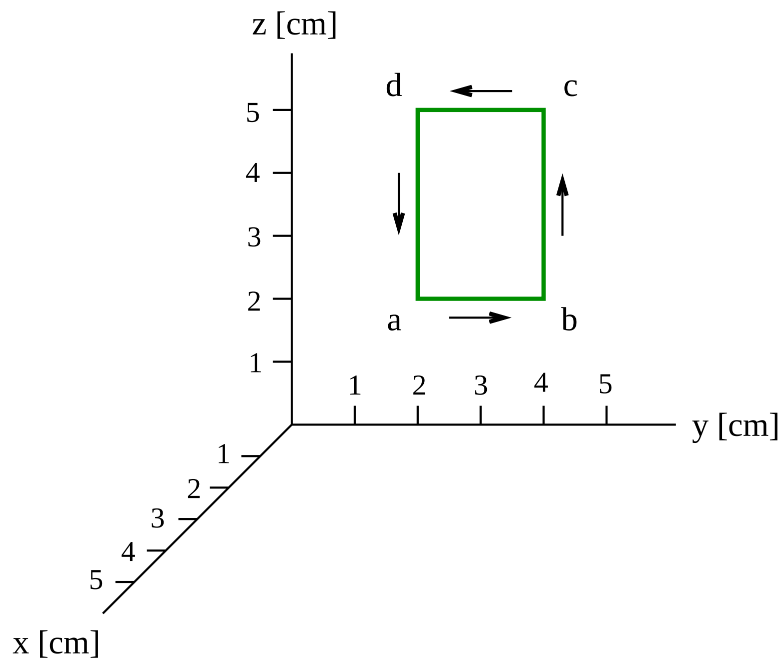


Solution:

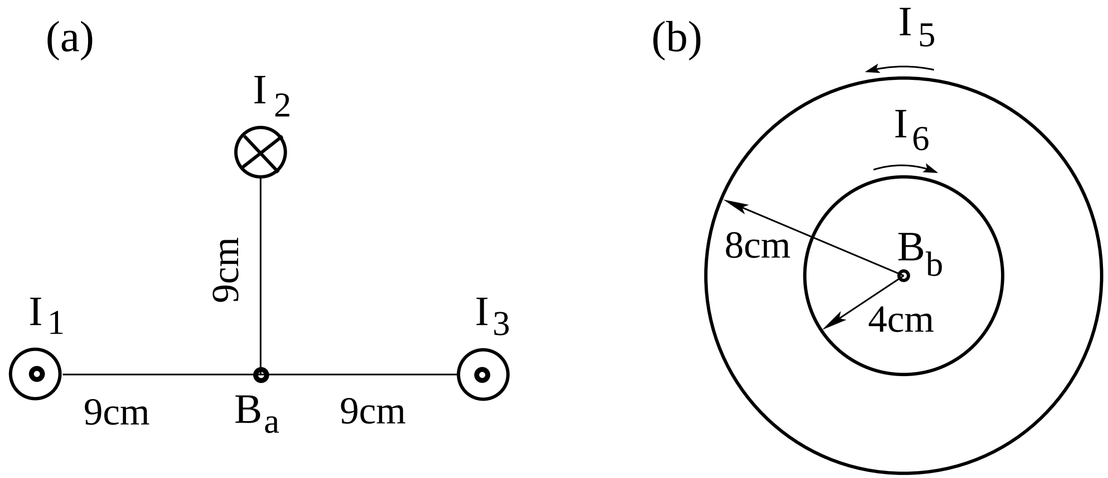
(a) $\mathbf{F}_{bc} = (1.7\text{A})(3\text{cm}\hat{\mathbf{k}}) \times (6\text{mT}\hat{\mathbf{j}}) = -3.06 \times 10^{-4}\text{N}\hat{\mathbf{i}}$
[$\mathbf{F}_{ab} = (1.3\text{A})(2\text{cm}\hat{\mathbf{j}}) \times (6\text{mT}\hat{\mathbf{k}}) = 1.56 \times 10^{-4}\text{N}\hat{\mathbf{i}}$]

(b) $\vec{\mu} = [(2\text{cm})(3\text{cm})\hat{\mathbf{i}}](1.7\text{A}) = 1.02 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}$
[$\vec{\mu} = [(2\text{cm})(3\text{cm})\hat{\mathbf{i}}](1.3\text{A}) = 7.8 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}$]

(c) $\vec{\tau} = (1.02 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}) \times (6\text{mT}\hat{\mathbf{j}}) = 6.12 \times 10^{-6}\text{Nm}\hat{\mathbf{k}}$
[$\vec{\tau} = (7.8 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}) \times (6\text{mT}\hat{\mathbf{k}}) = -4.68 \times 10^{-6}\text{Nm}\hat{\mathbf{j}}$]

Unit Exam III: Problem #2 (Spring ’14)

(a) Find the magnetic field $\mathbf{B}_a$ (magnitude and direction) generated by the three long, straight currents $I_1 = I_2 = I_3 = 1.8$mA [2.7mA]] in the directions shown.
(b) Find the magnetic field $\mathbf{B}_b$ (magnitude and direction) generated by the two circular currents $I_5 = I_6 = 1.5$mA [2.5mA] in the directions shown.



Unit Exam III: Problem #2 (Spring '14)

(a) Find the magnetic field $\mathbf{B}_a$ (magnitude and direction) generated by the three long, straight currents $I_1 = I_2 = I_3 = 1.8$mA [2.7mA]] in the directions shown.
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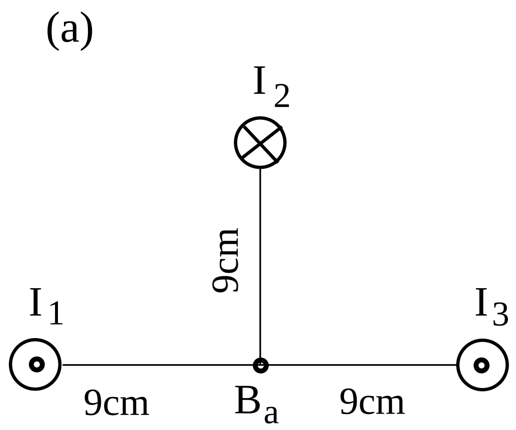
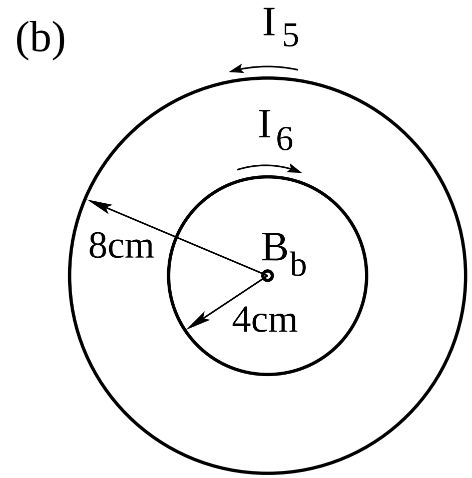




Solution:

(a) $B_a = \dfrac{\mu_0(1.8\text{mA})}{2\pi(9\text{cm})} = 4 \times 10^{-9}\text{T}$ (directed ←)

$[B_a = \dfrac{\mu_0(2.7\text{mA})}{2\pi(9\text{cm})} = 6 \times 10^{-9}\text{T}$ (directed ←)]

(b) $B_b = \dfrac{\mu_0(1.5\text{mA})}{2(4\text{cm})} - \dfrac{\mu_0(1.5\text{mA})}{2(8\text{cm})} = 1.18 \times 10^{-8}\text{T}$ (directed $\otimes$)

$[B_b = \dfrac{\mu_0(2.5\text{mA})}{2(4\text{cm})} - \dfrac{\mu_0(2.5\text{mA})}{2(8\text{cm})} = 1.96 \times 10^{-8}\text{T}$ (directed $\otimes$)]

Unit Exam III: Problem #3 (Spring '14)

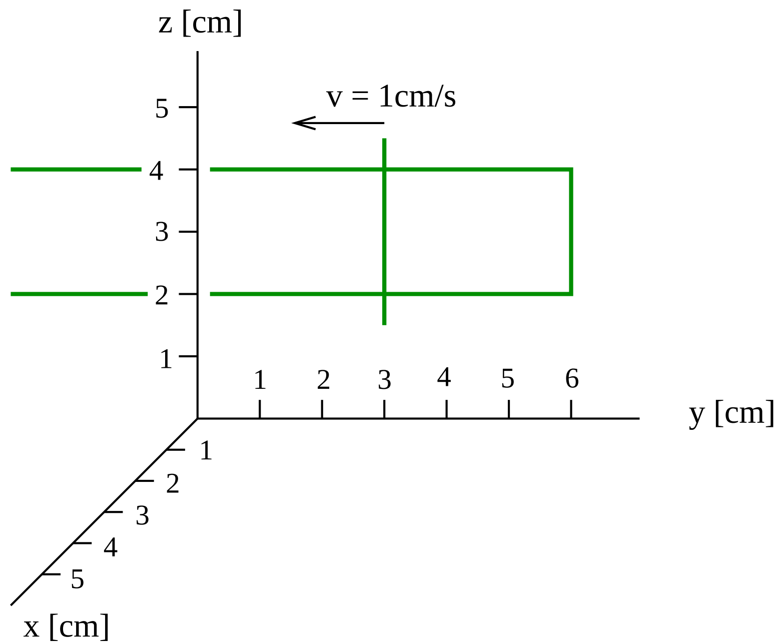
Consider a region of uniform magnetic field $\mathbf{B} = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 1\hat{\mathbf{k}})$mT [$\mathbf{B} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 1\hat{\mathbf{k}})$mT]. A conducting rod slides along conducting rails in the yz-plane as shown. The rails are connected on the right. The clock is set to $t = 0$ at the instant shown.

(a) Find the magnetic flux Φ_B through the conducting loop at $t = 0$.

(b) Find the magnetic flux Φ_B through the conducting loop at $t = 1$s.

(c) Find the induced EMF.

(d) Find the direction (cw/ccw) of the induced current.



Unit Exam III: Problem #3 (Spring ’14)

Consider a region of uniform magnetic field $\mathbf{B} = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 1\hat{\mathbf{k}})$mT [$\mathbf{B} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 1\hat{\mathbf{k}})$mT]. A conducting rod slides along conducting rails in the yz-plane as shown. The rails are connected on the right. The clock is set to $t = 0$ at the instant shown.

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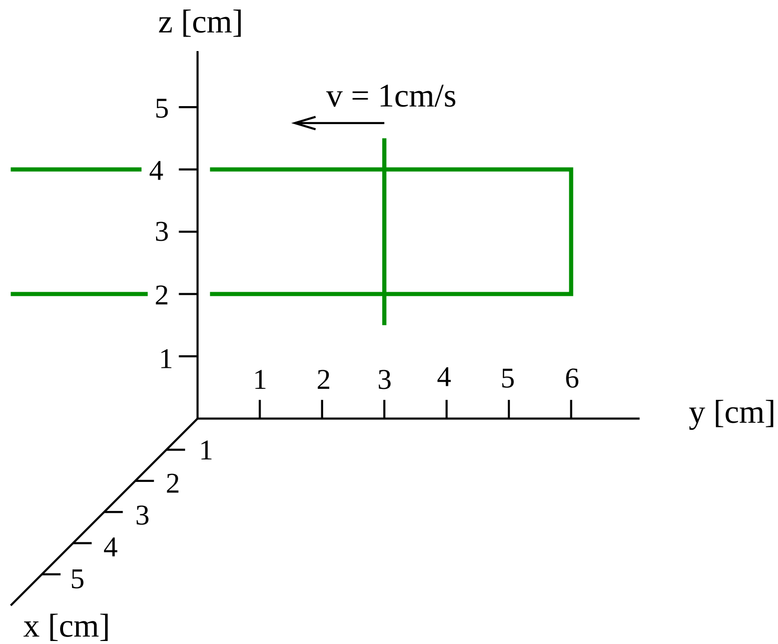


Solution:

(a) $\Phi_B = (3\text{cm})(2\text{cm})(3\text{mT}) = 1.8 \times 10^{-6}\text{Wb}$
$[\Phi_B = (3\text{cm})(2\text{cm})(2\text{mT}) = 1.2 \times 10^{-6}\text{Wb}]$

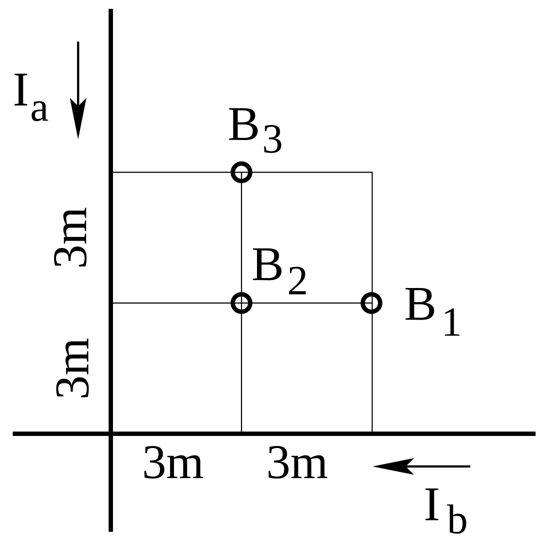
(b) $\Phi_B = (4\text{cm})(2\text{cm})(3\text{mT}) = 2.4 \times 10^{-6}\text{Wb}$
$[\Phi_B = (4\text{cm})(2\text{cm})(2\text{mT}) = 1.6 \times 10^{-6}\text{Wb}]$

(c) $\mathcal{E} = (1\text{cm/s})(3\text{mT})(2\text{cm}) = 6 \times 10^{-7}\text{V}$
$[\mathcal{E} = (1\text{cm/s})(2\text{mT})(2\text{cm}) = 4 \times 10^{-7}\text{V}]$

(d) cw [cw]

Unit Exam III: Problem #1 (Fall ’14)

Consider two infinitely long, straight wires with currents $I_a = 7$A, $I_b = 9$A in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$ at the points marked in the graph.



Unit Exam III: Problem #1 (Fall '14)

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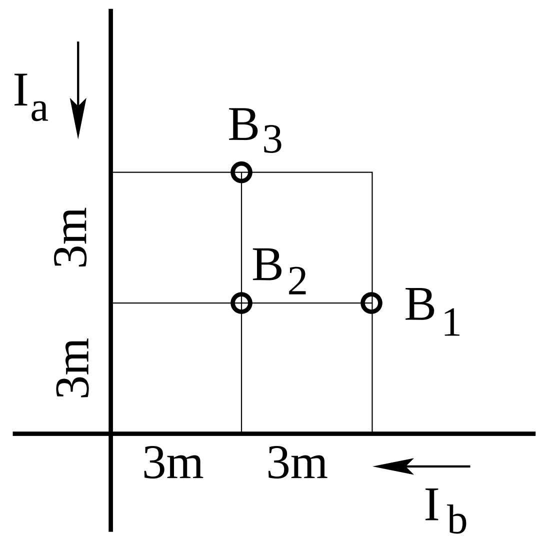
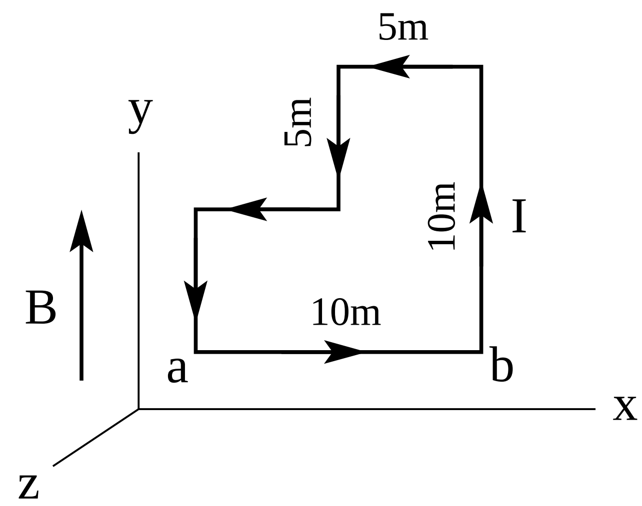


Solution:

- Convention used: out = positive, in = negative
- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{6\text{m}} - \frac{9\text{A}}{3\text{m}}\right) = -0.367\mu\text{T}$ (in).
- $B_2 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{3\text{m}} - \frac{9\text{A}}{3\text{m}}\right) = -0.133\mu\text{T}$ (in).
- $B_3 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{3\text{m}} - \frac{9\text{A}}{6\text{m}}\right) = +0.167\mu\text{T}$ (out).

Unit Exam III: Problem #2 (Fall ’14)

Consider the (piecewise rectangular) conducting loop in the xy-plane as shown with a counterclockwise current $I = 4\text{A}$ in a uniform magnetic field $\vec{B} = 2\text{T}\hat{j}$.
(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
(b) Find the force $\vec{F}$ (magnitude and direction) acting on the side ab of the rectangle.
(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



Consider the (piecewise rectangular) conducting loop in the xy-plane as shown with a counterclockwise current $I = 4$A in a uniform magnetic field $\vec{B} = 2\mathrm{T}\hat{j}$.
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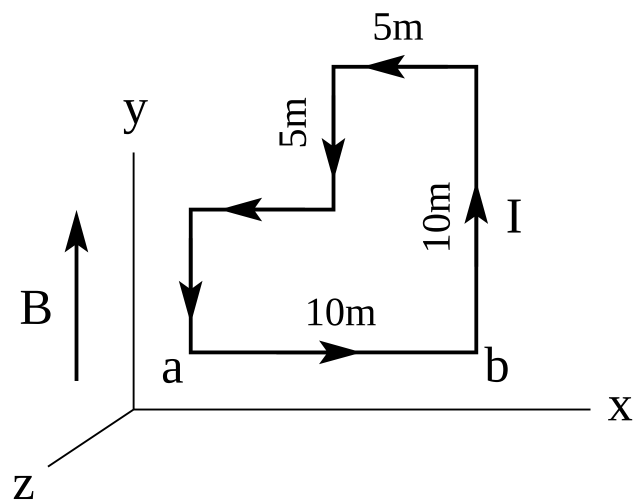


Solution:

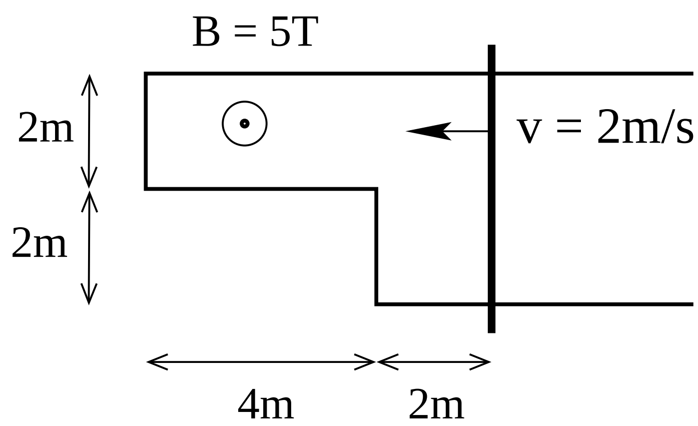
(a) $\vec{\mu} = (4\mathrm{A})(75\mathrm{m}^2)\hat{k} = 300\mathrm{Am}^2\hat{k}.$

(b) $\vec{F} = I\vec{L} \times \vec{B} = (4\mathrm{A})(10\mathrm{m}\hat{i}) \times (2\mathrm{T}\hat{j}) = 80\mathrm{N}\hat{k}.$

(c) $\vec{\tau} = \vec{\mu} \times \vec{B} = (300\mathrm{Am}^2\hat{k}) \times (2\mathrm{T}\hat{j}) = -600\mathrm{Nm}\hat{i}$

Unit Exam III: Problem #3 (Fall ’14)

A conducting frame with a moving conducting rod is located in a uniform magnetic field directed out of the plane as shown. The rod moves at constant velocity.
(a) Find the magnetic flux Φ_B through the frame and the induced emf $\mathcal{E}$ around the frame at the instant shown.
(b) Find the magnetic flux Φ_B through the frame and the induced emf $\mathcal{E}$ around the frame two seconds later.
Write magnitudes only (in SI units), no directions.



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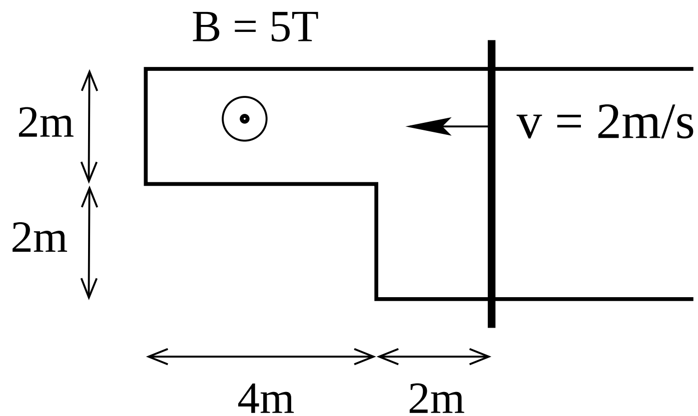


Solution:

(a) $\Phi_B = (16\mathrm{m}^2)(5\mathrm{T}) = 80\mathrm{Wb}, \quad \mathcal{E} = (2\mathrm{m/s})(5\mathrm{T})(4\mathrm{m}) = 40\mathrm{V}.$

(b) $\Phi_B = (4\mathrm{m}^2)(5\mathrm{T}) = 20\mathrm{Wb}, \quad \mathcal{E} = (2\mathrm{m/s})(5\mathrm{T})(2\mathrm{m}) = 20\mathrm{V}.$

Unit Exam III: Problem #1 (Spring ’15)

A clockwise current $I = 2.1$A is flowing around the conducting triangular frame shown in a region of uniform magnetic field $\vec{B} = -3$mT$\hat{\mathbf{j}}$.

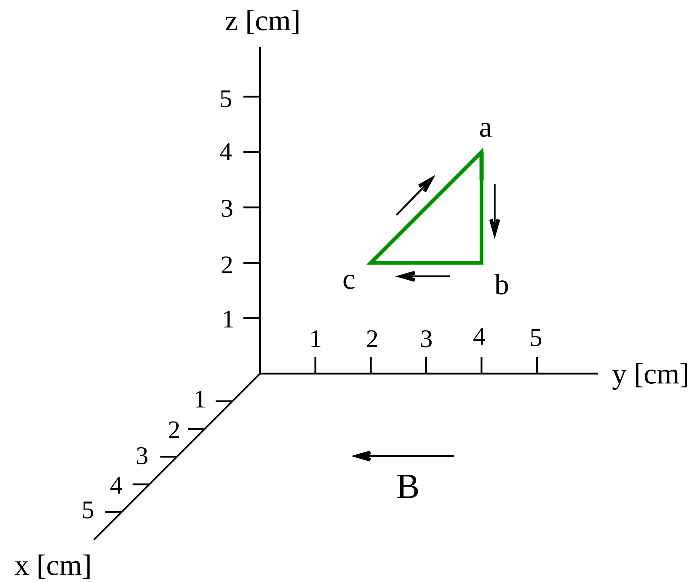
(a) Find the force $\vec{F}_{ab}$ acting on side ab of the triangle.

(b) Find the force $\vec{F}_{bc}$ acting on side bc of the triangle.

(c) Find the magnetic moment $\vec{\mu}$ of the current loop.

(d) Find the torque $\vec{\tau}$ acting on the current loop.

Remember that vectors have components or magnitude and direction.



Unit Exam III: Problem #1 (Spring ’15)

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(c) Find the magnetic moment $\vec{\mu}$ of the current loop.

(d) Find the torque $\vec{\tau}$ acting on the current loop.

Remember that vectors have components or magnitude and direction.

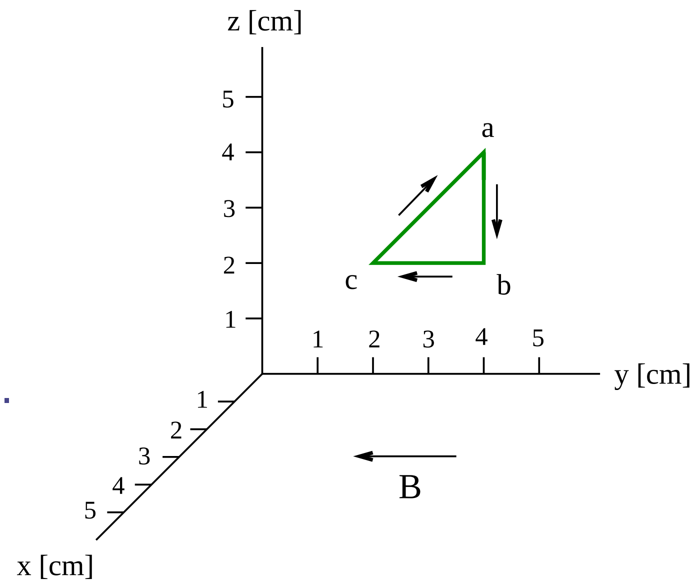
Solution:

(a) $\vec{F}_{ab} = (2.1\text{A})(-2\text{cm}\hat{\mathbf{k}}) \times (-3\text{mT}\hat{\mathbf{j}}) = -1.26 \times 10^{-4}\text{N}\hat{\mathbf{i}}.$

(b) $\vec{F}_{bc} = 0.$

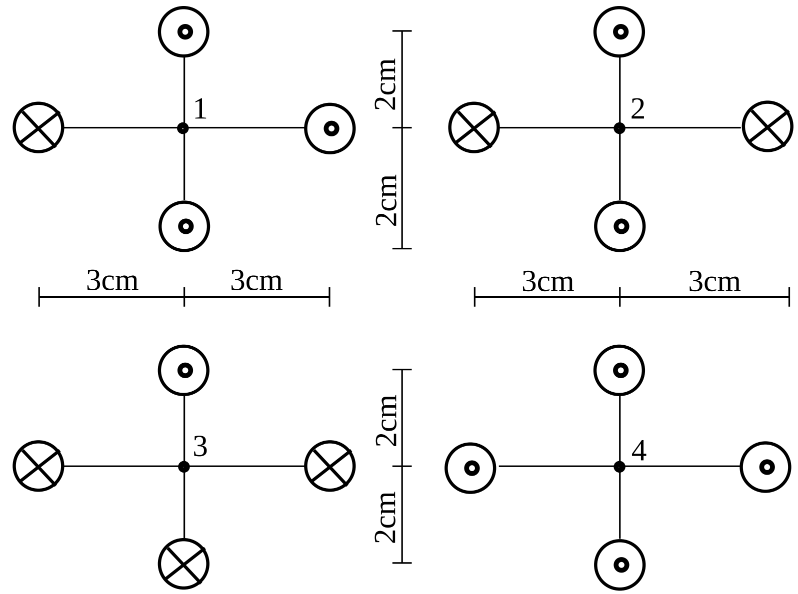
(c) $\vec{\mu} = \left[-\frac{1}{2}(2\text{cm})(2\text{cm})\hat{\mathbf{i}}\right](2.1\text{A}) = -4.2 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}.$

(d) $\vec{\tau} = (-4.2 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}) \times (-3\text{mT}\hat{\mathbf{j}}) = 1.26 \times 10^{-6}\text{Nm}\hat{\mathbf{k}}.$



Unit Exam III: Problem #2 (Spring ’15)

Consider four long, straight currents in four different configurations. All currents are $I = 4$mA in the directions shown ($\otimes =$ in, $\odot =$ out). Find the magnitude (in SI units) and the direction ($\leftarrow, \rightarrow, \uparrow, \downarrow$) of the magnetic fields $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{B}_4$ generated at the points $1, \ldots, 4$, respectively.



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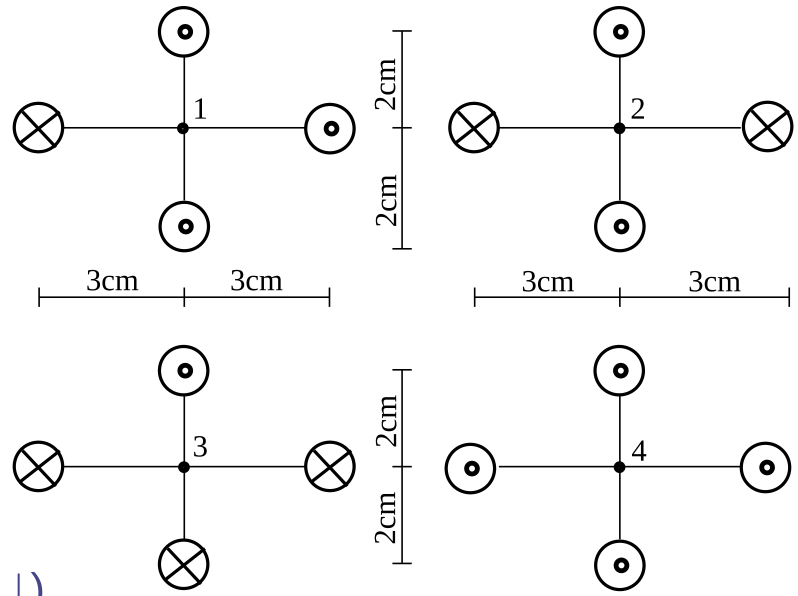


Solution:

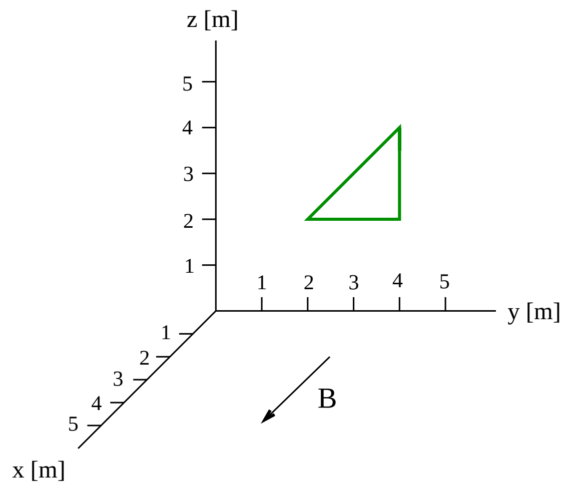
- $B_1 = 2\,\dfrac{\mu_0(4\text{mA})}{2\pi(3\text{cm})} = 5.33 \times 10^{-8}\text{T}$ (directed $\downarrow$).
- $B_2 = 0$ (no direction).
- $B_3 = 2\,\dfrac{\mu_0(4\text{mA})}{2\pi(2\text{cm})} = 8.00 \times 10^{-8}\text{T}$ (directed $\rightarrow$).
- $B_4 = 0$ (no direction).

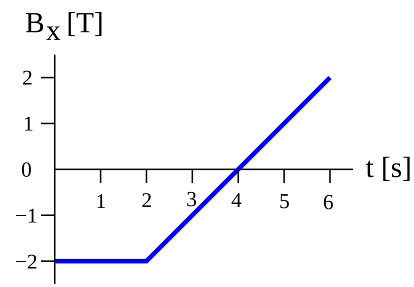
Unit Exam III: Problem #3 (Spring ’15)

A wire shaped into a triangle has resistance $R = 3.5\Omega$ and is placed in the yz-plane as shown. A uniform time-dependent magnetic field $\mathbf{B} = B_x(t)\hat{\mathbf{i}}$ is present. The dependence of B_x on time is shown graphically.

(a) Find magnitude $|\Phi_B^{(1)}|$ and $|\Phi_B^{(4)}|$ of the magnetic flux through the triangle at times $t = 1$s and $t = 4$s, respectively.

(b) Find magnitude I_1, I_4 and direction (cw/ccw) of the induced current at times $t = 1$s and $t = 4$s, respectively.





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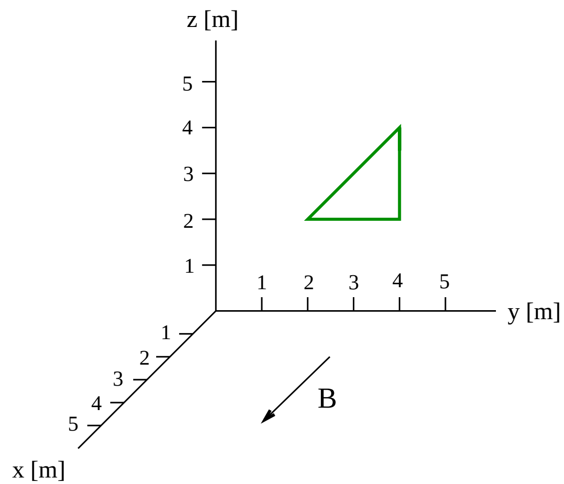
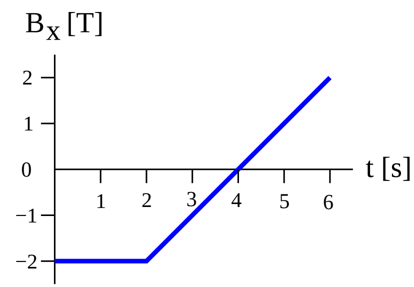




Solution:

(a) $|\Phi_B^{(1)}| = |(2\text{m}^2)(-2\text{T})| = 4.0\,\text{Wb},$

$|\Phi_B^{(4)}| = |(2\text{m}^2)(0\text{T}) = 0.$

(b) $$\left|\frac{d\Phi_B^{(1)}}{dt}\right| = \left|A\frac{dB}{dt}\right| = |(2\text{m}^2)(0\text{T/s}) = 0$$

$\Rightarrow\ I_1 = 0,$

$$\left|\frac{d\Phi_B^{(4)}}{dt}\right| = \left|A\frac{dB}{dt}\right| = |(2\text{m}^2)(1\text{T/s})| = 2.0\text{V}$$

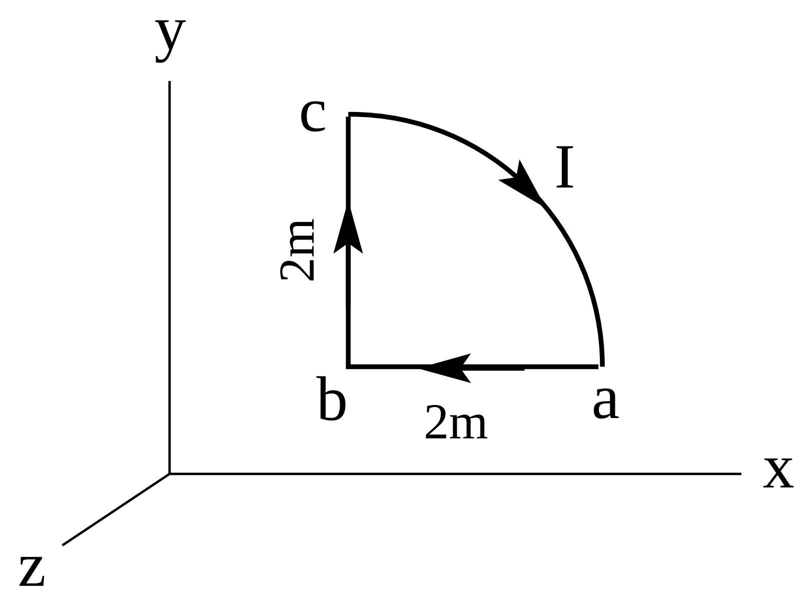
$\Rightarrow\ I_4 = \dfrac{2.0\text{V}}{3.5\Omega} = 0.571\text{A}$ (cw).

Consider a region with uniform magnetic field (i) $\vec{B} = 5\mathrm{T}\hat{j}$, (ii) $\vec{B} = -6\mathrm{T}\hat{i}$. A conducting loop in the xy-plane has the shape of a quarter circle with a clockwise current (i) $I = 4$A, (ii) $I = 3$A.

(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.

(b) Find the force $\vec{F}$ (magnitude and direction) acting on the side (i) ab, (ii) bc of the loop.

(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



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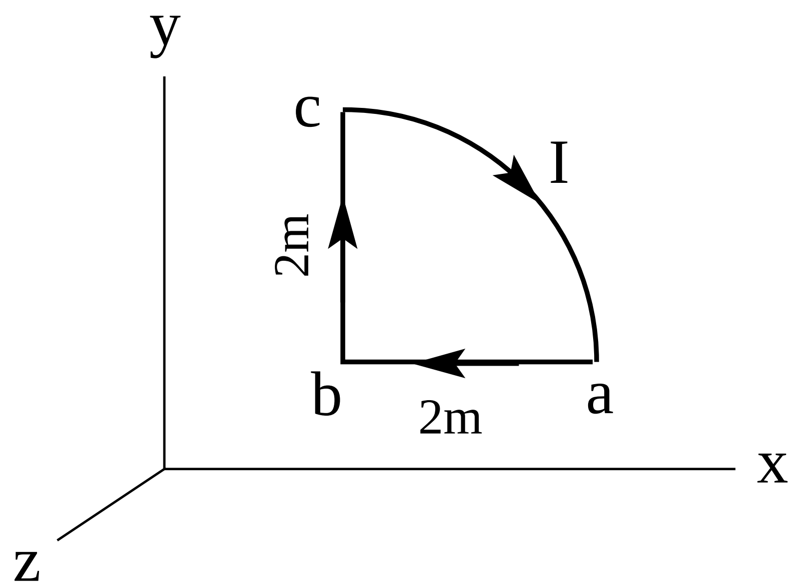


Solution:

(ia) $\vec{\mu} = (4\mathrm{A})(3.14\mathrm{m}^2)(-\hat{k}) = -12.6\mathrm{Am}^2\hat{k}.$

(ib) $\vec{F}_{ab} = (4\mathrm{A})(-2\mathrm{m}\hat{i}) \times (5\mathrm{T}\hat{j}) = -40\mathrm{N}\hat{k}.$

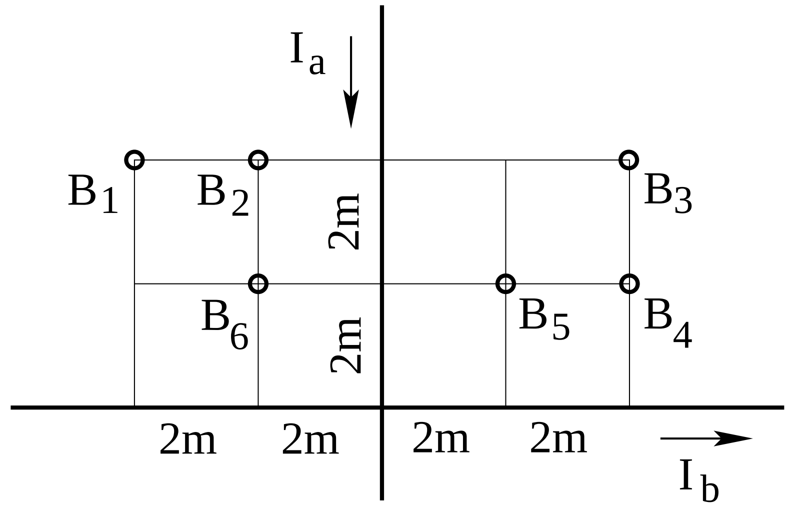
(ic) $\vec{\tau} = (-12.6\mathrm{Am}^2\hat{k}) \times (5\mathrm{T}\hat{j}) = 63.0\mathrm{Nm}\hat{i}$

(iia) $\vec{\mu} = (3\mathrm{A})(3.14\mathrm{m}^2)(-\hat{k}) = -9.42\mathrm{Am}^2\hat{k}.$

(iib) $\vec{F}_{bc} = (3\mathrm{A})(2\mathrm{m}\hat{j}) \times (-6\mathrm{T}\hat{i}) = 36\mathrm{N}\hat{k}.$

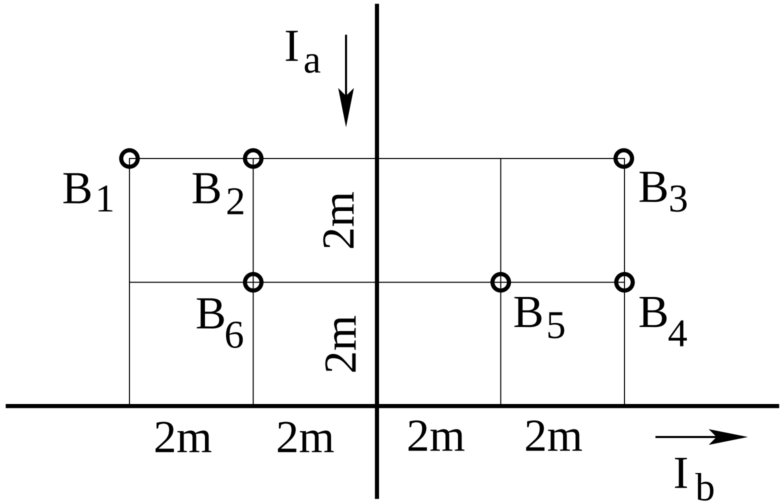
(iic) $\vec{\tau} = (-9.42\mathrm{Am}^2\hat{k}) \times (-6\mathrm{T}\hat{i}) = 56.5\mathrm{Nm}\hat{j}$

Consider two infinitely long, straight wires with currents of equal magnitude $I_a = I_b = 6$A in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, ..., $\mathbf{B}_6$ at the points marked in the graph.



Unit Exam III: Problem #2 (Fall ’15)

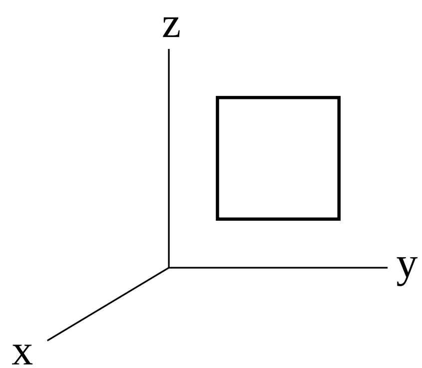
Consider two infinitely long, straight wires with currents of equal magnitude $I_a = I_b = 6\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, ..., $\mathbf{B}_6$ at the points marked in the graph.

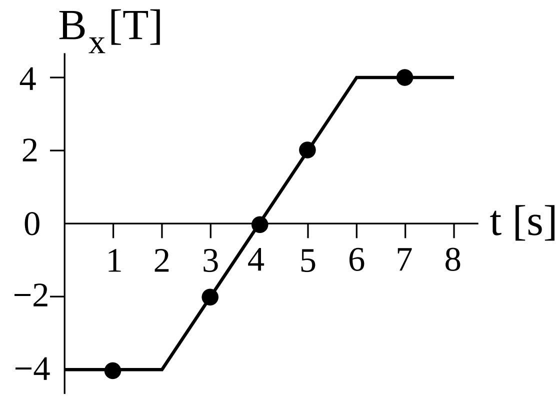


Solution:

- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} - \frac{6\text{A}}{4\text{m}}\right) = 0$ (no direction).
- $B_2 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} - \frac{6\text{A}}{2\text{m}}\right) = -0.3\mu\text{T}$ (into plane).
- $B_3 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{4\text{m}} + \frac{6\text{A}}{4\text{m}}\right) = +0.6\mu\text{T}$ (out of plane).
- $B_4 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{2\text{m}} + \frac{6\text{A}}{4\text{m}}\right) = 0.9\mu\text{T}$ (out of plane).
- $B_5 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{2\text{m}} + \frac{6\text{A}}{2\text{m}}\right) = 1.2\mu\text{T}$ (out of plane).
- $B_6 = \frac{\mu_0}{2\pi}\left(\frac{6\text{A}}{2\text{m}} - \frac{6\text{A}}{2\text{m}}\right) = 0$ (no direction).

A conducting wire bent into a square of side (i) 1.2m, (ii) 1.3m is placed in the yz-plane. The time-dependence of the magnetic field $\mathbf{B}(t) = B_x(t)\hat{\mathbf{i}}$ is shown graphically.
(a) Find the magnitude $|\Phi_B|$ of the magnetic flux through the square at times (i) $t = 1$s, $t = 3$s, and $t = 4$s, (ii) $t = 4$s, $t = 5$s, and $t = 7$s .
(b) Find the magnitude $|\mathcal{E}|$ of the induced EMF at the above times.
(c) Find the direction (cw, ccw, zero) of the induced current at the above times.

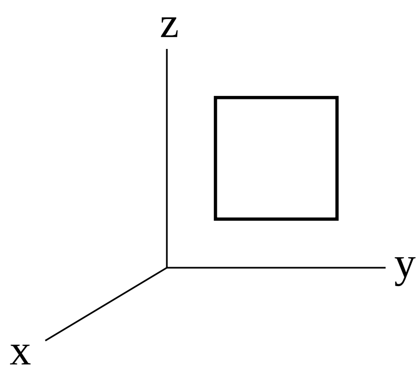


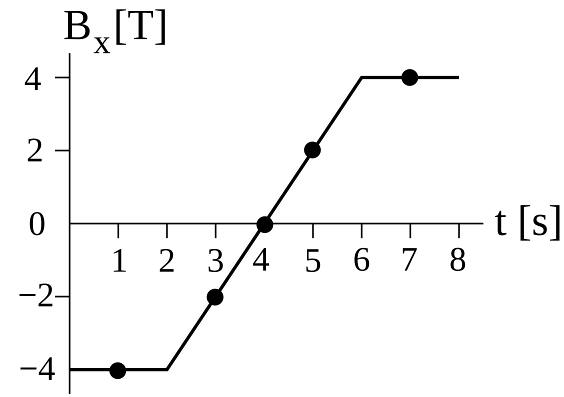


Solution:

Unit Exam III: Problem #3 (Fall ’15)

A conducting wire bent into a square of side (i) 1.2m, (ii) 1.3m is placed in the yz-plane. The time-dependence of the magnetic field $\mathbf{B}(t) = B_x(t)\hat{\mathbf{i}}$ is shown graphically.
(a) Find the magnitude $|\Phi_B|$ of the magnetic flux through the square at times (i) $t = 1$s, $t = 3$s, and $t = 4$s, (ii) $t = 4$s, $t = 5$s, and $t = 7$s .
(b) Find the magnitude $|\mathcal{E}|$ of the induced EMF at the above times.
(c) Find the direction (cw, ccw, zero) of the induced current at the above times.





Solution:

(ia) $|\Phi_B^{(1)}| = (1.44\text{m}^2)(4\text{T}) = 5.76\,\text{Wb}$

$|\Phi_B^{(3)}| = (1.44\text{m}^2)(2\text{T}) = 2.88\,\text{Wb}$

$|\Phi_B^{(4)}| = (1.44\text{m}^2)(0\text{T}) = 0$

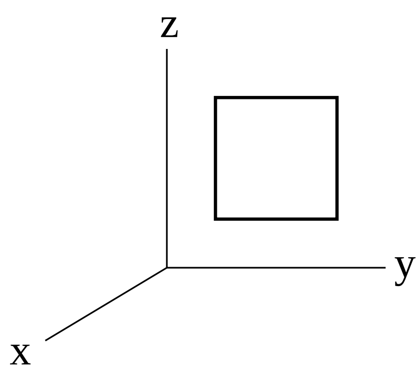
(ib) $\mathcal{E}^{(1)} = (1.44\text{m}^2)(0\text{T/s}) = 0$

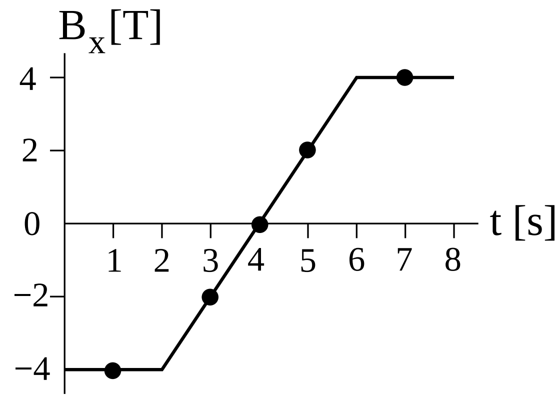
$\mathcal{E}^{(3)} = (1.44\text{m}^2)(2\text{T/s}) = 2.88\text{V}$

$\mathcal{E}^{(4)} = (1.44\text{m}^2)(2\text{T/s}) = 2.88\text{V}$

(ic) zero, cw, cw

A conducting wire bent into a square of side (i) 1.2m, (ii) 1.3m is placed in the yz-plane. The time-dependence of the magnetic field $\mathbf{B}(t) = B_x(t)\hat{\mathbf{i}}$ is shown graphically.
(a) Find the magnitude $|\Phi_B|$ of the magnetic flux through the square at times (i) $t = 1$s, $t = 3$s, and $t = 4$s, (ii) $t = 4$s, $t = 5$s, and $t = 7$s .
(b) Find the magnitude $|\mathcal{E}|$ of the induced EMF at the above times.
(c) Find the direction (cw, ccw, zero) of the induced current at the above times.





Solution:

(iia) $|\Phi_B^{(4)}| = (1.69\text{m}^2)(0\text{T}) = 0$

$|\Phi_B^{(5)}| = (1.69\text{m}^2)(2\text{T}) = 3.38\,\text{Wb}$

$|\Phi_B^{(7)}| = (1.69\text{m}^2)(4\text{T}) = 6.76\,\text{Wb}$

(iib) $\mathcal{E}^{(4)} = (1.69\text{m}^2)(2\text{T/s}) = 3.38\text{V}$

$\mathcal{E}^{(5)} = (1.69\text{m}^2)(2\text{T/s}) = 3.38\text{V}$

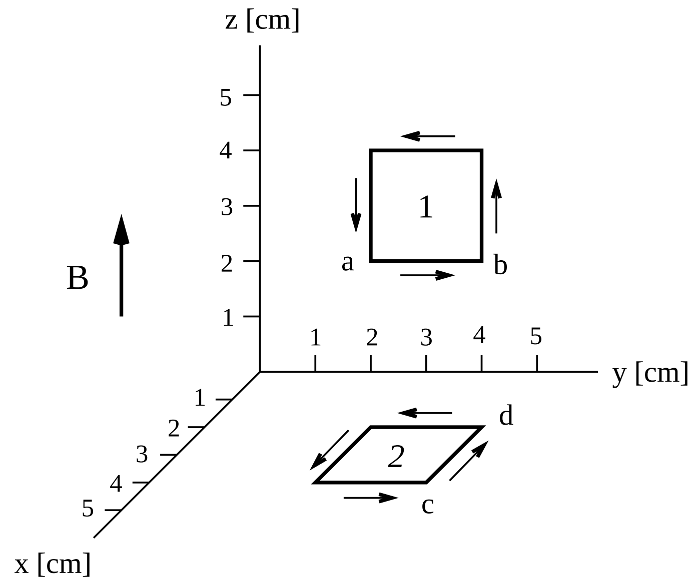
$\mathcal{E}^{(7)} = (1.69\text{m}^2)(0\text{T/s}) = 0$

(iic) cw, cw, zero

Unit Exam III: Problem #1 (Spring ’16)

Conducting squares 1 and 2, each of side 2cm, are positioned as shown. A current $I = 3$A is flowing around each square in the direction shown. A uniform magnetic field $\vec{B} = 5$mT$\hat{\mathbf{k}}$ exists in the entire region.

(a) Find the forces $\vec{F}_{ab}$ and $\vec{F}_{cd}$ acting on sides ab and cd, respectively.
(b) Find the magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ of squares 1 and 2, respectively.
(c) Find the torques $\vec{\tau}_1$ and $\vec{\tau}_2$ acting on squares 1 and 2, respectively.
Remember that vectors have components or magnitude and direction.



Unit Exam III: Problem #1 (Spring ’16)

Conducting squares 1 and 2, each of side 2cm, are positioned as shown. A current $I = 3\text{A}$ is flowing around each square in the direction shown. A uniform magnetic field $\vec{B} = 5\text{mT}\hat{\mathbf{k}}$ exists in the entire region.

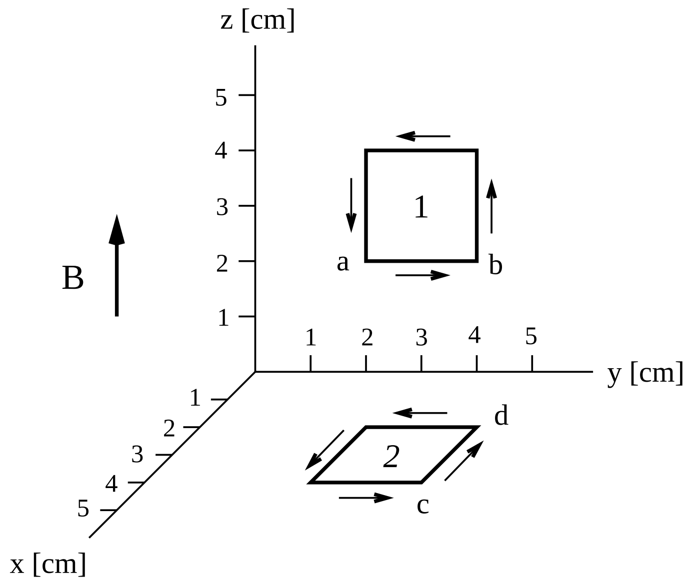
(a) Find the forces $\vec{F}_{ab}$ and $\vec{F}_{cd}$ acting on sides ab and cd, respectively.
(b) Find the magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ of squares 1 and 2, respectively.
(c) Find the torques $\vec{\tau}_1$ and $\vec{\tau}_2$ acting on squares 1 and 2, respectively.
Remember that vectors have components or magnitude and direction.

Solution:

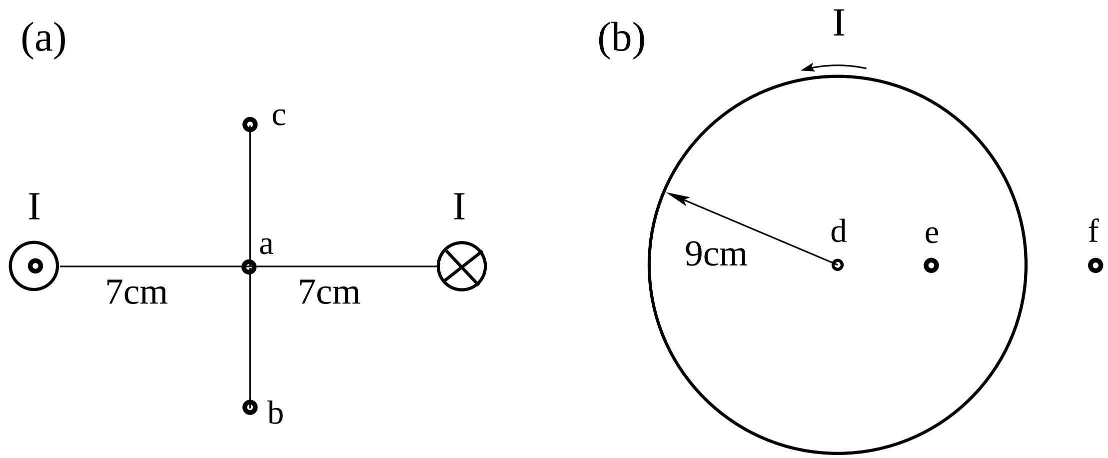
(a) $\vec{F}_{ab} = (3\text{A})(2\text{cm}\hat{\mathbf{j}}) \times (5\text{mT}\hat{\mathbf{k}}) = 3 \times 10^{-4}\text{N}\hat{\mathbf{i}}.$
$\vec{F}_{cd} = (3\text{A})(-2\text{cm}\hat{\mathbf{i}}) \times (5\text{mT}\hat{\mathbf{k}}) = 3 \times 10^{-4}\text{N}\hat{\mathbf{j}}.$

(b) $\vec{\mu}_1 = (2\text{cm})^2(3\text{A})\hat{\mathbf{i}} = 1.2 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}.$
$\vec{\mu}_2 = (2\text{cm})^2(3\text{A})\hat{\mathbf{k}} = 1.2 \times 10^{-3}\text{Am}^2\hat{\mathbf{k}}.$

(d) $\vec{\tau}_1 = (1.2 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}) \times (5\text{mT}\hat{\mathbf{k}}) = -6 \times 10^{-6}\text{Nm}\hat{\mathbf{j}}.$
$\vec{\tau}_2 = (1.2 \times 10^{-3}\text{Am}^2\hat{\mathbf{k}}) \times (5\text{mT}\hat{\mathbf{k}}) = 0.$



(a) Consider two long, straight currents $I = 3$mA in the directions shown. Find the magnitude of the magnetic field at point a. Find the directions ($\leftarrow, \rightarrow, \uparrow, \downarrow$) of the magnetic field at points b and c.
(b) Consider a circular current $I = 3$mA in the direction shown. Find the magnitude of the magnetic field at point d. Find the directions ($\otimes, \odot$) of the magnetic field at points e and f.



(a) Consider two long, straight currents $I = 3\text{mA}$ in the directions shown. Find the magnitude of the magnetic field at point a. Find the directions ($\leftarrow, \rightarrow, \uparrow, \downarrow$) of the magnetic field at points b and c.
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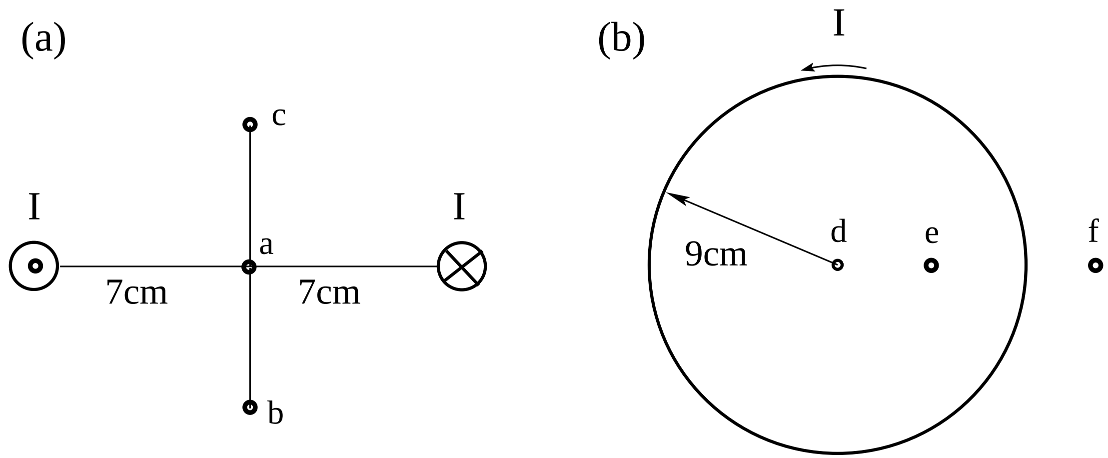


Solution:

(a) $B_a = 2\dfrac{\mu_0(3\text{mA})}{2\pi(7\text{cm})} = 1.71 \times 10^{-8}\text{T} \quad B_b \uparrow, \quad B_c \uparrow.$

(b) $B_d = \dfrac{\mu_0(3\text{mA})}{2(9\text{cm})} = 2.09 \times 10^{-8}\text{T}, \quad B_e \odot, \quad B_f \otimes.$

A wire shaped into a rectangular loop as shown is placed in the yz-plane. A uniform time-dependent magnetic field $\mathbf{B} = B_x(t)\hat{\mathbf{i}}$ is present. The dependence of B_x on time is shown graphically.

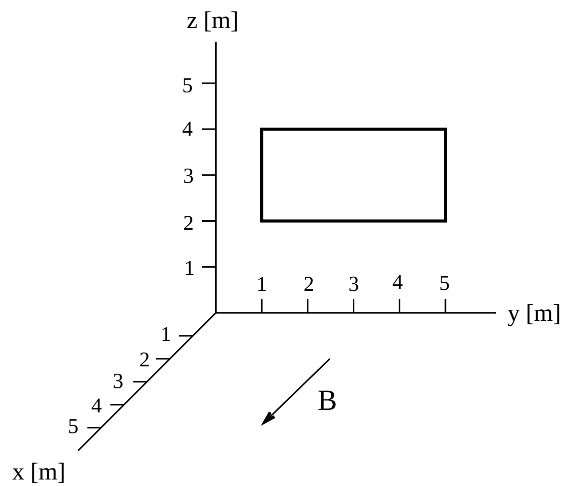
(a) Find magnitude $|\Phi_B^{(2)}|$ of the magnetic flux through the loop at time $t = 2$s.

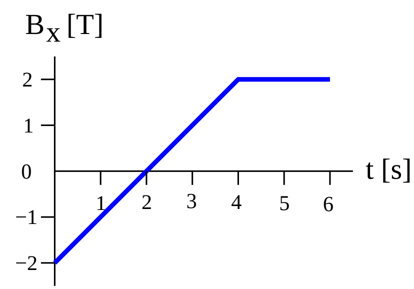
(b) Find magnitude $|\Phi_B^{(5)}|$ of the magnetic flux through the loop at time $t = 5$s.

(c) Find magnitude $|\mathcal{E}^{(2)}|$ of the induced EMF at time $t = 2$s.

(d) Find magnitude $|\mathcal{E}^{(5)}|$ of the induced EMF at time $t = 5$s.

(e) Find the direction (cw/ccw) and magnitude I of the induced current at time $t = 2$s if the wire has resistance 1Ω per meter of length.





Unit Exam III: Problem #3 (Spring '16)

A wire shaped into a rectangular loop as shown is placed in the yz-plane. A uniform time-dependent magnetic field $\mathbf{B} = B_x(t)\hat{\mathbf{i}}$ is present. The dependence of B_x on time is shown graphically.

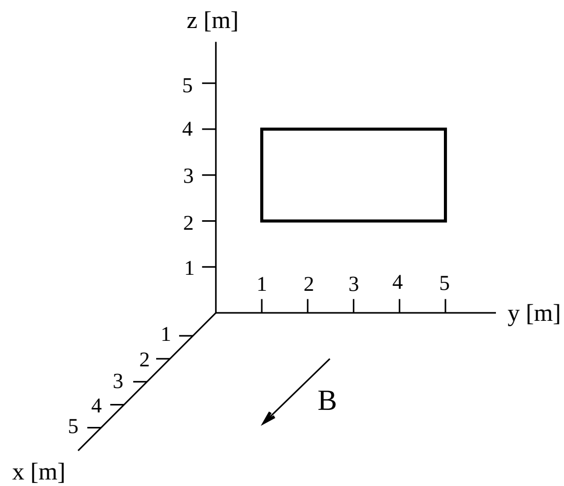
(a) Find magnitude $|\Phi_B^{(2)}|$ of the magnetic flux through the loop at time $t = 2$s.

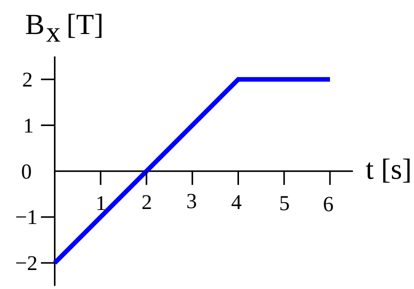
(b) Find magnitude $|\Phi_B^{(5)}|$ of the magnetic flux through the loop at time $t = 5$s.

(c) Find magnitude $|\mathcal{E}^{(2)}|$ of the induced EMF at time $t = 2$s.

(d) Find magnitude $|\mathcal{E}^{(5)}|$ of the induced EMF at time $t = 5$s.

(e) Find the direction (cw/ccw) and magnitude I of the induced current at time $t = 2$s if the wire has resistance 1Ω per meter of length.





Solution:

(a) $|\Phi_B^{(2)}| = |(8\text{m}^2)(0\text{T})| = 0,$

(b) $|\Phi_B^{(5)}| = |(8\text{m}^2)(2\text{T})| = 16\,\text{Wb},$

(c) $|\mathcal{E}^{(2)}| = \left|A\frac{dB}{dt}\right| = |(8\text{m}^2)(1\text{T/s}) = 8\text{V}$

(d) $|\mathcal{E}^{(5)}| = \left|A\frac{dB}{dt}\right| = |(8\text{m}^2)(0\text{T/s}) = 0$

(e) $I^{(2)} = \frac{8\text{V}}{12\Omega} = 0.667\text{A}.$ (cw).

Unit Exam III: Problem #1 (Fall '16)

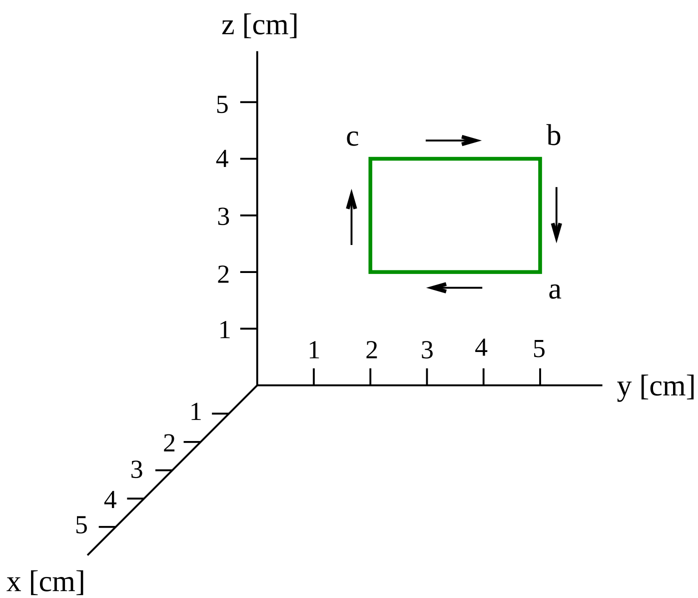
A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field $\mathbf{B}$.

(a) Find the force $\mathbf{F}_{ab}$ (magnitude and direction) acting on side ab.

(b) Find the force $\mathbf{F}_{bc}$ (magnitude and direction) acting on side bc.

(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.

(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.



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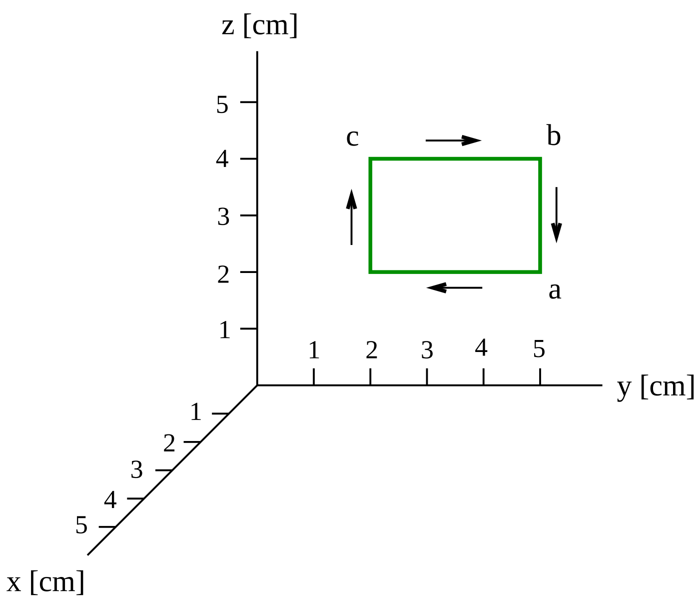
(a) Find the force $\mathbf{F}_{ab}$ (magnitude and direction) acting on side ab.
(b) Find the force $\mathbf{F}_{bc}$ (magnitude and direction) acting on side bc.
(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.



Solution for $I = 1.2\text{A}, \quad \mathbf{B} = 0.7\text{mT}\hat{\mathbf{k}}$:

Unit Exam III: Problem #1 (Fall '16)

A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field $\mathbf{B}$.

(a) Find the force $\mathbf{F}_{ab}$ (magnitude and direction) acting on side ab.

(b) Find the force $\mathbf{F}_{bc}$ (magnitude and direction) acting on side bc.

(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.

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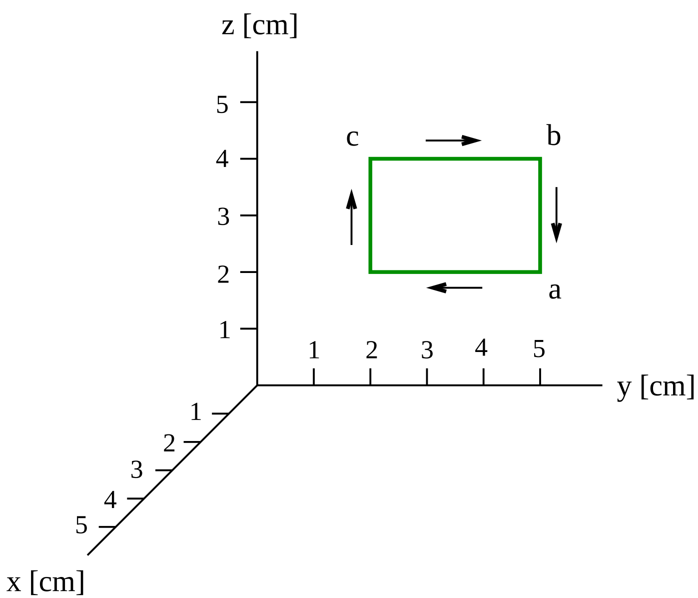


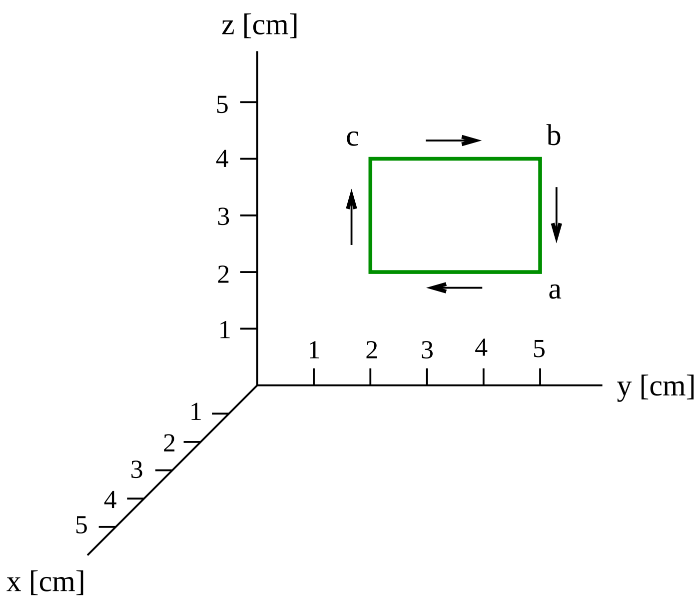
(a) $\mathbf{F}_{ab} = (1.2\text{A})(-2\text{cm}\hat{\mathbf{k}}) \times (0.7\text{mT}\hat{\mathbf{k}}) = 0.$

(b) $\mathbf{F}_{bc} = (1.2\text{A})(3\text{cm}\hat{\mathbf{j}}) \times (0.7\text{mT}\hat{\mathbf{k}}) = 2.52 \times 10^{-5}\text{N}\hat{\mathbf{i}}.$

(c) $\vec{\mu} = (2\text{cm})(3\text{cm})(1.2\text{A})(-\hat{\mathbf{i}}) = -7.2 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}.$

(d) $\vec{\tau} = (-7.2 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}) \times (0.7\text{mT}\hat{\mathbf{k}}) = 5.04 \times 10^{-7}\text{Nm}\hat{\mathbf{j}}.$

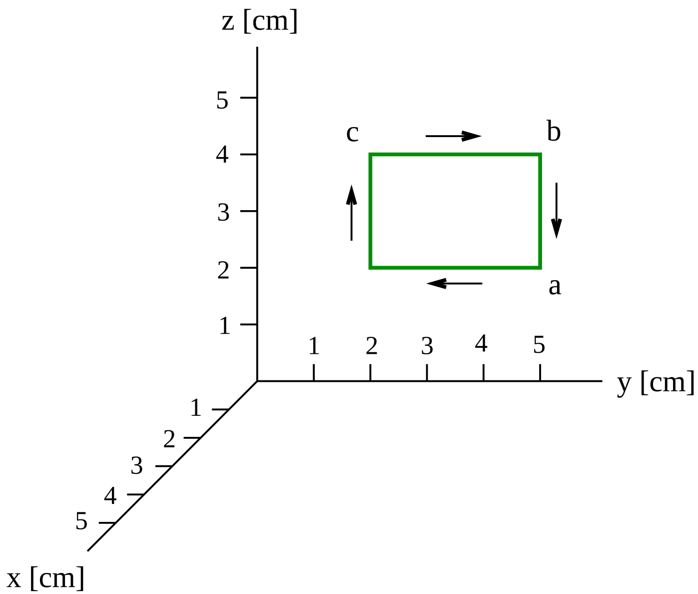
A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field $\mathbf{B}$.
(a) Find the force $\mathbf{F}_{ab}$ (magnitude and direction) acting on side ab.
(b) Find the force $\mathbf{F}_{bc}$ (magnitude and direction) acting on side bc.
(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.



Solution for $I = 2.1\text{A}, \quad \mathbf{B} = 0.8\text{mT}\hat{\mathbf{j}}$

Unit Exam III: Problem #1 (Fall ’16)

A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field $\mathbf{B}$.
(a) Find the force $\mathbf{F}_{ab}$ (magnitude and direction) acting on side ab.
(b) Find the force $\mathbf{F}_{bc}$ (magnitude and direction) acting on side bc.
(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.



(a) $\mathbf{F}_{ab} = (2.1\text{A})(-2\text{cm}\hat{\mathbf{k}}) \times (0.8\text{mT}\hat{\mathbf{j}}) = 3.36 \times 10^{-5}\text{N}\hat{\mathbf{i}}.$

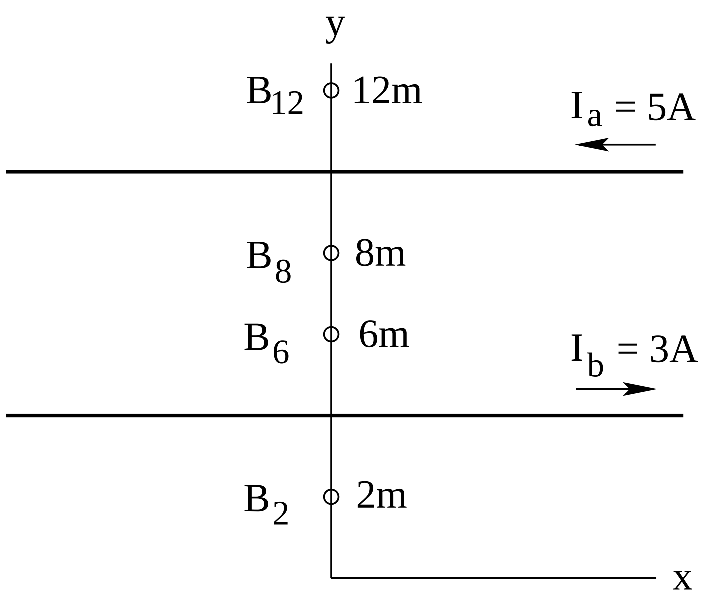
(b) $\mathbf{F}_{bc} = (2.1\text{A})(3\text{cm}\hat{\mathbf{j}}) \times (0.8\text{mT}\hat{\mathbf{j}}) = 0.$

(c) $\vec{\mu} = (2\text{cm})(3\text{cm})(2.1\text{A})(-\hat{\mathbf{i}}) = -1.26 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}.$

(d) $\vec{\tau} = (-1.26 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}) \times (0.8\text{mT}\hat{\mathbf{j}}) = -1.01 \times 10^{-6}\text{Nm}\hat{\mathbf{k}}.$

Unit Exam III: Problem #2 (Fall ’16)

Two infinitely long, straight wires at positions $y = 10\text{m}$ and $y = 4\text{m}$ carry currents I_a and I_b, respectively. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_{12}$, $\mathbf{B}_8$, $\mathbf{B}_6$, and $\mathbf{B}_2$ at the points marked in the graph.



Unit Exam III: Problem #2 (Fall '16)

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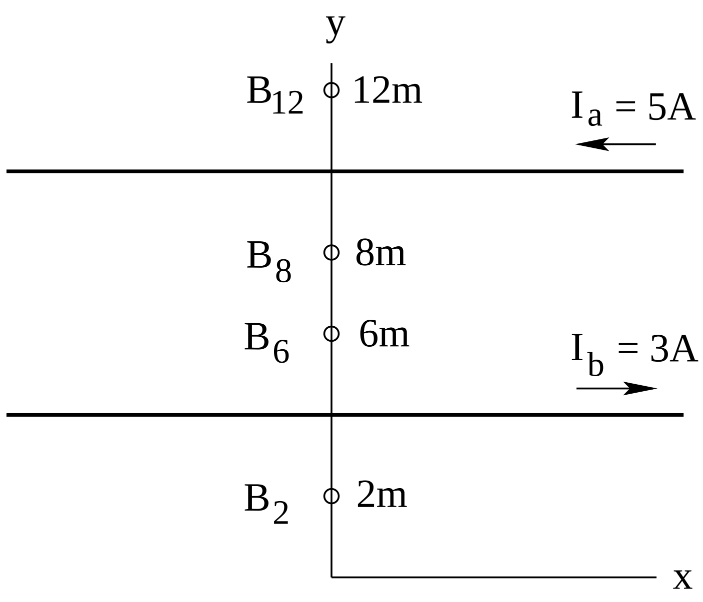
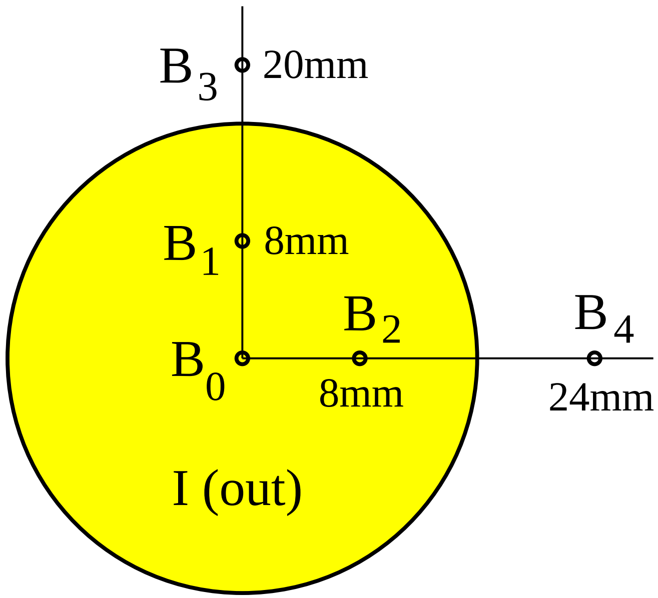


Solution:

- $B_{12} = \frac{\mu_0}{2\pi}\left(-\frac{5\text{A}}{2\text{m}} + \frac{3\text{A}}{8\text{m}}\right) = -4.25 \times 10^{-7}\text{T}$ (in).
- $B_8 = \frac{\mu_0}{2\pi}\left(\frac{5\text{A}}{2\text{m}} + \frac{3\text{A}}{4\text{m}}\right) = 6.50 \times 10^{-7}\text{T}$ (out).
- $B_6 = \frac{\mu_0}{2\pi}\left(\frac{5\text{A}}{4\text{m}} + \frac{3\text{A}}{2\text{m}}\right) = 5.50 \times 10^{-7}\text{T}$ (out).
- $B_2 = \frac{\mu_0}{2\pi}\left(\frac{5\text{A}}{8\text{m}} - \frac{3\text{A}}{2\text{m}}\right) = -1.75 \times 10^{-7}\text{T}$ (in).

Unit Exam III: Problem #3 (Fall ’16)

A conducting wire of 16mm radius carries a current I that is uniformly distributed over its cross section and directed out of the plane. Find direction (left/right/up/down) and magnitude of the magnetic fields $\mathbf{B}_0$, $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, and $\mathbf{B}_4$ at the positions indicated if the current is $I = 2.5$A.



Unit Exam III: Problem #3 (Fall ’16)

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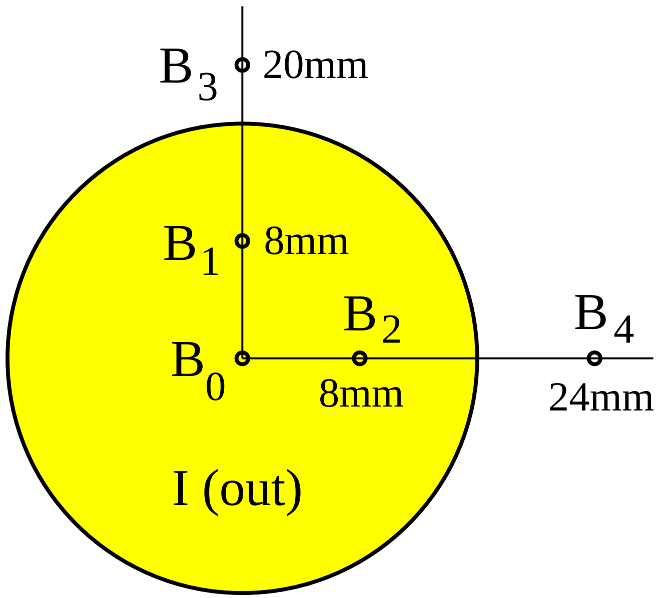
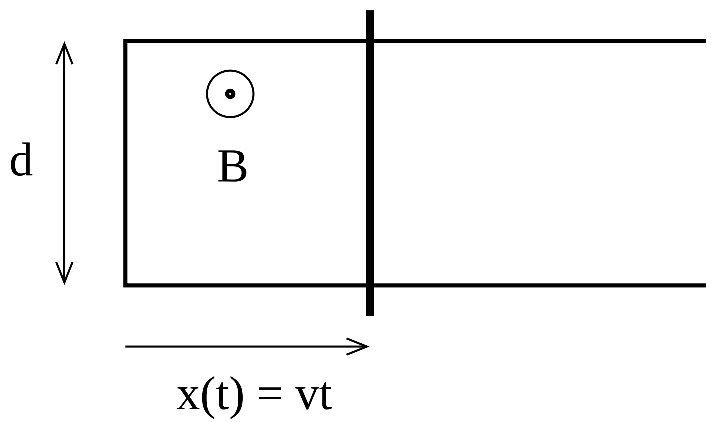


Solution:

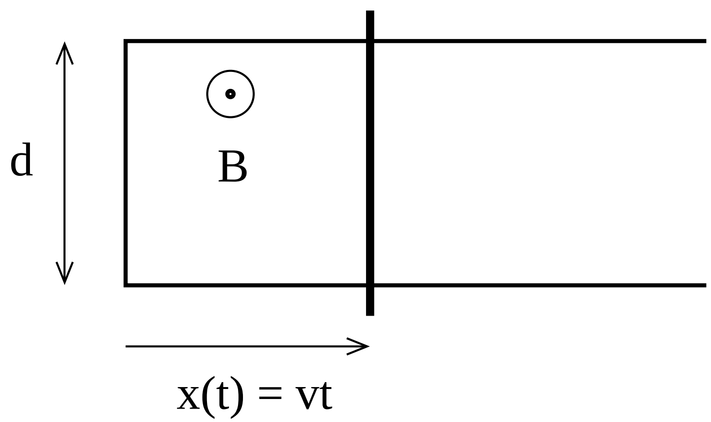
- $B_0 = 0$
- $(B_1)(2\pi)(8\text{mm}) = \mu_0(I/4) \quad \Rightarrow \; B_1 = 1.56 \times 10^{-5}\text{T}$ (left)
- $(B_2)(2\pi)(8\text{mm}) = \mu_0(I/4) \quad \Rightarrow \; B_2 = 1.56 \times 10^{-5}\text{T}$ (up)
- $(B_3)(2\pi)(20\text{mm}) = \mu_0 I \quad \Rightarrow \; B_3 = 2.5 \times 10^{-5}\text{T}$ (left)
- $(B_4)(2\pi)(24\text{mm}) = \mu_0 I \quad \Rightarrow \; B_4 = 2.08 \times 10^{-5}\text{T}$ (up)

A conducting frame of width $d = 1.6$m with a moving conducting rod is located in a uniform magnetic field $B = 3$T directed out of the plane. The rod moves at constant velocity $v = 0.4$m/s toward the right. Its instantaneous position is $x(t) = vt$. Find the magnetic flux Φ_B through the frame and the induced emf $\mathcal{E}$ around the frame at times $t_2 = 2$s, $t_3 = 3$s, $t_4 = 4$s, and $t_5 = 5$s. Write magnitudes only (in SI units), no directions. Is the induced current directed clockwise or counterclockwise?



Unit Exam III: Problem #4 (Fall ’16)

A conducting frame of width $d = 1.6$m with a moving conducting rod is located in a uniform magnetic field $B = 3$T directed out of the plane. The rod moves at constant velocity $v = 0.4$m/s toward the right. Its instantaneous position is $x(t) = vt$. Find the magnetic flux Φ_B through the frame and the induced emf $\mathcal{E}$ around the frame at times $t_2 = 2$s, $t_3 = 3$s, $t_4 = 4$s, and $t_5 = 5$s. Write magnitudes only (in SI units), no directions. Is the induced current directed clockwise or counterclockwise?

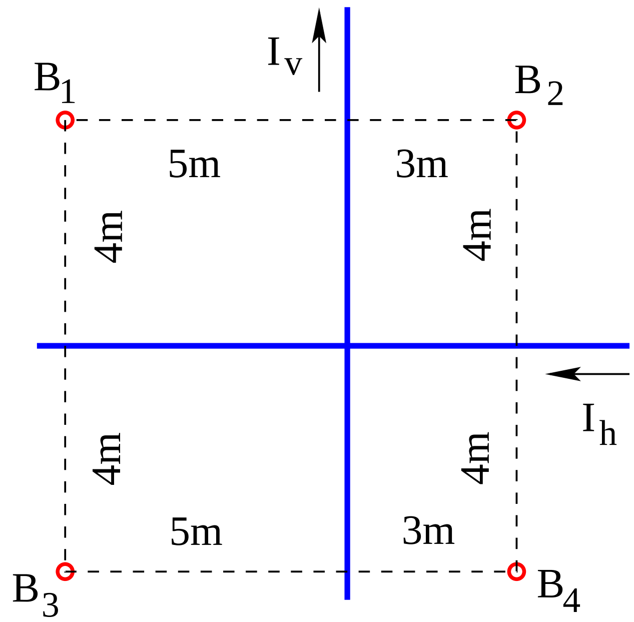


Solution:

- $\Phi_B^{(2)} = (1.6\text{m})(0.8\text{m})(3\text{T}) = 3.84\text{Wb}$,
  $\mathcal{E}^{(2)} = (0.4\text{m/s})(3\text{T})(1.6\text{m}) = 1.92\text{V}$.
- $\Phi_B^{(3)} = (1.6\text{m})(1.2\text{m})(3\text{T}) = 5.76\text{Wb}$,
  $\mathcal{E}^{(3)} = (0.4\text{m/s})(3\text{T})(1.6\text{m}) = 1.92\text{V}$.
- $\Phi_B^{(4)} = (1.6\text{m})(1.6\text{m})(3\text{T}) = 7.68\text{Wb}$,
  $\mathcal{E}^{(4)} = (0.4\text{m/s})(3\text{T})(1.6\text{m}) = 1.92\text{V}$.
- $\Phi_B^{(5)} = (1.6\text{m})(2.0\text{m})(3\text{T}) = 9.60\text{Wb}$,
  $\mathcal{E}^{(5)} = (0.4\text{m/s})(3\text{T})(1.6\text{m}) = 1.92\text{V}$.
- Clockwise current.

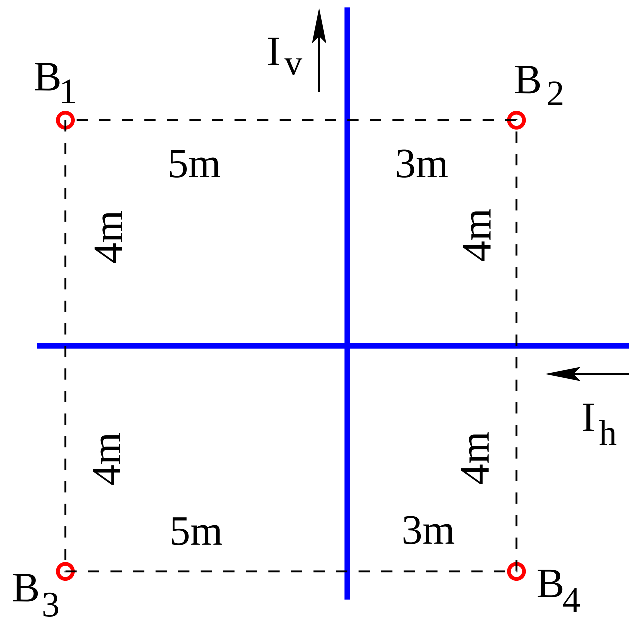
Unit Exam III: Problem #1 (Spring '17)

Consider two infinitely long, straight wires with currents $I_v = 6.9$A, $I_h = 7.2$A in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{B}_4$, at the points marked in the graph.



Unit Exam III: Problem #1 (Spring ’17)

Consider two infinitely long, straight wires with currents $I_v = 6.9\text{A}$, $I_h = 7.2\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{B}_4$, at the points marked in the graph.

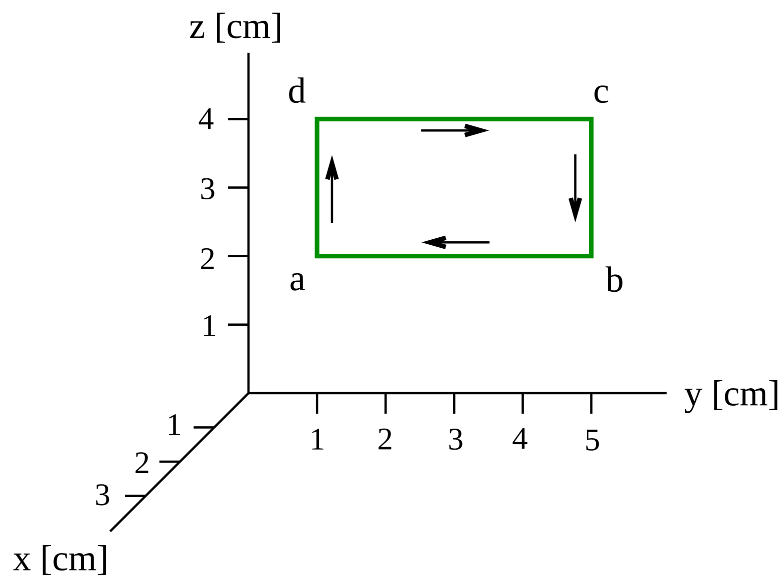


Solution:

- Convention used: out = positive, in = negative
- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{6.9\text{A}}{5\text{m}} - \frac{7.2\text{A}}{4\text{m}}\right) = -0.84 \times 10^{-7}\text{T}$ (in).
- $B_2 = \frac{\mu_0}{2\pi}\left(-\frac{6.9\text{A}}{3\text{m}} - \frac{7.2\text{A}}{4\text{m}}\right) = -8.20 \times 10^{-7}\text{T}$ (in).
- $B_3 = \frac{\mu_0}{2\pi}\left(\frac{6.9\text{A}}{5\text{m}} + \frac{7.2\text{A}}{4\text{m}}\right) = 6.36 \times 10^{-7}\text{T}$ (out).
- $B_4 = \frac{\mu_0}{2\pi}\left(-\frac{6.9\text{A}}{3\text{m}} + \frac{7.2\text{A}}{4\text{m}}\right) = -1.00 \times 10^{-7}\text{T}$ (in).

Unit Exam III: Problem #2 (Spring ’17)

In a region of uniform magnetic field $\mathbf{B} = 4\text{mT}\hat{\mathbf{k}}$ [$\mathbf{B} = 5\text{mT}\hat{\mathbf{j}}$] a clockwise current $I = 1.4$A [$I = 1.5$A] is flowing through the conducting rectangular frame.
(i) Find the force $\mathbf{F}_{dc}$ (magnitude and direction) acting on side dc of the rectangle. (ii) Find the force $\mathbf{F}_{ad}$ (magnitude and direction) acting on side ad of the rectangle.
(iii) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
(iv) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the current loop.



In a region of uniform magnetic field $\mathbf{B} = 4\text{mT}\hat{\mathbf{k}}$ [$\mathbf{B} = 5\text{mT}\hat{\mathbf{j}}$] a clockwise current $I = 1.4\text{A}$ [$I = 1.5\text{A}$] is flowing through the conducting rectangular frame.
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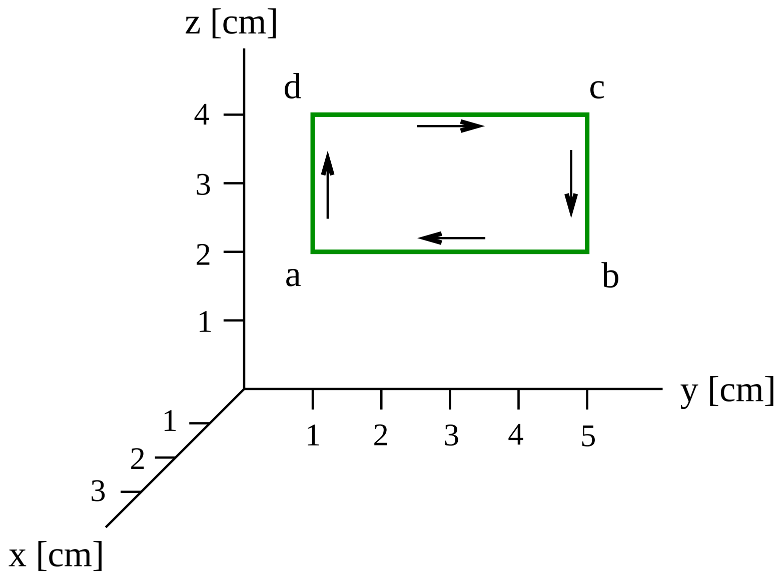
(iv) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the current loop.



Solution:

(i) $\mathbf{F}_{dc} = (1.4\text{A})(4\text{cm}\hat{\mathbf{j}}) \times (4\text{mT}\hat{\mathbf{k}}) = 2.24 \times 10^{-4}\text{N}\hat{\mathbf{i}}.$

$[\mathbf{F}_{dc} = (1.5\text{A})(4\text{cm}\hat{\mathbf{j}}) \times (5\text{mT}\hat{\mathbf{j}}) = 0.]$

(ii) $\mathbf{F}_{ad} = (1.4\text{A})(2\text{cm}\hat{\mathbf{k}}) \times (4\text{mT}\hat{\mathbf{k}}) = 0.$

$[\mathbf{F}_{ad} = (1.5\text{A})(2\text{cm}\hat{\mathbf{k}}) \times (5\text{mT}\hat{\mathbf{j}}) = -1.50 \times 10^{-4}\text{N}\hat{\mathbf{i}}.]$

(iii) $\vec{\mu} = [-(4\text{cm})(2\text{cm})\hat{\mathbf{i}}](1.4\text{A}) = -1.12 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}.$

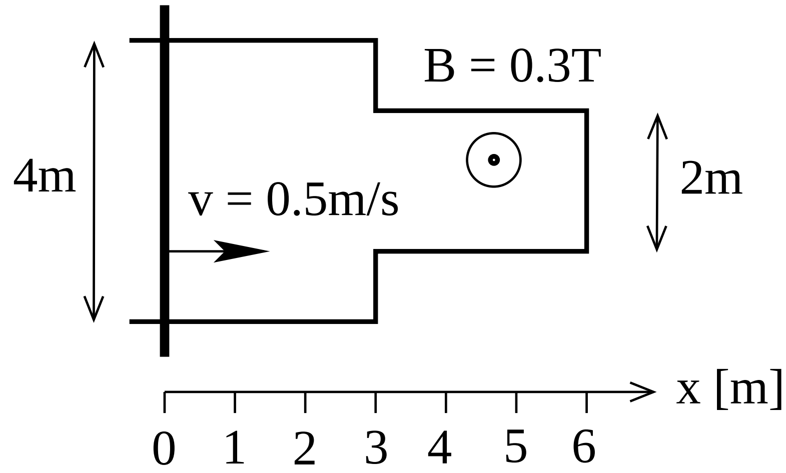
$[\vec{\mu} = [-(4\text{cm})(2\text{cm})\hat{\mathbf{i}}](1.5\text{A}) = -1.20 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}.]$

(iv) $\vec{\tau} = (-1.12 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}) \times (4\text{mT}\hat{\mathbf{k}}) = 4.48 \times 10^{-6}\text{Nm}\hat{\mathbf{j}}.$

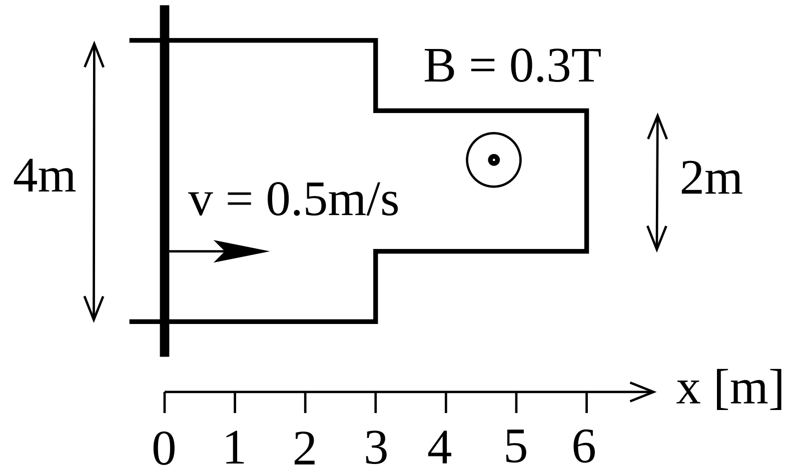
$[\vec{\tau} = (-1.20 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}) \times (5\text{mT}\hat{\mathbf{j}}) = -6.00 \times 10^{-6}\text{Nm}\hat{\mathbf{k}}.]$

Unit Exam III: Problem #3 (Spring ’17)

A conducting frame with a moving conducting rod is located in a uniform magnetic field directed out of the plane as shown. The rod moves at constant velocity.
Find the magnetic flux Φ_B through the frame and the induced emf $\mathcal{E}$ around the frame when the rod is
(a) at position $x = 1$m,
(b) at position $x = 4$m.
(c) at position $x = 2$m,
(d) at position $x = 5$m.
Write magnitudes only (in SI units), no directions.



A conducting frame with a moving conducting rod is located in a uniform magnetic field directed out of the plane as shown. The rod moves at constant velocity.
Find the magnetic flux Φ_B through the frame and the induced emf $\mathcal{E}$ around the frame when the rod is
(a) at position $x = 1$m,
(b) at position $x = 4$m.
(c) at position $x = 2$m,
(d) at position $x = 5$m.
Write magnitudes only (in SI units), no directions.



Solution:

(a) $\Phi_B = (8+6)\text{m}^2(0.3\text{T}) = 4.2\text{Wb}, \quad \mathcal{E} = (0.5\text{m/s})(0.3\text{T})(4\text{m}) = 0.6\text{V}.$

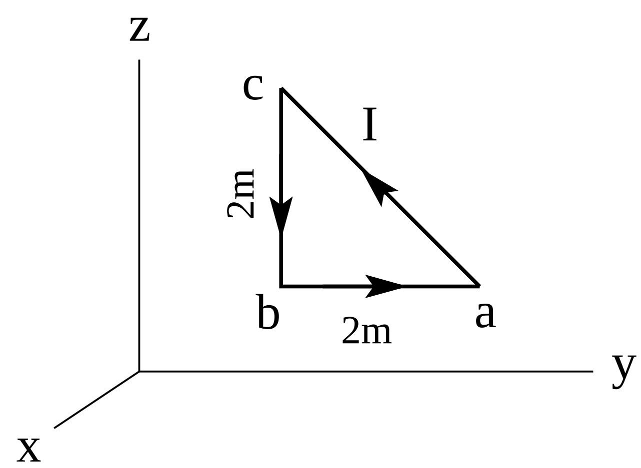
(b) $\Phi_B = (4\text{m}^2)(0.3\text{T}) = 1.2\text{Wb}, \quad \mathcal{E} = (0.5\text{m/s})(0.3\text{T})(2\text{m}) = 0.3\text{V}.$

(c) $\Phi_B = (4+6)\text{m}^2(0.3\text{T}) = 3.0\text{Wb}, \quad \mathcal{E} = (0.5\text{m/s})(0.3\text{T})(4\text{m}) = 0.6\text{V}.$

(d) $\Phi_B = (2\text{m}^2)(0.3\text{T}) = 0.6\text{Wb}, \quad \mathcal{E} = (0.5\text{m/s})(0.3\text{T})(2\text{m}) = 0.3\text{V}.$

Unit Exam III: Problem #1 (Fall ’17)

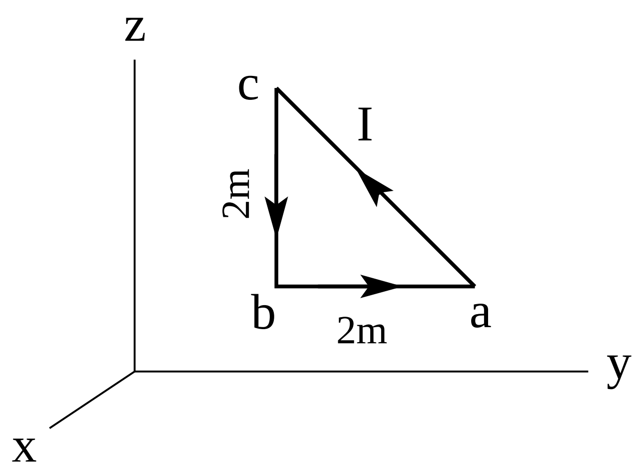
Consider a region with uniform magnetic field $\vec{B} = 4\mathrm{T}\hat{j}$ [$\vec{B} = 5\mathrm{T}\hat{k}$]. A conducting loop in the yz-plane has the shape of a right-angled triangle as shown with a counterclockwise current $I = 0.7$A [$I = 0.9$A].
(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
(b) Find the force $\vec{F}_{ab}$ (magnitude and direction) acting on the side ab of the loop.
(c) Find the force $\vec{F}_{bc}$ (magnitude and direction) acting on the side bc of the loop.
(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



Unit Exam III: Problem #1 (Fall ’17)

Consider a region with uniform magnetic field $\vec{B} = 4\mathrm{T}\hat{j}$ [$\vec{B} = 5\mathrm{T}\hat{k}$]. A conducting loop in the yz-plane has the shape of a right-angled triangle as shown with a counterclockwise current $I = 0.7$A [$I = 0.9$A].

(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
(b) Find the force $\vec{F}_{ab}$ (magnitude and direction) acting on the side ab of the loop.
(c) Find the force $\vec{F}_{bc}$ (magnitude and direction) acting on the side bc of the loop.
(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



Solution:

(a) $\vec{\mu} = (0.7\mathrm{A})(2\mathrm{m}^2)\hat{i} = 1.4\mathrm{Am}^2\hat{i}$
$[\vec{\mu} = (0.9\mathrm{A})(2\mathrm{m}^2)\hat{i} = 1.8\mathrm{Am}^2\hat{i}]$

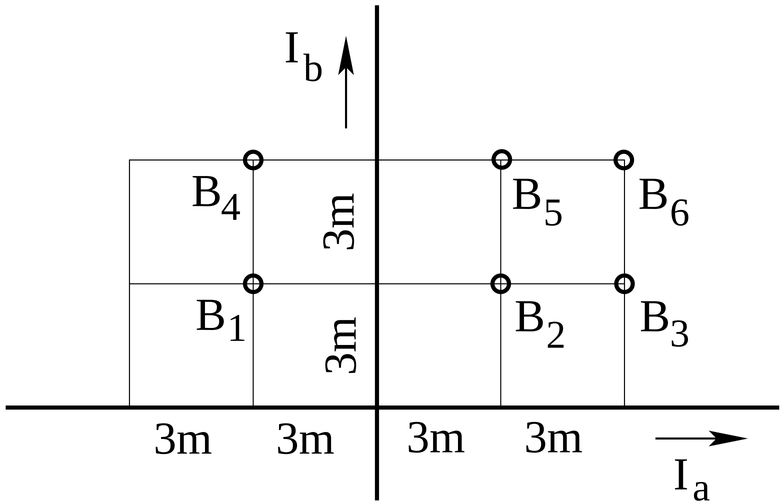
(b) $\vec{F}_{ab} = 0 \quad [\vec{F}_{ab} = (0.9\mathrm{A})(2\mathrm{m}\hat{j}) \times (5\mathrm{T}\hat{k}) = 9.0\mathrm{N}\hat{i}]$

(c) $\vec{F}_{bc} = (0.7\mathrm{A})(-2\mathrm{m}\hat{k}) \times (4\mathrm{T}\hat{j}) = 5.6\mathrm{N}\hat{i} \quad [\vec{F}_{bc} = 0]$

(d) $\vec{\tau} = (1.4\mathrm{Am}^2\hat{i}) \times (4\mathrm{T}\hat{j}) = 5.6\mathrm{Nm}\hat{k}$
$[\vec{\tau} = (1.8\mathrm{Am}^2\hat{i}) \times (5\mathrm{T}\hat{k}) = -9.0\mathrm{Nm}\hat{j}]$

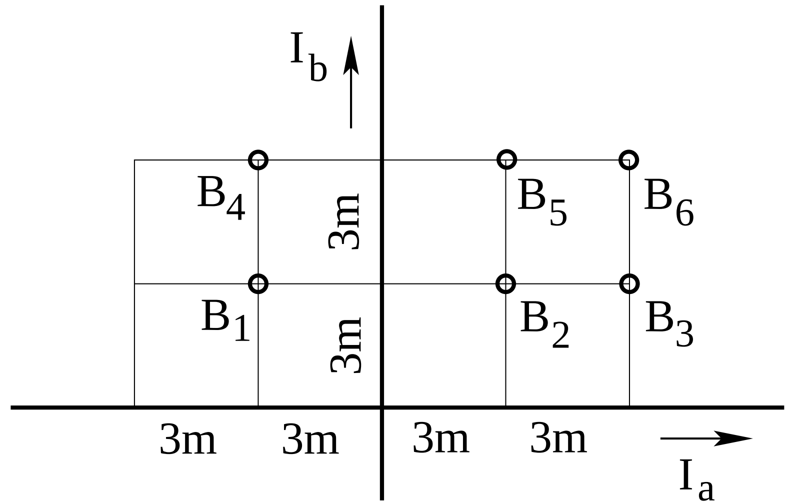
Unit Exam III: Problem #2 (Fall '17)

Consider two infinitely long, straight wires with currents $I_a = I_b = 7$A in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{B}_4$, $\mathbf{B}_5$, $\mathbf{B}_6$ at the points marked in the graph.



Unit Exam III: Problem #2 (Fall ’17)

Consider two infinitely long, straight wires with currents $I_a = I_b = 7\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{B}_4$, $\mathbf{B}_5$, $\mathbf{B}_6$ at the points marked in the graph.



Solution:

- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{3\text{m}} + \frac{7\text{A}}{3\text{m}}\right) = +0.933\mu\text{T}$ (out of plane).
- $B_2 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{3\text{m}} - \frac{7\text{A}}{3\text{m}}\right) = 0$ (no direction).
- $B_3 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{3\text{m}} - \frac{7\text{A}}{6\text{m}}\right) = +0.233\mu\text{T}$ (out of plane).
- $B_4 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{6\text{m}} + \frac{7\text{A}}{3\text{m}}\right) = 0.7\mu\text{T}$ (out of plane).
- $B_5 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{6\text{m}} - \frac{7\text{A}}{3\text{m}}\right) = -0.233\mu\text{T}$ (into plane).
- $B_6 = \frac{\mu_0}{2\pi}\left(\frac{7\text{A}}{6\text{m}} - \frac{7\text{A}}{6\text{m}}\right) = 0$ (no direction).

Unit Exam III: Problem #3 (Fall '17)

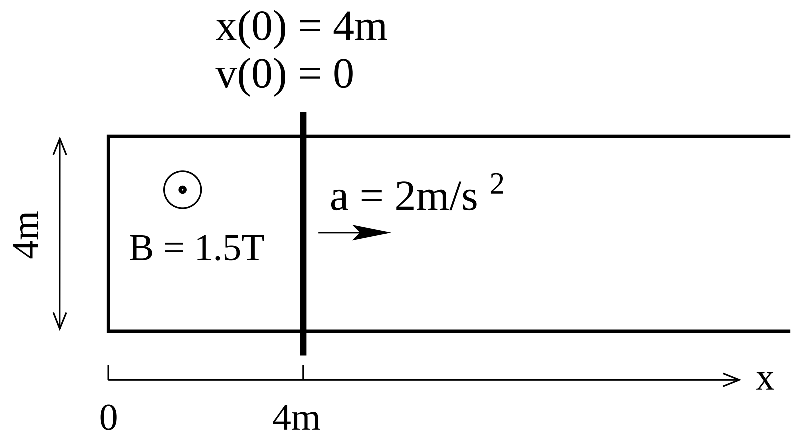
A conducting frame with a moving conducting rod is placed in a uniform magnetic field directed out of the plane. The rod starts from rest at time $t = 0$ at the position shown and moves with constant acceleration to the right.

(a) Find the magnetic flux Φ_B through the conducting loop and the induced emf $\mathcal{E}$ around the loop at $t = 0$.

(b) Find the position $x(3\mathrm{s})$ and velocity $v(3\mathrm{s})$ of the rod at time $t = 3$s.

(c) Find the magnetic flux Φ_B through the loop and the induced emf $\mathcal{E}$ around the loop at time $t = 3$s.

Write magnitudes only (in SI units), no directions.



Unit Exam III: Problem #3 (Fall '17)

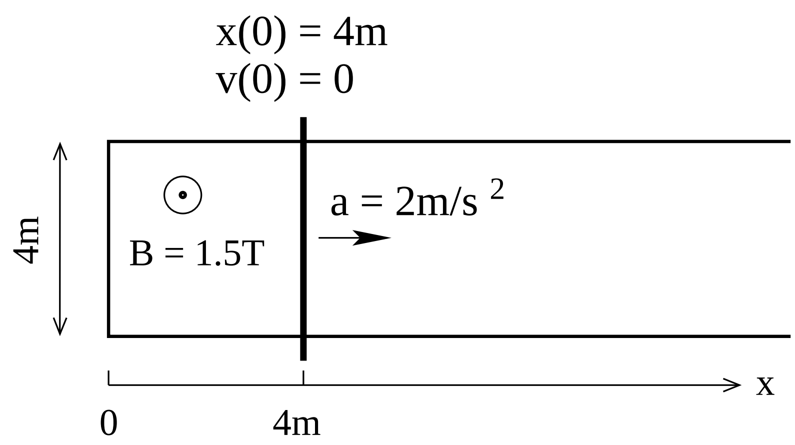
A conducting frame with a moving conducting rod is placed in a uniform magnetic field directed out of the plane. The rod starts from rest at time $t = 0$ at the position shown and moves with constant acceleration to the right.

(a) Find the magnetic flux Φ_B through the conducting loop and the induced emf $\mathcal{E}$ around the loop at $t = 0$.

(b) Find the position $x(3\text{s})$ and velocity $v(3\text{s})$ of the rod at time $t = 3\text{s}$.

(c) Find the magnetic flux Φ_B through the loop and the induced emf $\mathcal{E}$ around the loop at time $t = 3\text{s}$.

Write magnitudes only (in SI units), no directions.



Solution:

(a) $\Phi_B = (16\text{m}^2)(1.5\text{T}) = 24\text{Wb}, \quad \mathcal{E} = 0.$

(b) $x(2\text{s}) = 4\text{m} + \frac{1}{2}(2\text{m/s}^2)(3\text{s})^2 = 13\text{m}, \quad v(3\text{s}) = (2\text{m/s}^2)(3\text{s}) = 6\text{m/s}.$

(b) $\Phi_B = (52\text{m}^2)(1.5\text{T}) = 78\text{Wb}, \quad \mathcal{E} = (6\text{m/s})(1.5\text{T})(4\text{m}) = 36\text{V}.$

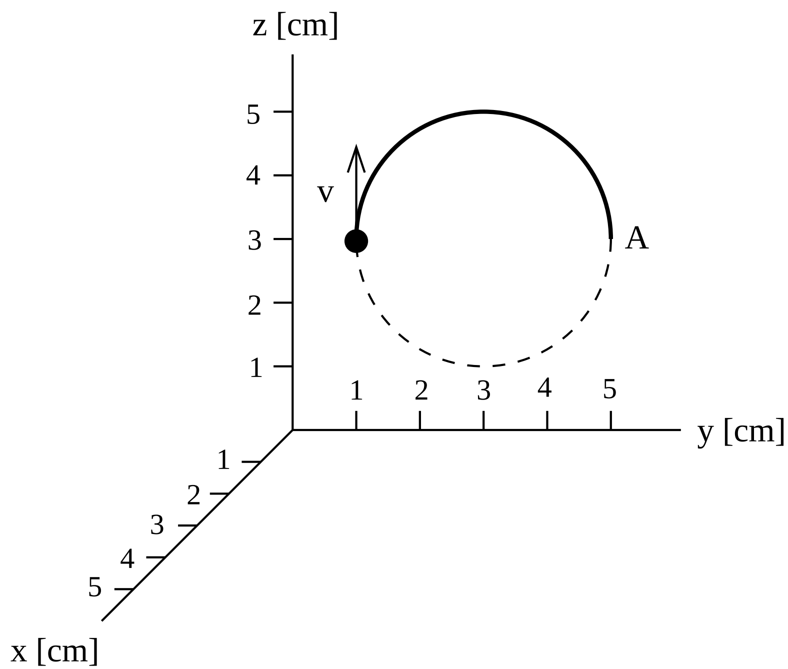
Unit Exam III: Problem #1 (Spring '18)

In a uniform magnetic field of strength $B = 3.5\text{mT}$ [$B = 5.3\text{mT}$], a proton with specifications ($m = 1.67 \times 10^{-27}\text{kg},\ \ q = 1.60 \times 10^{-19}\text{C}$) moves at speed v around a circle in the yz-plane as shown.

(a) Show that the direction of the magnetic field must be $+\hat{\mathbf{i}}$

(b) What is the speed of the proton?

(c) How long does it take the proton to reach point A from its current position?



Unit Exam III: Problem #1 (Spring ’18)

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(a) Show that the direction of the magnetic field must be $+\hat{\mathbf{i}}$

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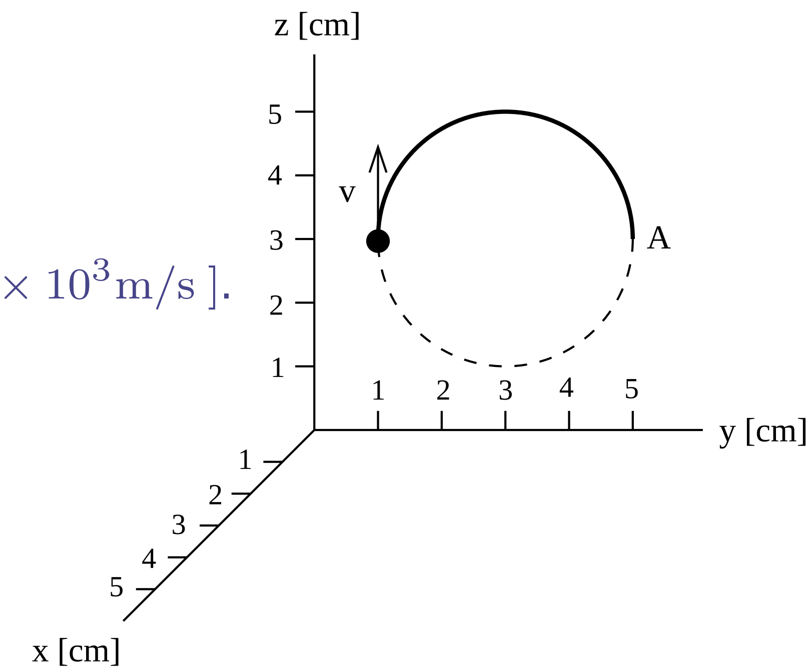


Solution:

(a) $F\hat{\mathbf{j}} = qv\hat{\mathbf{k}} \times B\hat{\mathbf{i}}$.

(b) $\dfrac{mv^2}{r} = qvB \quad \Rightarrow\ v = \dfrac{qBr}{m} = 6.71 \times 10^3\text{m/s} \quad [\,10.2 \times 10^3\text{m/s}\,]$.

(c) $t = \dfrac{\pi r}{v} = \dfrac{\pi m}{qB} = 9.37 \times 10^{-6}\text{s} \quad [\,6.19 \times 10^{-6}\text{s}\,]$.

Unit Exam III: Problem #2a (Spring '18)

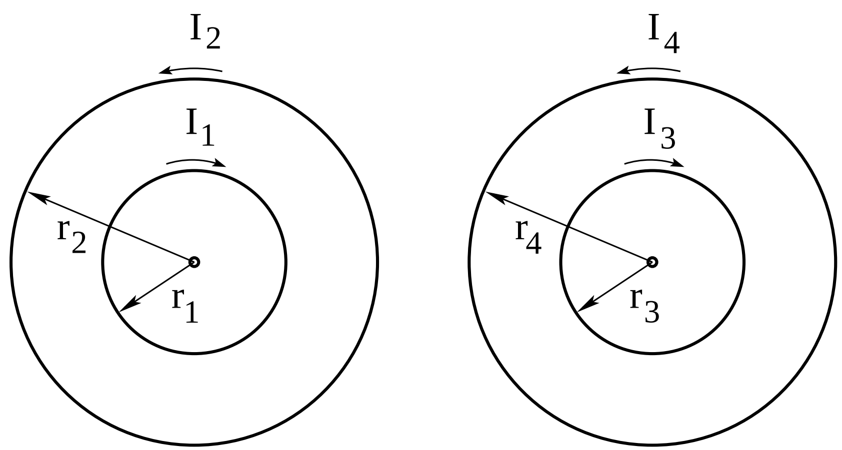
Consider two pairs of concentric circular currents in separate regions. The current directions are indicated by arrows. The radii are $r_1 = r_3 = 5$cm and $r_2 = r_4 = 10$cm

(a) Find magnitude B_1 and direction $(\odot, \otimes)$ of the magnetic field produced by current $I_1 = 1.5$A at the center.

(b) Find magnitude μ_4 and direction $(\odot, \otimes)$ of the magnetic dipole moment produced by current $I_4 = 4.5$A.

(c) What must be the ratio I_2/I_1 such that the magnetic field at the center is zero?

(d) What must be the ratio I_4/I_3 such that the magnetic dipole moment is zero?



Unit Exam III: Problem #2a (Spring '18)

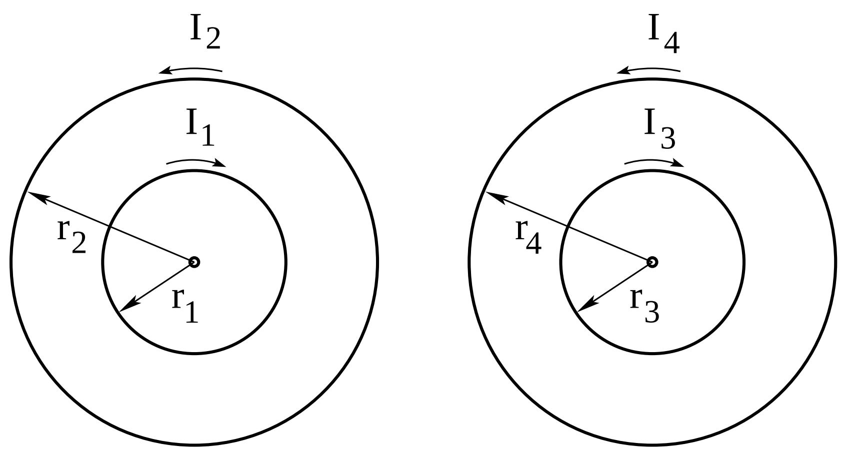
Consider two pairs of concentric circular currents in separate regions. The current directions are indicated by arrows. The radii are $r_1 = r_3 = 5$cm and $r_2 = r_4 = 10$cm

(a) Find magnitude B_1 and direction $(\odot, \otimes)$ of the magnetic field produced by current $I_1 = 1.5$A at the center.

(b) Find magnitude μ_4 and direction $(\odot, \otimes)$ of the magnetic dipole moment produced by current $I_4 = 4.5$A.

(c) What must be the ratio I_2/I_1 such that the magnetic field at the center is zero?

(d) What must be the ratio I_4/I_3 such that the magnetic dipole moment is zero?



Solution:

(a) $B_1 = \dfrac{\mu_0(1.5\text{A})}{2(5\text{cm})} = 1.88 \times 10^{-5}\text{T} \quad \otimes$

(b) $\mu_4 = \pi(10\text{cm})^2(4.5\text{A}) = 1.41 \times 10^{-1}\text{Am}^2 \quad \odot$

(c) $B_1 = B_2 \quad \Rightarrow \dfrac{I_2}{I_1} = \dfrac{r_2}{r_1} = 2.$

(d) $\mu_3 = \mu_4 \quad \Rightarrow \dfrac{I_4}{I_3} = \dfrac{r_3^2}{r_4^2} = 0.25.$

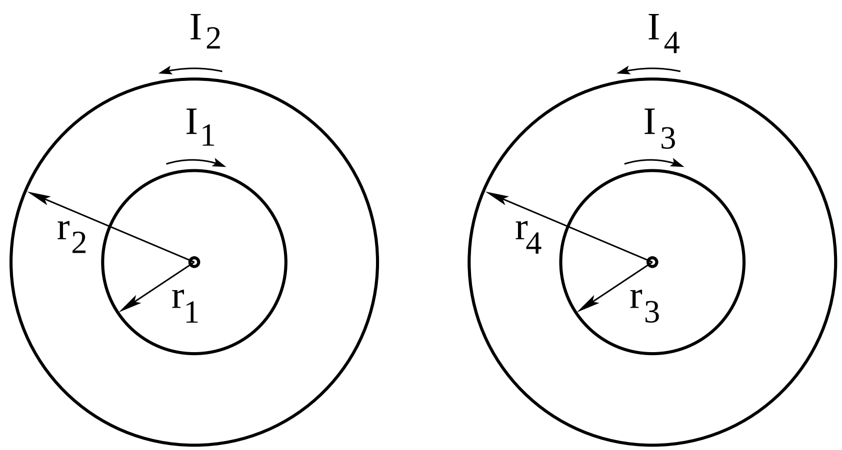
Consider two pairs of concentric circular currents in separate regions. The current directions are indicated by arrows. The radii are $r_1 = r_3 = 5$cm and $r_2 = r_4 = 10$cm

(a) Find magnitude B_2 and direction $(\odot, \otimes)$ of the magnetic field produced by current $I_2 = 2.5$A at the center.

(b) Find magnitude μ_3 and direction $(\odot, \otimes)$ of the magnetic dipole moment produced by current $I_3 = 3$A.

(c) What must be the ratio I_2/I_1 such that the magnetic field at the center is zero?

(d) What must be the ratio I_4/I_3 such that the magnetic dipole moment is zero?



Unit Exam III: Problem #2b (Spring '18)

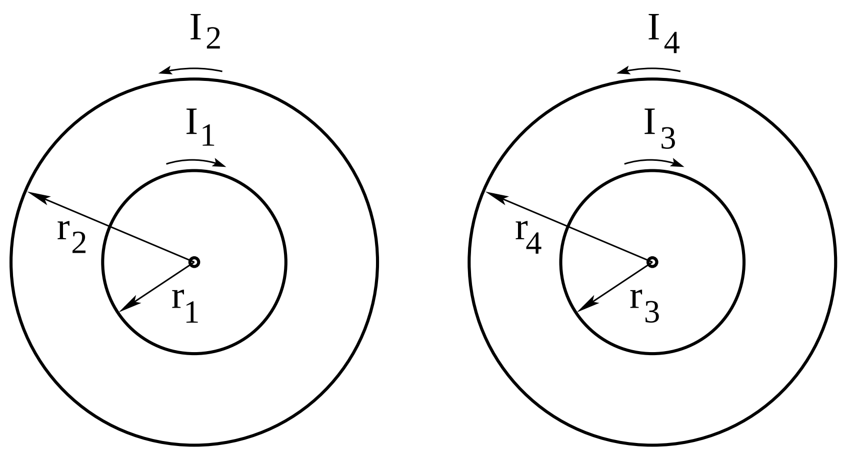
Consider two pairs of concentric circular currents in separate regions. The current directions are indicated by arrows. The radii are $r_1 = r_3 = 5$cm and $r_2 = r_4 = 10$cm

(a) Find magnitude B_2 and direction $(\odot, \otimes)$ of the magnetic field produced by current $I_2 = 2.5$A at the center.

(b) Find magnitude μ_3 and direction $(\odot, \otimes)$ of the magnetic dipole moment produced by current $I_3 = 3$A.

(c) What must be the ratio I_2/I_1 such that the magnetic field at the center is zero?

(d) What must be the ratio I_4/I_3 such that the magnetic dipole moment is zero?



Solution:

(a) $B_2 = \dfrac{\mu_0(2.5\text{A})}{2(10\text{cm})} = 1.57 \times 10^{-5}\text{T} \quad \odot$

(b) $\mu_3 = \pi(5\text{cm})^2(3\text{A}) = 2.36 \times 10^{-2}\text{Am}^2 \quad \otimes$

(c) $B_1 = B_2 \quad \Rightarrow \dfrac{I_2}{I_1} = \dfrac{r_2}{r_1} = 2.$

(d) $\mu_3 = \mu_4 \quad \Rightarrow \dfrac{I_4}{I_3} = \dfrac{r_3^2}{r_4^2} = 0.25.$

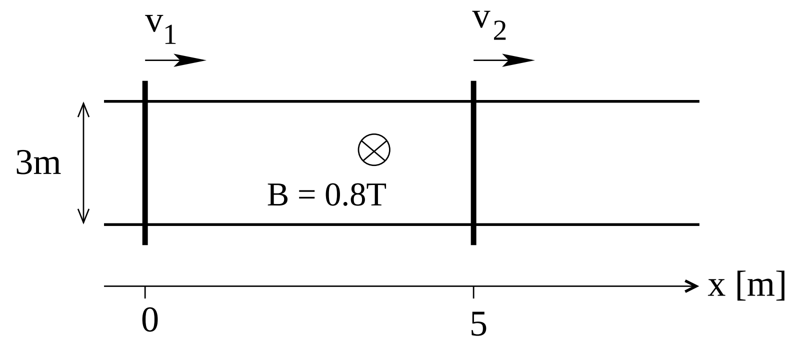
Unit Exam III: Problem #3 (Spring ’18)

A pair of fixed rails are connected by two moving rods. A uniform magnetic field B is present. The positions of the rods at time $t = 0$ are as shown. The (constant) velocities are $v_1 = 0.5$m/s, $v_2 = 2.5$m/s [$v_1 = 1.5$m/s, $v_2 = 0.5$m/s].

(a) Find the magnetic flux Φ_0 at time $t = 0$ and Φ_1 at $t = 2$s (magnitude only).

(b) Find the induced emf $\mathcal{E}_0$ at time $t = 0$ and $\mathcal{E}_1$ at $t = 2$s (magnitude only).

(c) Find the direction (cw/ccw) of the induced current at $t = 0$.



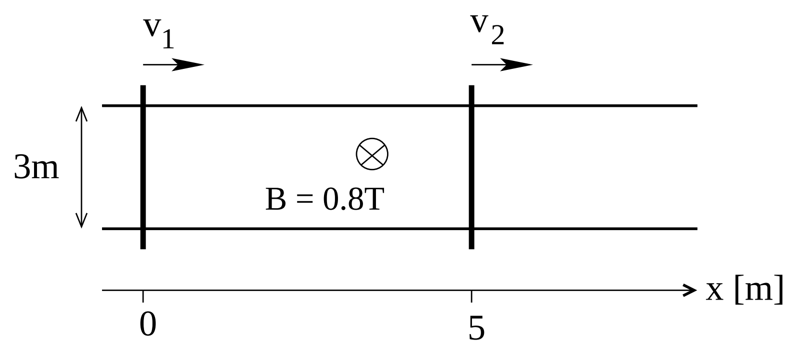
Unit Exam III: Problem #3 (Spring '18)

A pair of fixed rails are connected by two moving rods. A uniform magnetic field B is present. The positions of the rods at time $t = 0$ are as shown. The (constant) velocities are $v_1 = 0.5$m/s, $v_2 = 2.5$m/s [$v_1 = 1.5$m/s, $v_2 = 0.5$m/s].

(a) Find the magnetic flux Φ_0 at time $t = 0$ and Φ_1 at $t = 2$s (magnitude only).

(b) Find the induced emf $\mathcal{E}_0$ at time $t = 0$ and $\mathcal{E}_1$ at $t = 2$s (magnitude only).

(c) Find the direction (cw/ccw) of the induced current at $t = 0$.



Solution:

(a) $\Phi_0 = (5\text{m} - 0\text{m})(3\text{m})(0.8\text{T}) = 12\text{Wb}, \quad \Phi_1 = (10\text{m} - 1\text{m})(3\text{m})(0.8\text{T}) = 21.6\text{Wb}$
[$\Phi_0 = (5\text{m} - 0\text{m})(3\text{m})(0.8\text{T}) = 12\text{Wb}, \quad \Phi_1 = (6\text{m} - 3\text{m})(3\text{m})(0.8\text{T}) = 7.2\text{Wb}$]

(b) $|\mathcal{E}_0| = |\mathcal{E}_1| = (2.5\text{m/s} - 0.5\text{m/s})(0.8\text{T})(3\text{m}) = 4.8\text{V}$
[$|\mathcal{E}_0| = |\mathcal{E}_1| = (1.5\text{m/s} - 0.5\text{m/s})(0.8\text{T})(3\text{m}) = 2.4\text{V}$]

(c) ccw [cw]

Unit Exam III: Problem #1 (Fall '18)

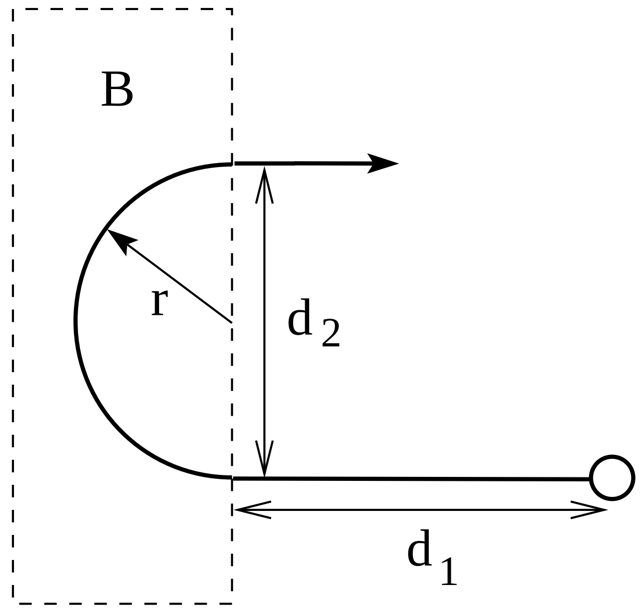
A proton $(m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C})$, launched with initial speed $v_0 = 4000\mathrm{m/s}$ $[3000\mathrm{m/s}]$ a distance $d_1 = 25$cm [32cm] from a region of magnetic field, exits that region after a half-circle turn of diameter $d_2 = 30$cm [35cm].

(a) Find the centripetal force F provided by the magnetic field.

(b) Find magnitude and direction $(\odot, \otimes)$ of the magnetic field $\mathbf{B}$.

(c) Find the time t_1 elapsed between launch and entrance into the region of field.

(d) Find the time t_2 elapsed between entrance and exit.



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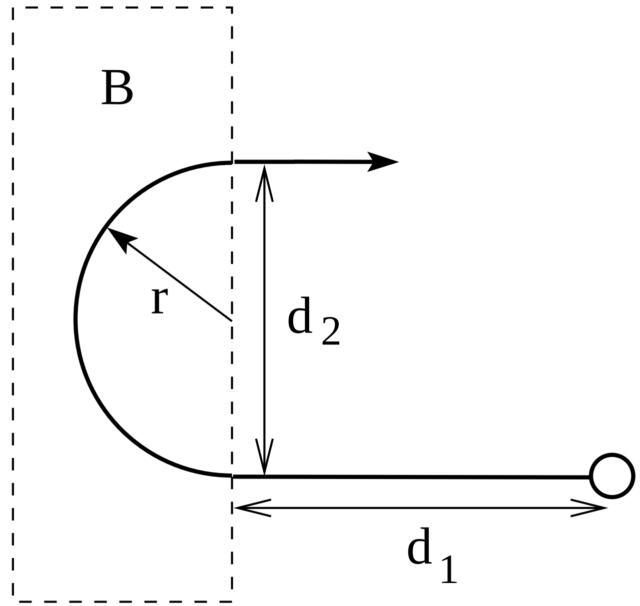


Solution:

(a) $\dfrac{mv_0^2}{d_2/2} = 1.78 \times 10^{-19}\mathrm{N} \quad [8.59 \times 10^{-20}\mathrm{N}]$.

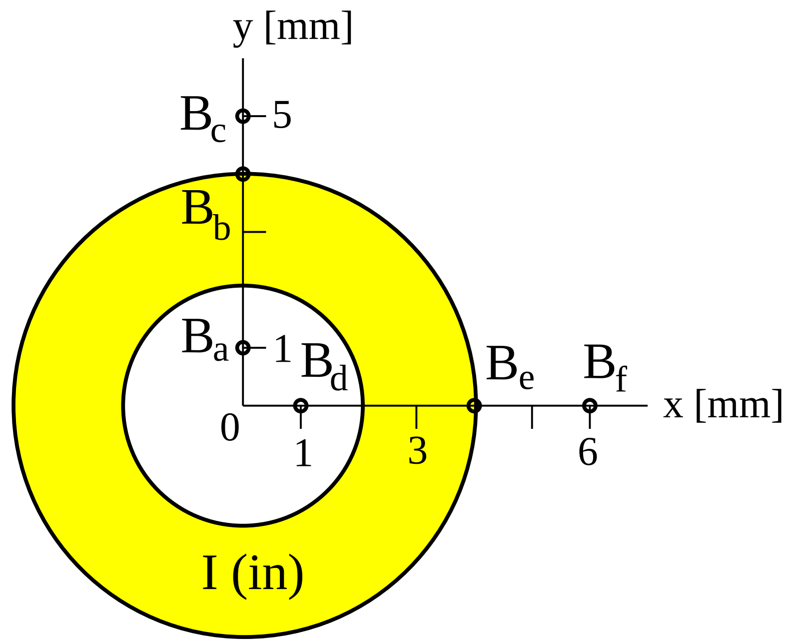
(b) $B = \dfrac{F}{qv_0} = 2.78 \times 10^{-4}\mathrm{T} \quad [1.79 \times 10^{-4}\mathrm{T}] \quad \odot$

(c) $t_1 = \dfrac{d_1}{v_0} = 6.25 \times 10^{-5}\mathrm{s} \quad [1.07 \times 10^{-4}\mathrm{s}]$.

(d) $t_2 = \dfrac{\pi d_2}{2v_0} = 1.18 \times 10^{-4}\mathrm{s} \quad [1.83 \times 10^{-4}\mathrm{s}]$.

Unit Exam III: Problem #2 (Fall ’18)

A wire in the shape of a cylindrical shell with a 2mm inner radius and 4mm outer radius carries a current $I = 3.7$A [4.1A] that is uniformly distributed over its cross section and directed into the plane. Find direction (left/right/up/down/in/out) and magnitude of the magnetic fields $\mathbf{B}_a$, $\mathbf{B}_b$, $\mathbf{B}_c$ [$\mathbf{B}_d$, $\mathbf{B}_e$, $\mathbf{B}_f$] at the positions indicated.



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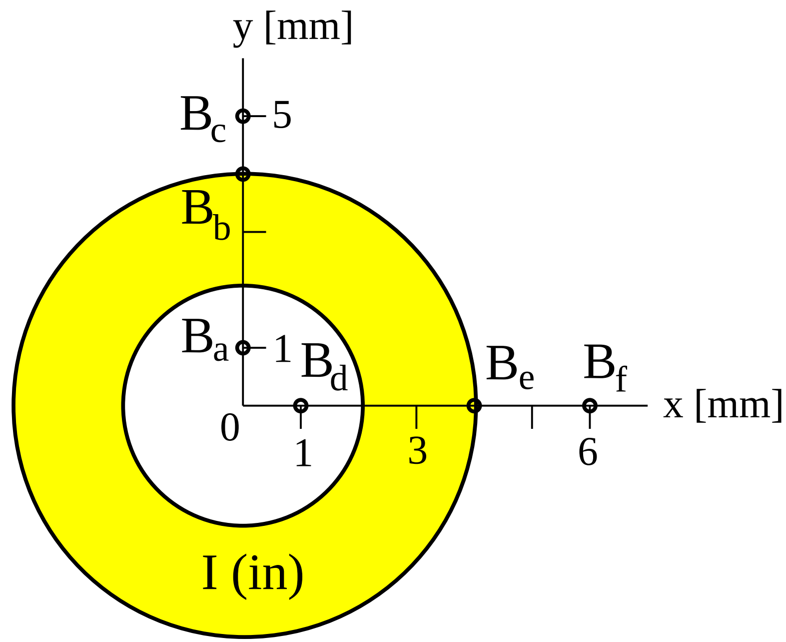
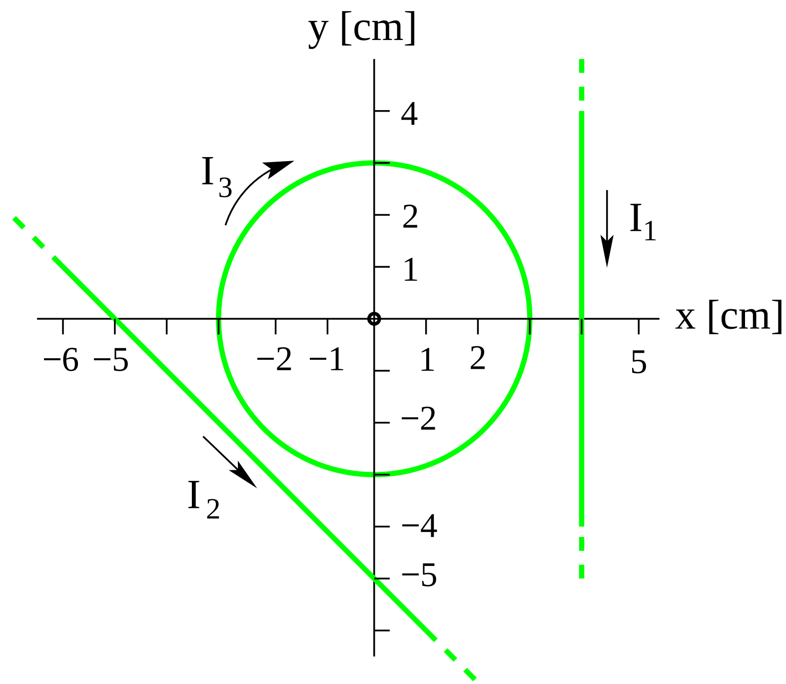


Solution:

- $B_a = 0$
- $(B_b)(2\pi)(4\text{mm}) = \mu_0(3.7\text{A})$
  $\Rightarrow \; B_b = 1.85 \times 10^{-4}\text{T}$ (right)
- $(B_c)(2\pi)(5\text{mm}) = \mu_0(3.7\text{A})$
  $\Rightarrow \; B_c = 1.48 \times 10^{-4}\text{T}$ (right)
- $[B_d = 0]$
- $[(B_e)(2\pi)(4\text{mm}) = \mu_0(4.1\text{A}) \quad \Rightarrow \; B_e = 2.05 \times 10^{-4}\text{T}$ (down)]
- $[(B_f)(2\pi)(6\text{mm}) = \mu_0(4.1\text{A}) \quad \Rightarrow \; B_f = 1.37 \times 10^{-4}\text{T}$ (down)]

Unit Exam III: Problem #3 (Fall '18)

Two very long straight wires and a circular wire positioned in the xy-plane carry electric currents $I_1 = I_2 = I_3 = 1.3$A [1.7A] in the directions shown.
(a) Calculate magnitude (B_1, B_2, B_2) and direction (left/right/up/down/in/out) of the magnetic field produced by each current at the origin of the coordinate system.
(b) Calculate magnitude μ and direction (left/right/up/down/in/out) of the magnetic dipole moment produced by the circular current.



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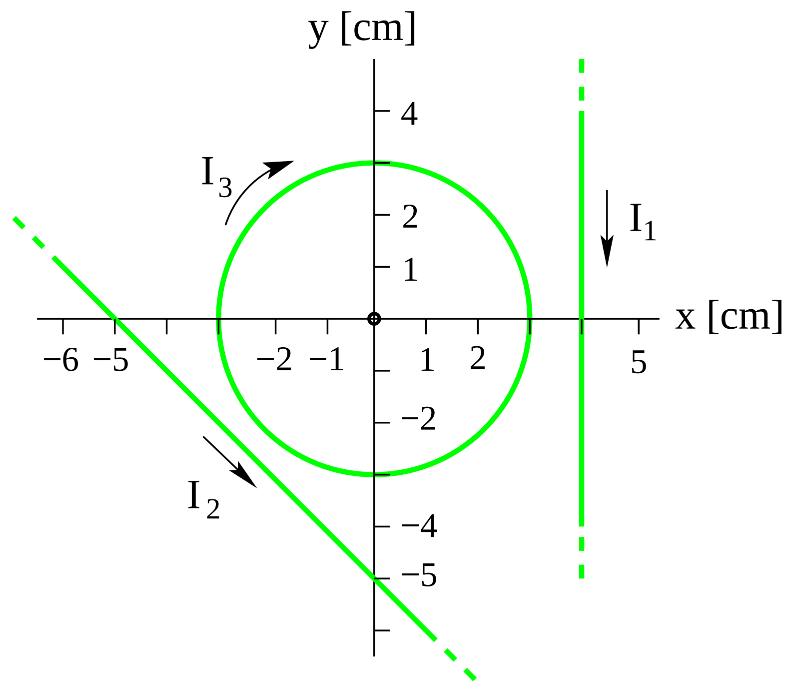


Solution:

(a) $B_1 = \dfrac{\mu_0(I_1)}{2\pi(4\text{cm})} = 6.5\mu\text{T} \quad [8.5\mu\text{T}]$. (in)

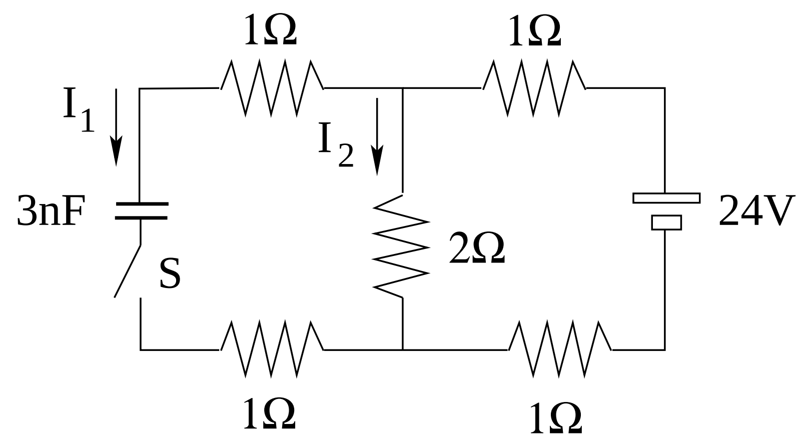
$B_2 = \dfrac{\mu_0(I_2)}{2\pi(5\text{cm}/\sqrt{2})} = 7.35\mu\text{T} \quad [9.62\mu\text{T}]$ (out)

$B_3 = \dfrac{\mu_0(I_3)}{2(3\text{cm})} = 27.2\mu\text{T} \quad [35.6\mu\text{T}]$ (in)

(b) $\mu = \pi(3\text{cm})^2(I_3) = 3.68 \times 10^{-3}\text{Am}^2 \quad [4.81 \times 10^{-3}\text{Am}^2]$ (in)

Unit Exam III: Problem #1 (Spring ’19)

This circuit is in a steady state with the switch open and the capacitor discharged.
(a) Find the currents I_1 and I_2 while the switch is still open.
(b) Find the currents I_1 and I_2 right after the switch has been closed.
(c) Find the currents I_1 and I_2 a long time later.
(d) Find the voltage V across the capacitor, also a long time later.



This circuit is in a steady state with the switch open and the capacitor discharged.
(a) Find the currents I_1 and I_2 while the switch is still open.
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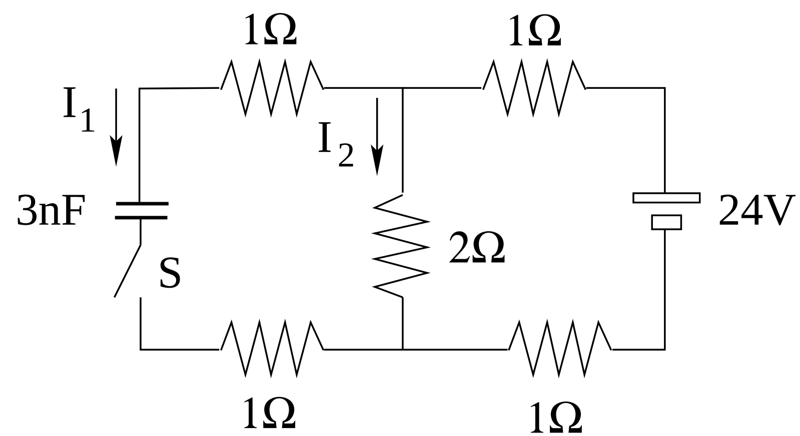


Solution:

(a) $I_1 = 0, \quad I_2 = \dfrac{24\text{V}}{1\Omega + 2\Omega + 1\Omega} = 6\text{A}.$

(b) $R_{eq} = 1\Omega + \left(\dfrac{1}{2\Omega} + \dfrac{1}{1\Omega + 1\Omega}\right)^{-1} + 1\Omega = 3\Omega$ (capacitor discharged)

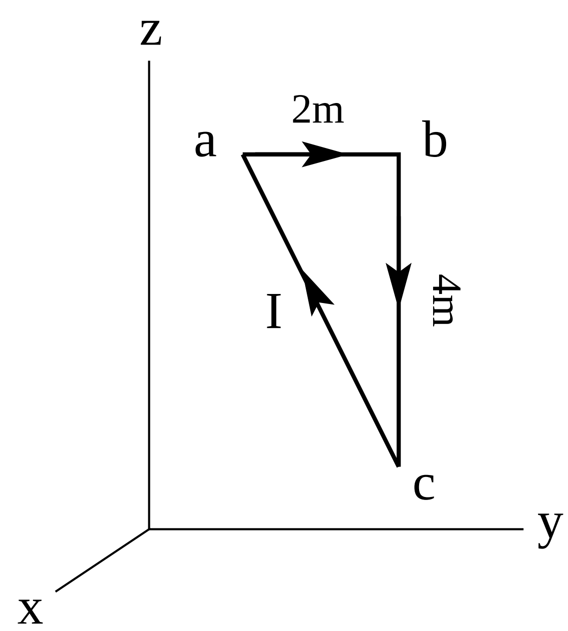
$$\Rightarrow I_1 + I_2 = \frac{24\text{V}}{3\Omega} = 8\text{A}, \quad I_1 = I_2 = 4\text{A}.$$

(c) capacitor fully charged: $I_1 = 0, \quad I_2 = \dfrac{24\text{V}}{1\Omega + 2\Omega + 1\Omega} = 6\text{A}.$

(d) loop rule: $(2\Omega)(6\text{A}) - (1\Omega)(0\text{A}) - V - (1\Omega)(0\text{A}) = 0 \quad \Rightarrow V = 12V.$

Unit Exam III: Problem #2 (Spring ’19)

Consider a region with uniform magnetic field $\vec{B} = 3\mathrm{T}\hat{\mathbf{j}} + 5\mathrm{T}\hat{\mathbf{k}}$. A conducting loop positioned in the yz-plane has the shape of a right-angled triangle and carries a clockwise current $I = 2$A.
(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
(b) Find the force $\vec{F}_{ab}$ (magnitude and direction) acting on side ab.
(c) Find the force $\vec{F}_{bc}$ (magnitude and direction) acting on side bc.
(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



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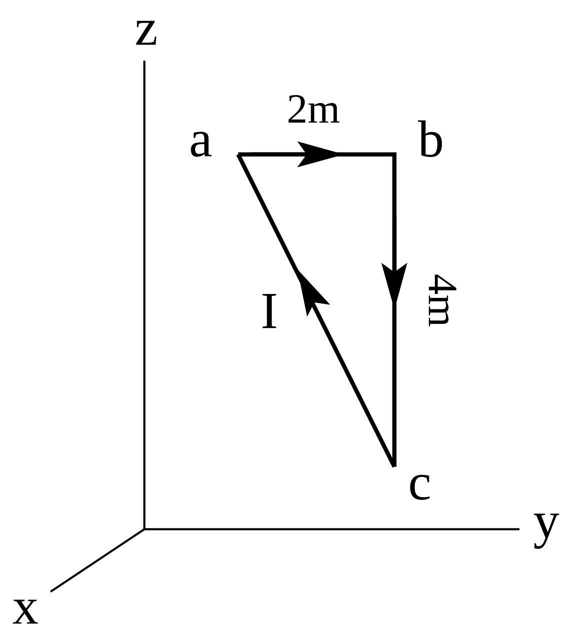


Solution:

(a) $\vec{\mu} = -(2\mathrm{A})(4\mathrm{m}^2)\hat{\mathbf{i}} = -8\mathrm{Am}^2\hat{\mathbf{i}}$.

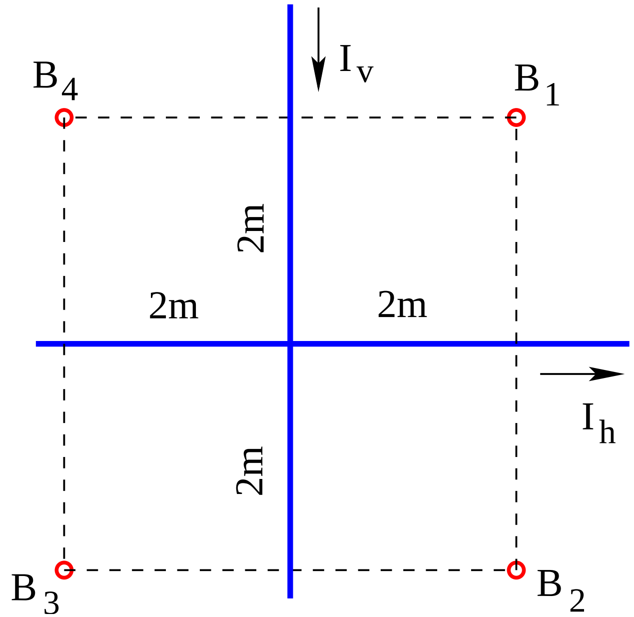
(b) $\vec{F}_{ab} = (2\mathrm{A})(2\mathrm{m}\hat{\mathbf{j}}) \times \left[3\mathrm{T}\hat{\mathbf{j}} + 5\mathrm{T}\hat{\mathbf{k}}\right] = 20\mathrm{N}\hat{\mathbf{i}}$.

(c) $\vec{F}_{bc} = (2\mathrm{A})(-4\mathrm{m}\hat{\mathbf{k}}) \times \left[3\mathrm{T}\hat{\mathbf{j}} + 5\mathrm{T}\hat{\mathbf{k}}\right] = 24\mathrm{N}\hat{\mathbf{i}}$.

(d) $\vec{\tau} = (-8\mathrm{Am}^2\hat{\mathbf{i}}) \times \left[3\mathrm{T}\hat{\mathbf{j}} + 5\mathrm{T}\hat{\mathbf{k}}\right] = -24\mathrm{Nm}\hat{\mathbf{k}} + 40\mathrm{Nm}\hat{\mathbf{j}}$

Unit Exam III: Problem #3 (Spring ’19)

Consider two infinitely long, straight wires with currents $I_v = 3$A, $I_h = 3$A in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{B}_4$, at the points marked in the graph.



Unit Exam III: Problem #3 (Spring '19)

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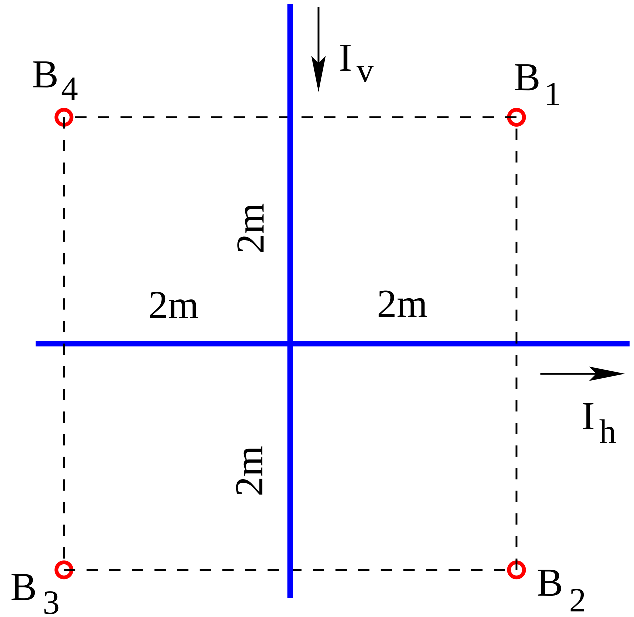

Solution:

- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{I_v}{2\text{m}} + \frac{I_h}{2\text{m}}\right) = +6 \times 10^{-7}\text{T}$ (out).
- $B_2 = \frac{\mu_0}{2\pi}\left(\frac{I_v}{2\text{m}} - \frac{I_h}{2\text{m}}\right) = 0.$
- $B_3 = \frac{\mu_0}{2\pi}\left(-\frac{I_v}{2\text{m}} - \frac{I_h}{2\text{m}}\right) = -6 \times 10^{-7}\text{T}$ (in).
- $B_4 = \frac{\mu_0}{2\pi}\left(-\frac{I_v}{2\text{m}} + \frac{I_h}{2\text{m}}\right) = 0.$