

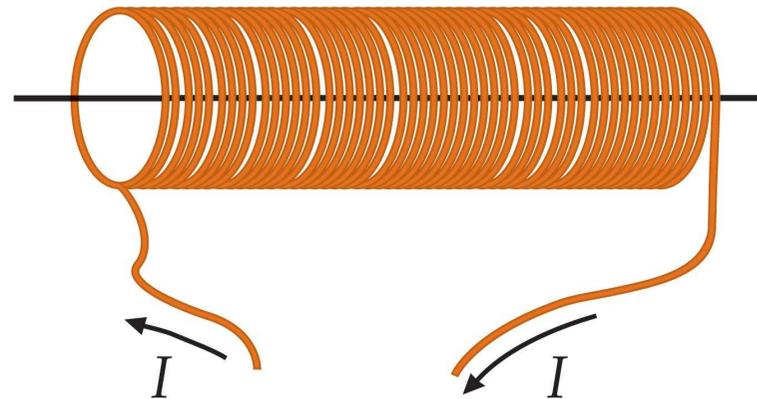


## Inductor (device):

- A wire that is wound into  $N$  turns of some shape and area.
- The current  $I$  flowing through the wire generates a magnetic field  $\vec{B}$  in its vicinity.
- The magnetic field  $\vec{B}$ , in turn, produces a magnetic flux  $\Phi_B$  through each turn.

## Inductance (device property):

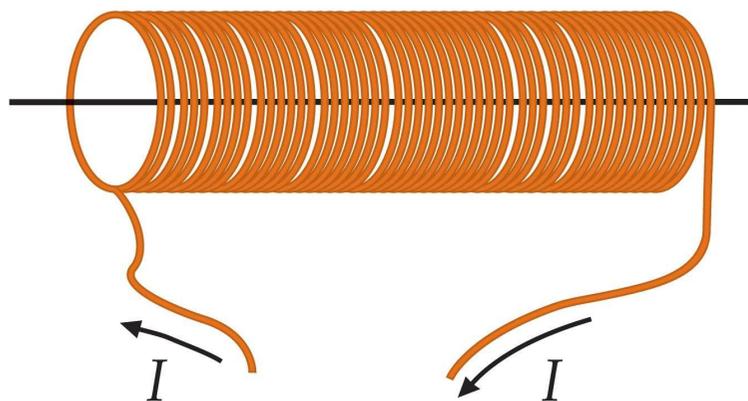
- Definition:  $L = \frac{N\Phi_B}{I}$
- SI unit:  $1\text{H} = 1\text{Wb/A}$  (one Henry)



# Inductance of a Solenoid



- $A$ : cross-sectional area
- $\ell$ : length
- $n$ : number of turns per unit length
- $N = n\ell$ : total number of turns
- $B = \mu_0 nI$ : magnetic field inside solenoid
- $\Phi_B = BA$ : magnetic flux through each turn
- $\Rightarrow$  Inductance of solenoid:  $L \equiv \frac{N\Phi_B}{I} = \mu_0 n^2 A\ell$



# Inductance of a Toroid



- Total number of turns:  $N$

- Magnetic field inside toroid:  $B = \frac{\mu_0 I}{2\pi r}$

- Magnetic flux through each turn (loop):

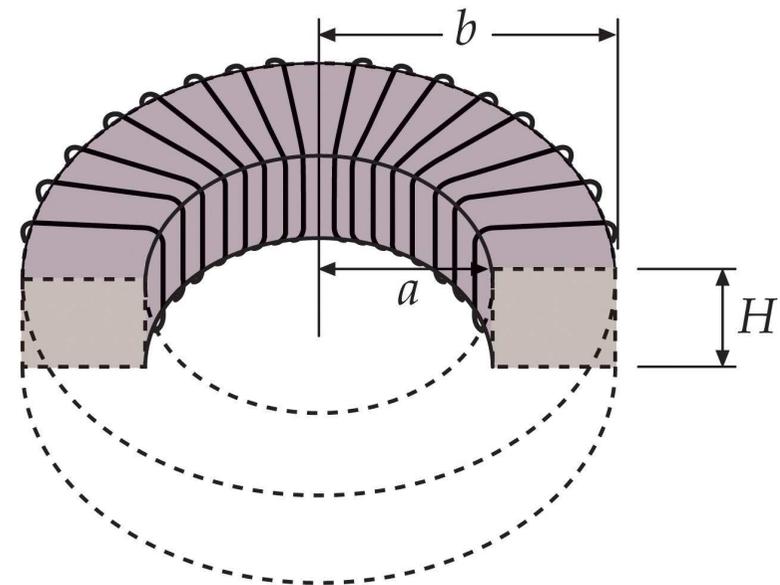
$$\Phi_B = \int_a^b BH dr = \frac{\mu_0 I N H}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 I N H}{2\pi} \ln \frac{b}{a}$$

- Inductance:  $L \equiv \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 H}{2\pi} \ln \frac{b}{a}$

- Narrow toroid:  $s \equiv b - a \ll a$

$$\ln \frac{b}{a} = \ln \left( 1 + \frac{s}{a} \right) \simeq \frac{s}{a}$$

- Inductance:  $L = \frac{\mu_0 N^2 (sH)}{2\pi a}$



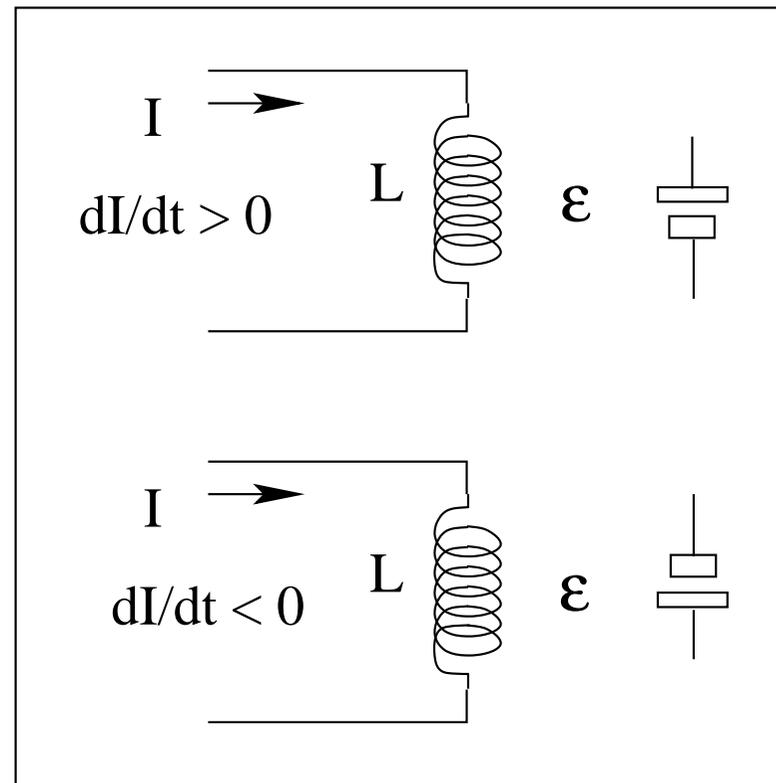
# Self-Induction



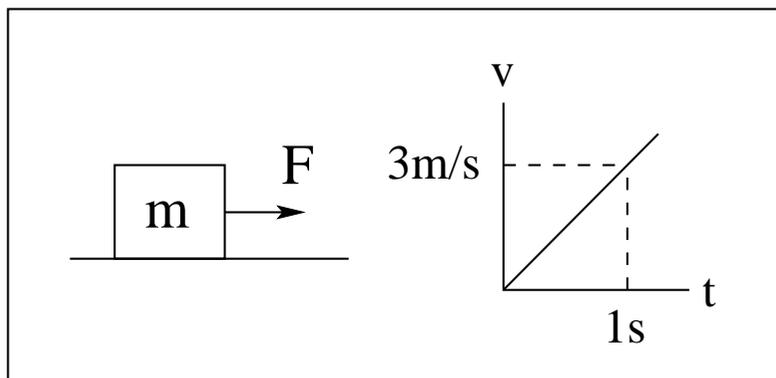
The induced electric EMF acts such as to oppose the change in the current that causes it (Lenz's law).

The presence of an inductance makes the electric current sluggish (resistant to change).

- Faraday's law:  $\mathcal{E} = -\frac{d}{dt}(N\Phi_B)$
- Inductance:  $L = \frac{N\Phi_B}{I}$
- Self-induced EMF:  $\mathcal{E} = -L\frac{dI}{dt}$



# Inertia: Mechanical vs Electrical

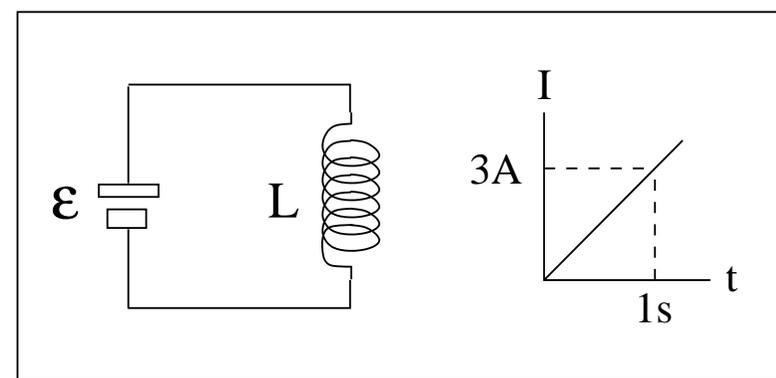


$$F = 6\text{N}, \quad m = 2\text{kg}$$

Newton's second law

$$F - m \frac{dv}{dt} = 0$$

$$\frac{dv}{dt} = \frac{F}{m} = 3\text{m/s}^2$$



$$\mathcal{E} = 6\text{V}, \quad L = 2\text{H}$$

Loop rule

$$\mathcal{E} - L \frac{dI}{dt} = 0$$

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L} = 3\text{A/s}$$

# Mutual Induction



$\Phi_{12}$ : magnetic flux through each loop of coil 2 caused by current  $I_1$  through coil 1

$\Phi_{21}$ : magnetic flux through each loop of coil 1 caused by current  $I_2$  through coil 2

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1} \quad (\text{mutual inductance})$$

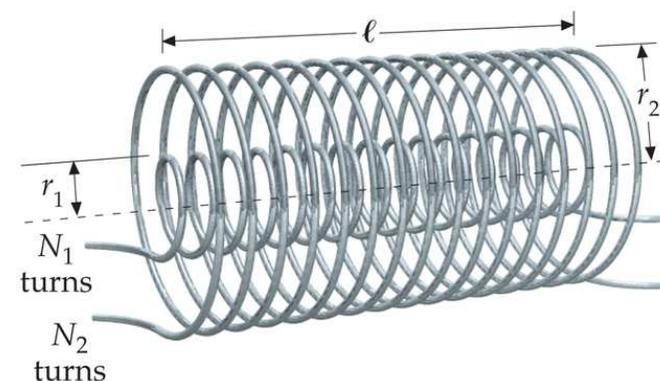
$$M_{21} = \frac{N_1 \Phi_{21}}{I_2} \quad (\text{mutual inductance})$$

$$\mathcal{E}_2 = -M_{12} \frac{dI_1}{dt} \quad (\text{emf induced in coil 2 due to current in coil 1})$$

$$\mathcal{E}_1 = -M_{21} \frac{dI_2}{dt} \quad (\text{emf induced in coil 1 due to current in coil 2})$$

$$M_{12} = M_{21} = M \quad (\text{symmetry property})$$

$$M = \mu_0 \frac{N_1}{\ell} \frac{N_2}{\ell} (\ell \pi r_1^2) \quad (\text{present configuration})$$



(a)

# Energy Stored in Inductor



Establishing a current in the inductor requires work.  
The work done is equal to the potential energy stored in the inductor.

- Current through inductor:  $I$  (increasing)
- Voltage induced across inductor:  $|\mathcal{E}| = L \frac{dI}{dt}$
- Power absorbed by inductor:  $P = |\mathcal{E}|I$
- Increment of potential energy:  $dU = Pdt = LI dI$
- Potential energy of inductor with current  $I$  established:

$$U = L \int_0^I I dI = \frac{1}{2} LI^2$$

Q: where is the potential energy stored?

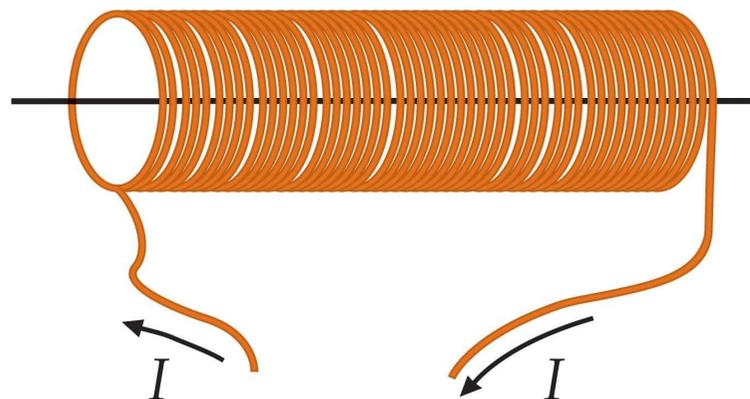
A: in the magnetic field.

# Energy Density Within Solenoid



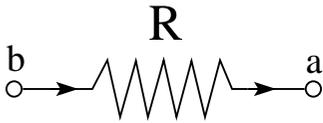
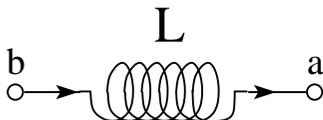
Energy is stored in the magnetic field inside the solenoid.

- Inductance:  $L = \mu_0 n^2 A \ell$
- Magnetic field:  $B = \mu_0 n I$
- Potential energy:  $U = \frac{1}{2} L I^2 = \frac{1}{2\mu_0} B^2 (A \ell)$
- Volume of solenoid interior:  $A \ell$
- Energy density of magnetic field:  $u_B = \frac{U}{A \ell} = \frac{1}{2\mu_0} B^2$



# Response of Device to Electric Current



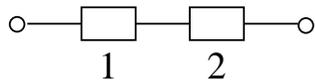
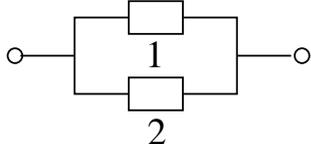
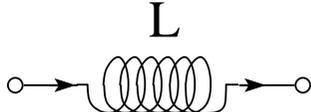
device	symbol	voltage
resistor		$V_{ab} = RI$
inductor		$V_{ab} = L \frac{dI}{dt}$
capacitor		$V_{ab} = \frac{1}{C} Q \quad \frac{dQ}{dt} = I$

Note that the response of each device to a current  $I$  is a voltage  $V_{ab} \equiv V_b - V_a$ .

- resistor: response proportional to current itself
- inductor: response proportional to derivative of current
- capacitor: response proportional to integral of current

# Devices in Series or in Parallel



		in series	in parallel
resistors		 $R_{eq} = R_1 + R_2$	 $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$
inductors		$L_{eq} = L_1 + L_2$	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$
capacitors		$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$	$C_{eq} = C_1 + C_2$

## Magnetic Induction: Application (13)

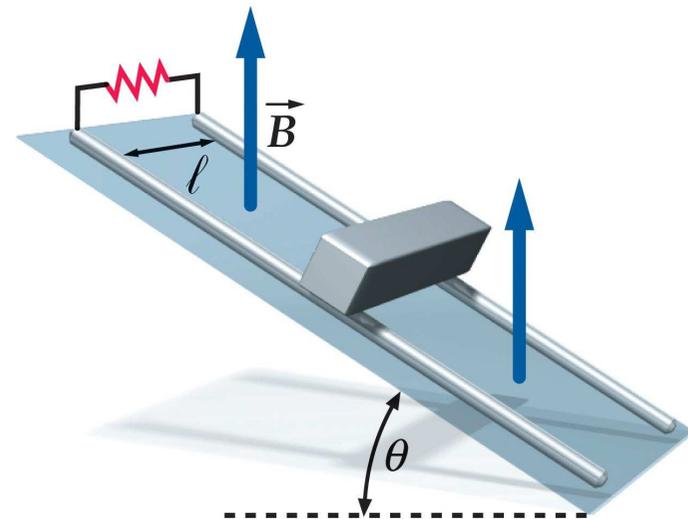


A rod of length  $\ell$ , mass  $m$ , and negligible resistance slides without friction down a pair of parallel conducting rails, which are connected at the top of the incline by a resistor with resistance  $R$ . A uniform vertical magnetic field  $\vec{B}$  exists throughout the region.

- (a) Identify the forces acting on the rod when it slides down with velocity  $v$ .
- (b) Determine the velocity for which all forces acting on the rod are in balance.

Determine the direction of the induced current from

- (c) the magnetic force acting on the charge carriers in the rod,
- (d) from the change in magnetic flux through the conducting loop,
- (e) from Lenz's law.



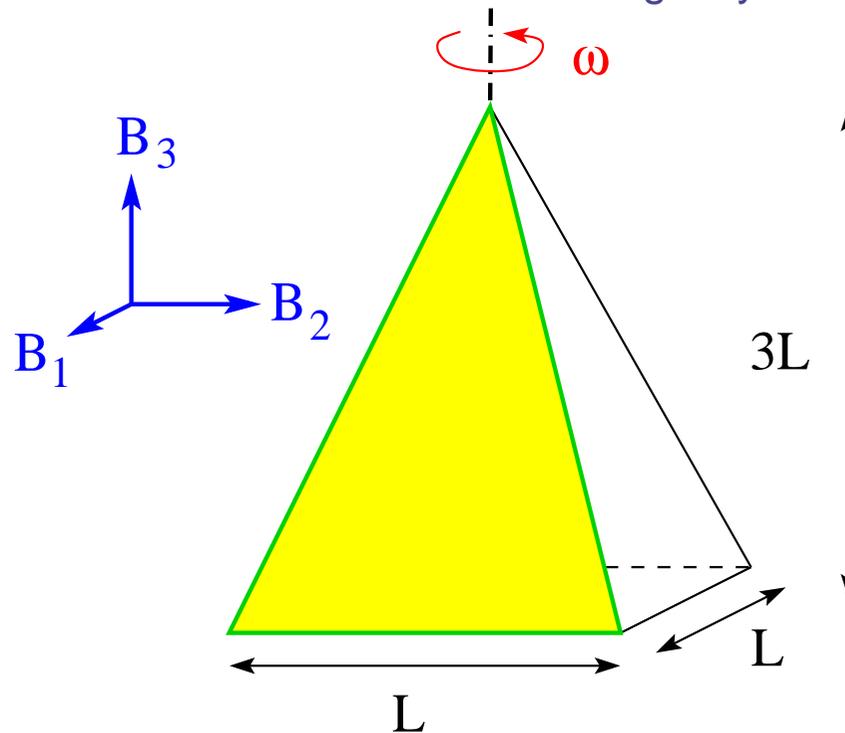
# Magnetic Induction: Application (11)



The pyramid is positioned in one of three magnetic fields  $\vec{B}_1, \vec{B}_2, \vec{B}_3$  of 1T magnitude in the directions shown and rotates about its vertical axis with angular velocity  $\omega$ . The green triangular face is a conducting frame.

At the instant pictured here, determine...

- (a) which field produces the largest/smallest magnetic flux through the triangle,
- (b) what the direction of the current induced in the triangle by each field is (cw,ccw,zero).

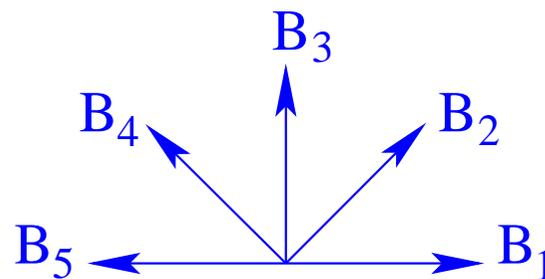
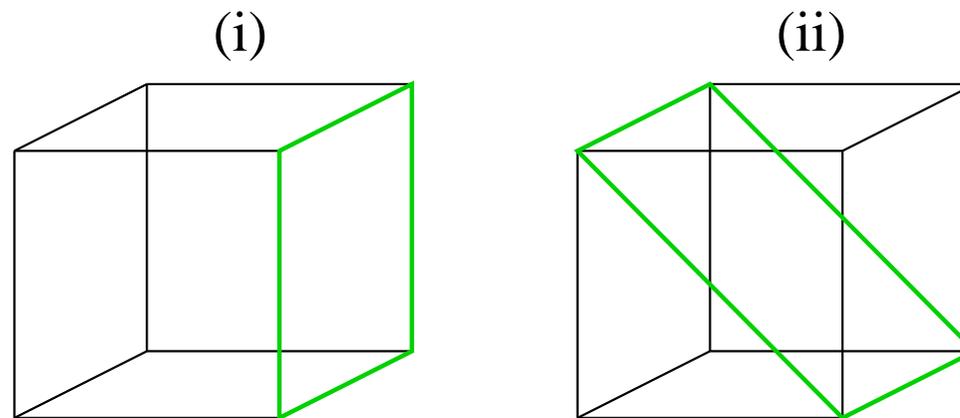


# Magnetic Induction: Application (12)



Consider two conducting loops (i) and (ii) (indicated by green lines in cubes of sides  $L = 2\text{m}$ ). Each loop is placed in a region of uniform magnetic field with linearly increasing magnitude,  $B(t) = bt, b = 2\text{T/s}$ , and one of the five directions indicated.

- (a) Find the magnetic flux through each loop as produced by each field.
- (b) Find the magnitude and direction of the emf induced by each field in each loop.



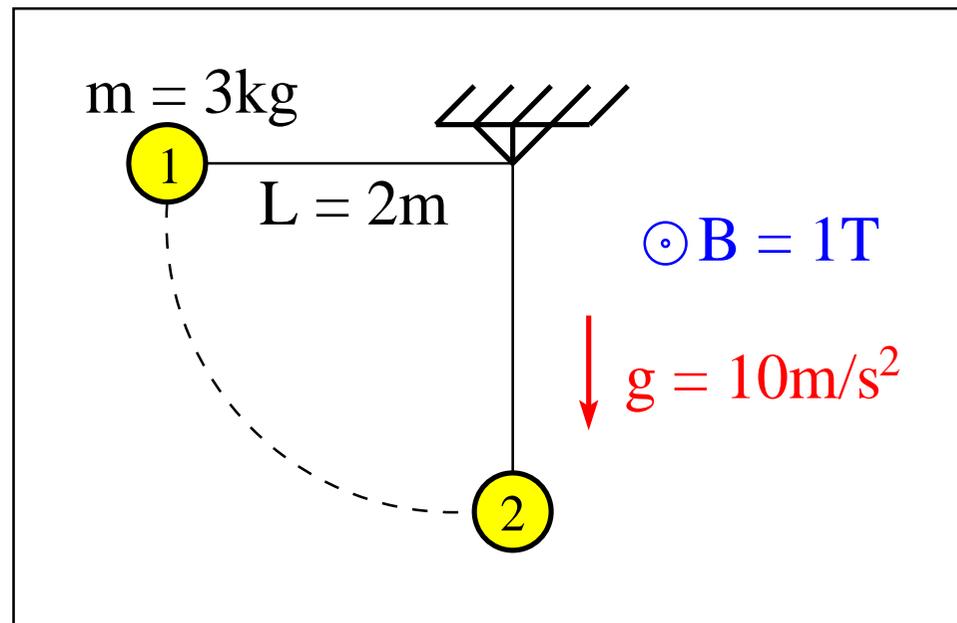
## Magnetic Induction: Application (2)



A conducting sphere attached to the pivot by a thin conducting wire is released from rest in position 1. There is a uniform magnetic field pointing out of the plane.

When the sphere has reached position 2...

- (a) which is at the higher gravitat. potential: pivot/sphere?
- (b) which is at the higher electrical potential: pivot/sphere?
- (c) what is the induced EMF between pivot and sphere?



# Magnetic Induction: Application (14)



Consider a conducting frame moving in the magnetic field of a straight current-carrying wire.

- magnetic field:  $B = \frac{\mu_0 I}{2\pi r}$
- magnetic flux:  $\Phi_B = \int \vec{B} \cdot \vec{A}, \quad dA = adr$

$$\Phi_B = \frac{\mu_0 I a}{2\pi} \int_x^{x+b} \frac{dr}{r} = \frac{\mu_0 I a}{2\pi} [\ln(x+b) - \ln x] = \frac{\mu_0 I a}{2\pi} \ln \frac{x+b}{x}$$

- induced EMF:  $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d\Phi_B}{dx} \frac{dx}{dt} = -\frac{d\Phi_B}{dx} v$

$$\mathcal{E} = -\frac{\mu_0 I a v}{2\pi} \left[ \frac{1}{x+b} - \frac{1}{x} \right] = \frac{\mu_0 I a b v}{2\pi x(x+b)}$$

- induced current:  $I_{ind} = \frac{\mathcal{E}}{R}$  clockwise

