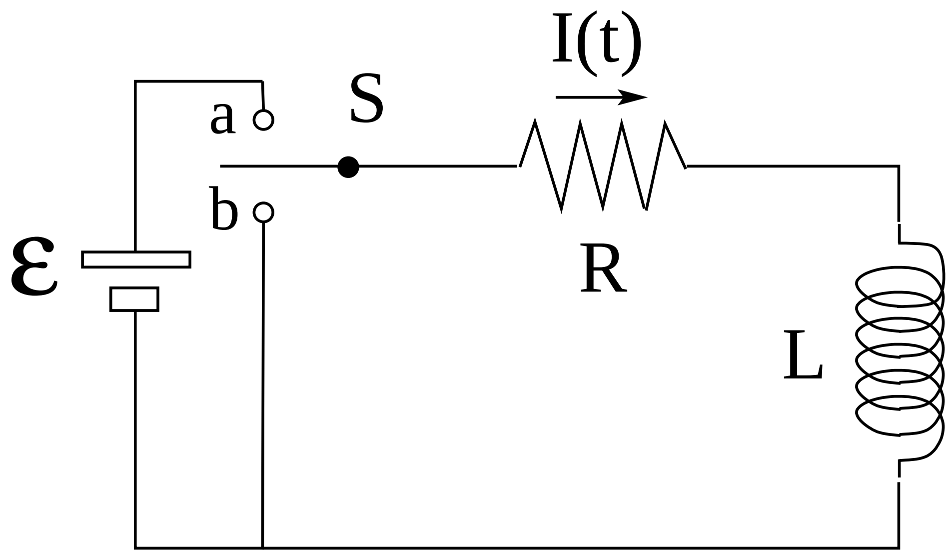
RL Circuit: Fundamentals

Specifications:

- $\mathcal{E}$ (emf)
- R (resistance)
- L (inductance)

Switch S:

- a: current buildup
- b: current shutdown



Time-dependent quantities:

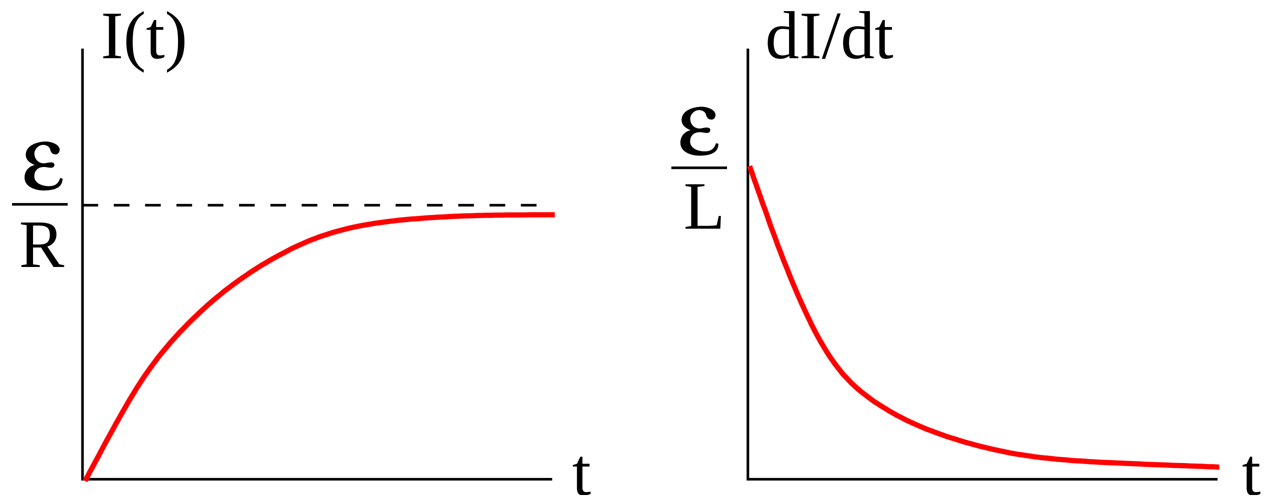
- $I(t)$: instantaneous current through inductor
- $\frac{dI}{dt}$: rate of change of instantaneous current
- $V_R(t) = I(t)R$: instantaneous voltage across resistor
- $V_L(t) = L\,\frac{dI}{dt}$: instantaneous voltage across inductor

RL Circuit: Current Buildup in Inductor

- Loop rule: $\mathcal{E} - IR - L\frac{dI}{dt} = 0$
- Differential equation: $L\frac{dI}{dt} = \mathcal{E} - IR \quad \Rightarrow \frac{dI}{dt} = \frac{\mathcal{E}/R - I}{L/R}$

$$\int_0^I \frac{dI}{\mathcal{E}/R - I} = \int_0^t \frac{dt}{L/R} \quad \Rightarrow \; -\ln\left(\frac{\mathcal{E}/R - I}{\mathcal{E}/R}\right) = \frac{t}{L/R} \quad \Rightarrow \frac{\mathcal{E}/R - I}{\mathcal{E}/R} = e^{-Rt/L}$$

- Current through inductor: $I(t) = \frac{\mathcal{E}}{R}\left[1 - e^{-Rt/L}\right]$
- Rate of current change: $\frac{dI}{dt} = \frac{\mathcal{E}}{L}\, e^{-Rt/L}$

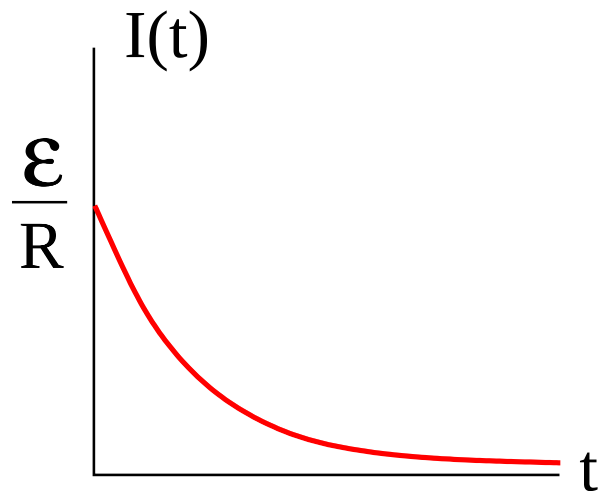


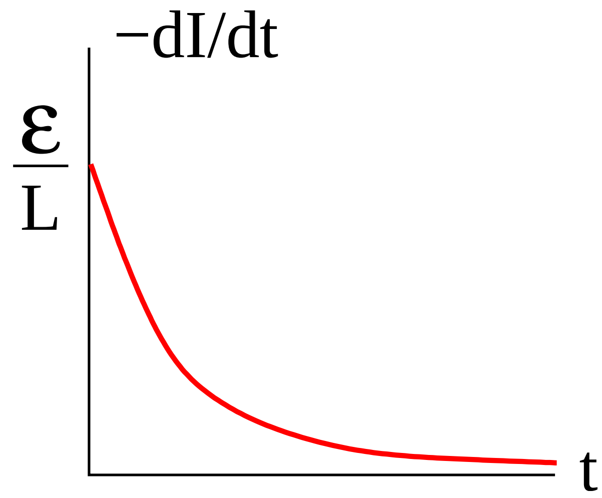
RL Circuit: Current Shutdown in Inductor

- Loop rule: $-IR - L\frac{dI}{dt} = 0$
- Differential equation: $L\frac{dI}{dt} + IR = 0 \quad \Rightarrow \quad \frac{dI}{dt} = -\frac{R}{L}I$

$$\Rightarrow \int_{\mathcal{E}/R}^{I} \frac{dI}{I} = -\frac{R}{L}\int_0^t dt \quad \Rightarrow \quad \ln\frac{I}{\mathcal{E}/R} = -\frac{R}{L}t \quad \Rightarrow \quad \frac{I}{\mathcal{E}/R} = e^{-Rt/L}$$

- Current: $I(t) = \frac{\mathcal{E}}{R}\, e^{-Rt/L}$
- Rate of current change: $\frac{dI}{dt} = -\frac{\mathcal{E}}{L}\, e^{-Rt/L}$



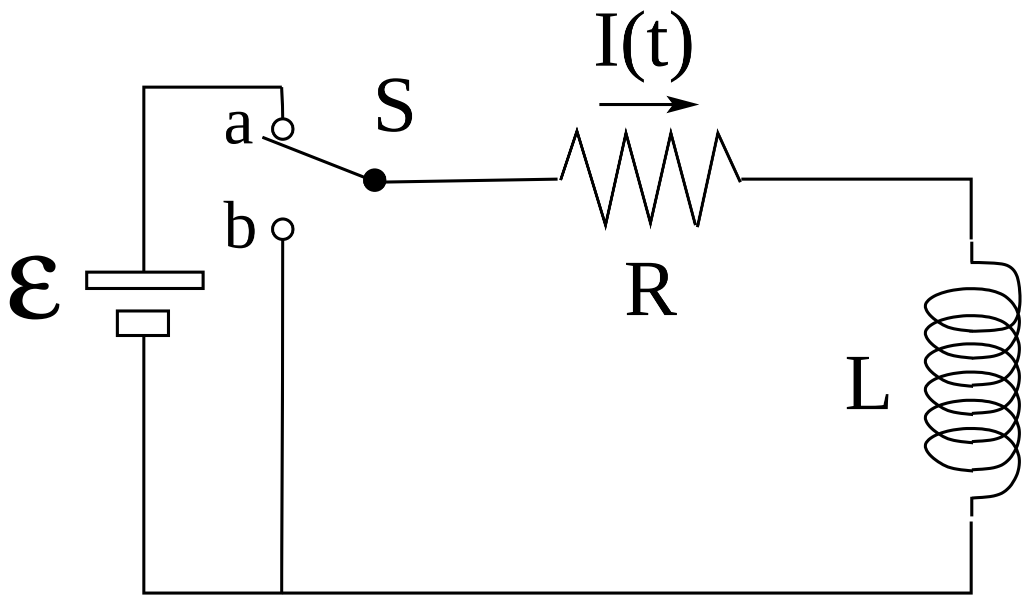


RL Circuit: Energy Transfer During Current Buildup

Loop rule: $IR + L\frac{dI}{dt} = \mathcal{E} \quad (I > 0,\ \frac{dI}{dt} > 0)$

- $I\mathcal{E}$: rate at which EMF source delivers energy
- $IV_R = I^2R$: rate at which energy is dissipated in resistor
- $IV_L = LI\frac{dI}{dt}$: rate at which energy is stored in inductor

Balance of energy transfer: $I^2R + LI\frac{dI}{dt} = I\mathcal{E}$

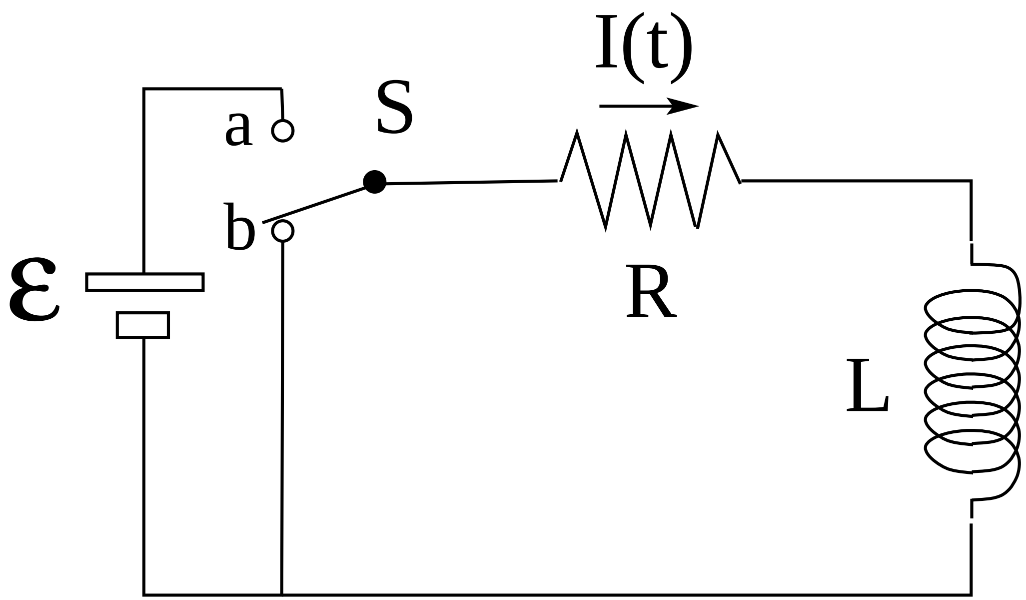


RL Circuit: Energy Transfer During Current Shutdown

Loop rule: $IR + L\frac{dI}{dt} = 0 \quad \left(I > 0,\ \frac{dI}{dt} < 0\right)$

- $IV_L = LI\frac{dI}{dt}$: rate at which inductor releases energy
- $IV_R = I^2R$: rate at which energy is dissipated in resistor

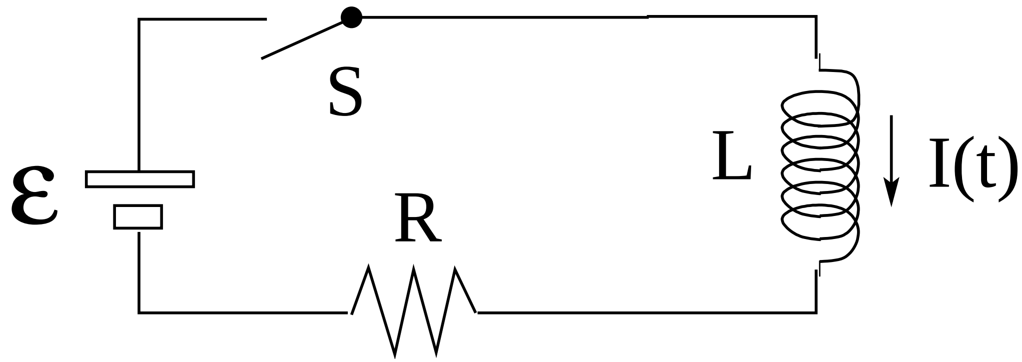
Balance of energy transfer: $I^2R + LI\frac{dI}{dt} = 0$



RL Circuit: Some Physical Properties

Specification of RL circuit by 3 device properties:

- $\mathcal{E}$ [V] (emf)
- R $[\Omega]$ (resistance)
- L [H] (inductance)



Physical properties of RL circuit during current buildup determined by 3 combinations of the device properties:

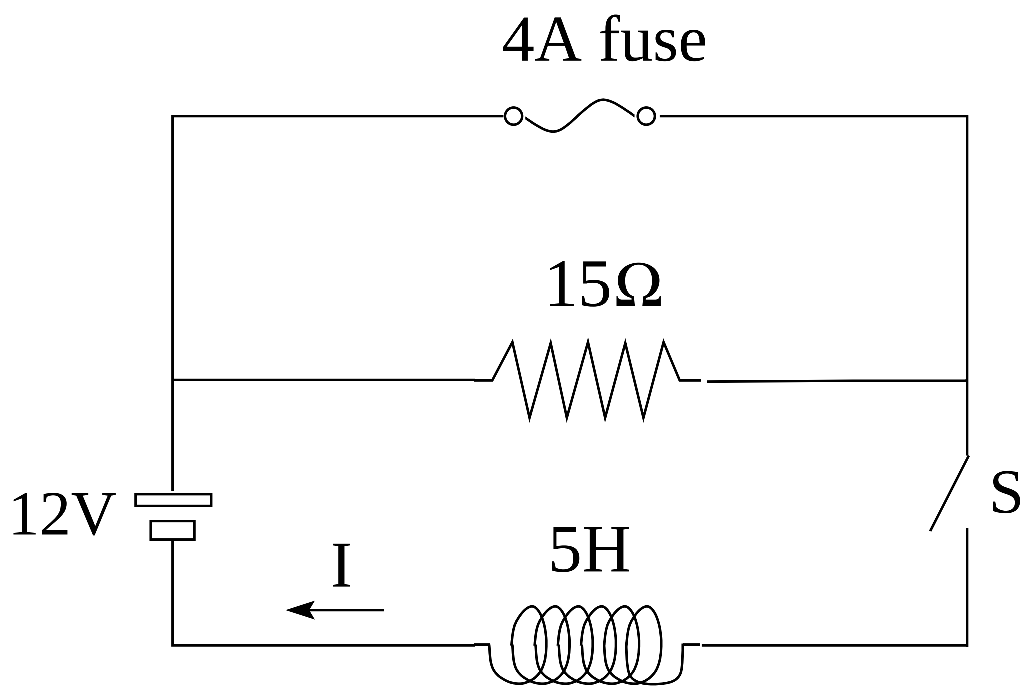
- $\frac{\mathcal{E}}{L} = \left.\frac{dI}{dt}\right|_{t=0}$: initial rate at which current increases
- $\frac{\mathcal{E}}{R} = I(t = \infty)$: final value of current
- $L/R = \tau$: time it takes to build up 63% of the current through the circuit
  $[1 - e^{-1} = 0.632\ldots]$

RL Circuit: Application (7)

In the circuit shown the switch S is closed at time $t = 0$.

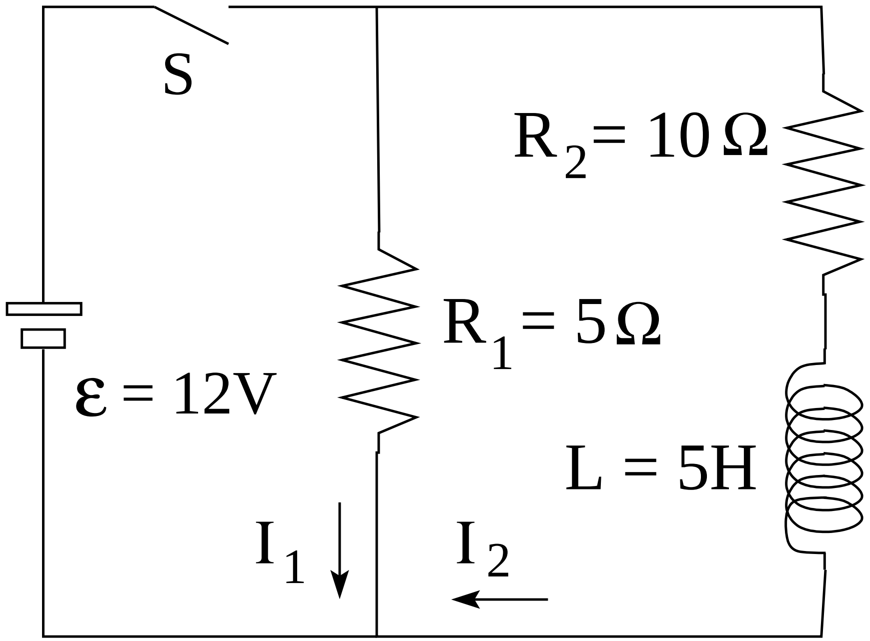
(a) Find the current I as a function of time for $0 < t < t_F$, where t_F marks the instant the fuse breaks.

(b) Find the current I as a function of time for $t > t_F$.



In the circuit shown the switch has been open for a long time.
Find the currents I_1 and I_2

- just after the switch has been closed,
- a long time later,
- as functions of time for $0 < t < \infty$.



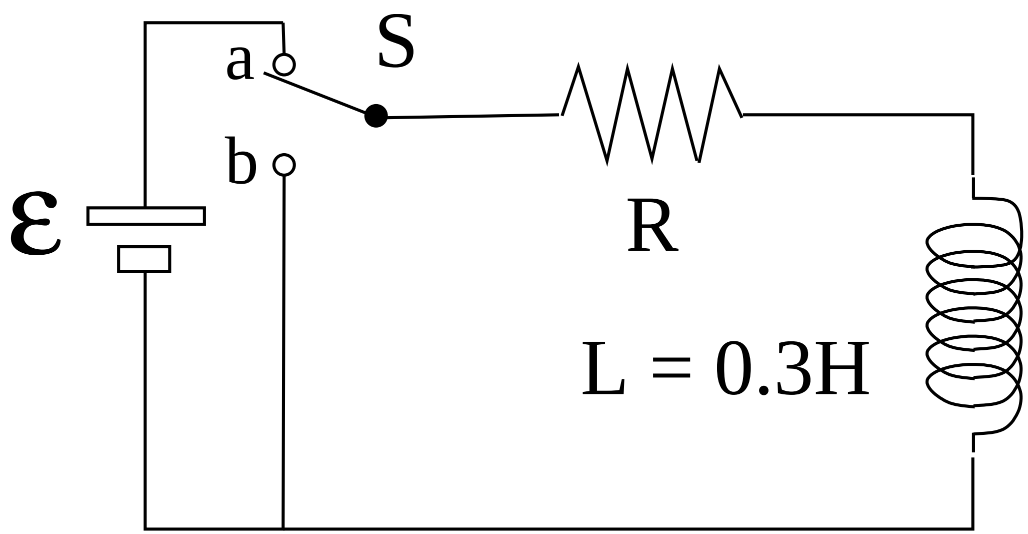
In the RL circuit shown the switch has been at position a for a long time and is thrown to position b at time $t = 0$. At that instant the current has the value $I_0 = 0.7$A and decreases at the rate $dI/dt = -360$A/s.

(a) Find the EMF $\mathcal{E}$ of the battery.

(b) Find the resistance R of the resistor.

(c) At what time t_1 has the current decreased to the value $I_1 = 0.2$A?

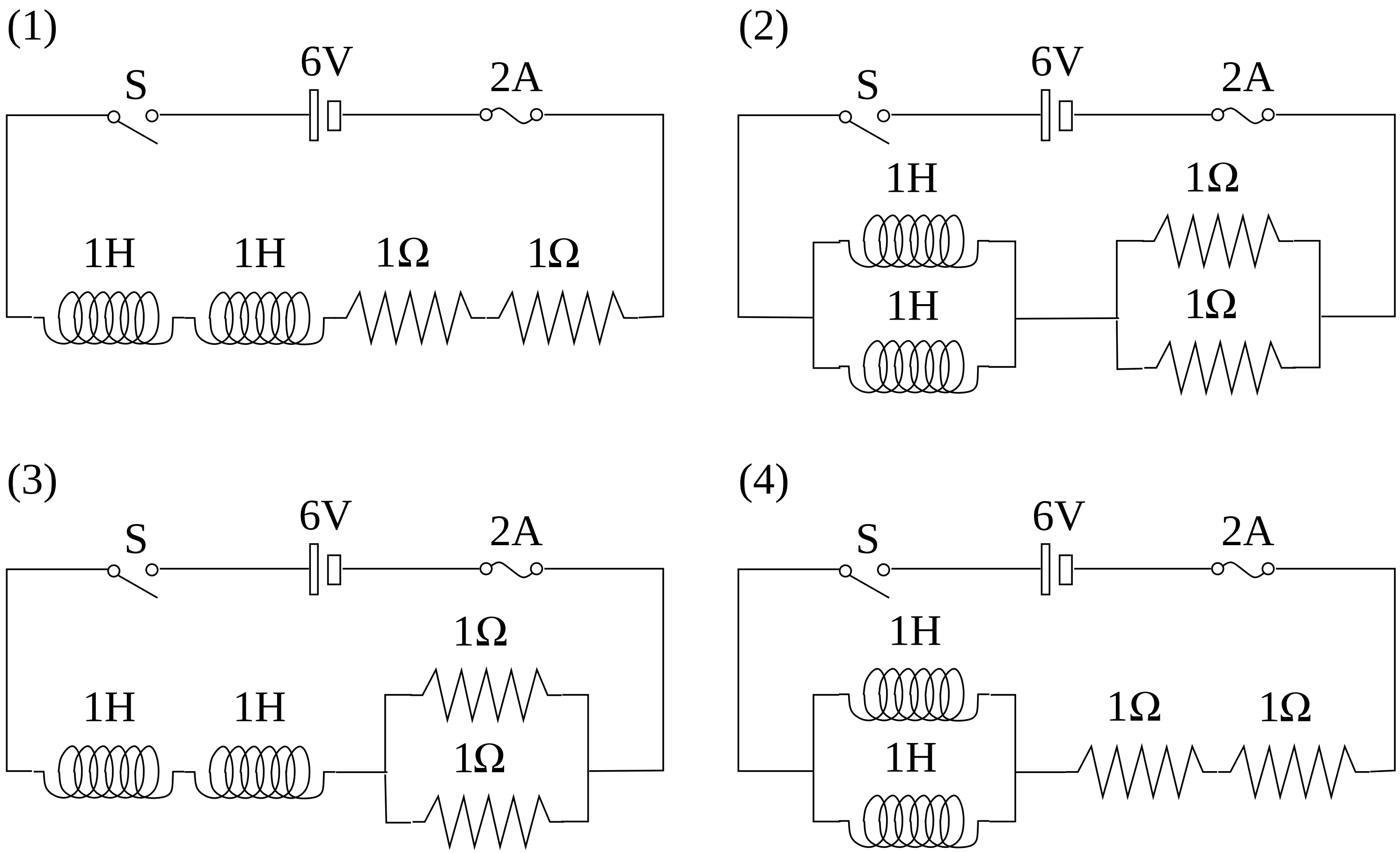
(d) Find the voltage across the inductor at time t_1.



RL Circuit: Application (5)

Each RL circuit contains a 2A fuse. The switches are closed at $t = 0$.

- In what sequence are the fuses blown?



RL Circuit: Application (3)

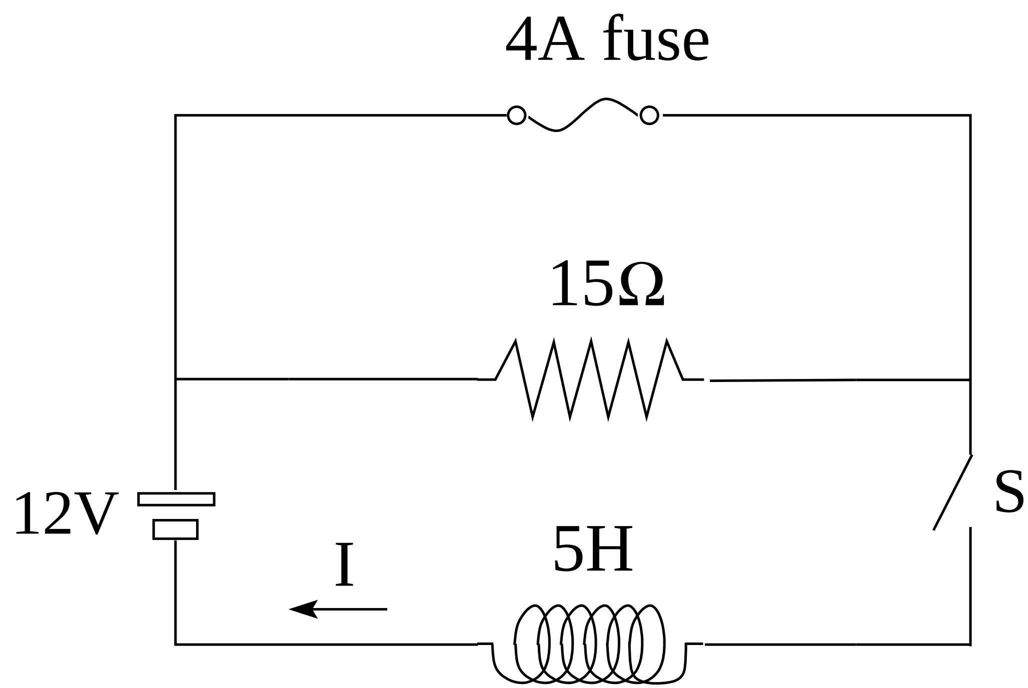
The switch is closed at $t = 0$. Find the current I

(a) immediately after the switch has been closed,

(b) immediately before the fuse breaks,

(c) immediately after the fuse has broken,

(d) a very long time later.

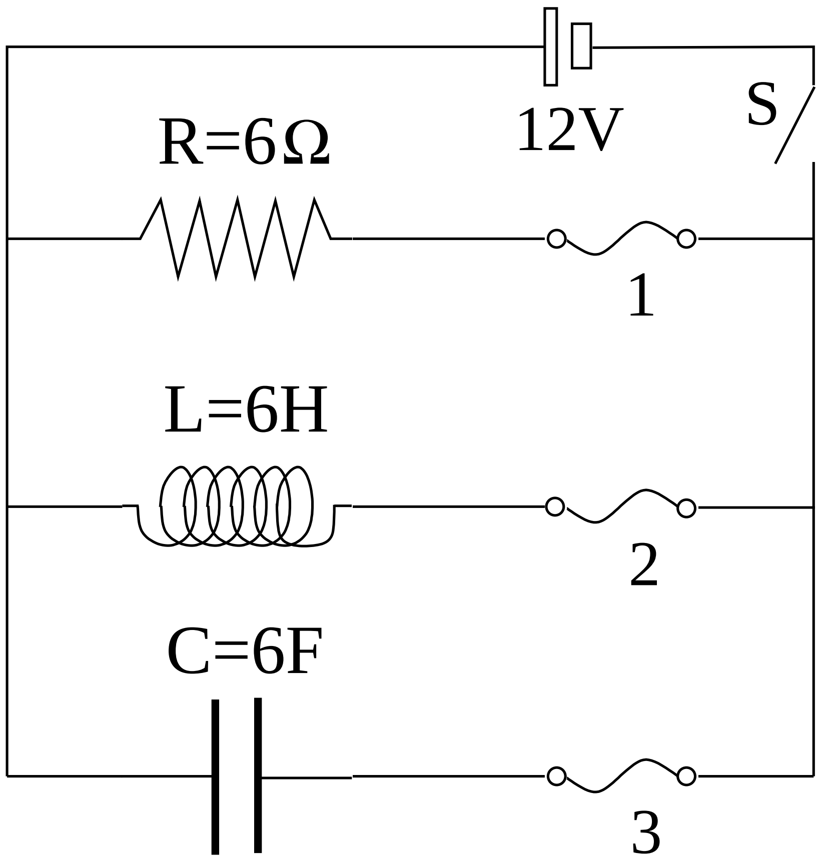


RL Circuit: Application (1)

Each branch in the circuit shown contains a 3A fuse. The switch is closed at time $t = 0$.

(a) Which fuse is blown in the shortest time?

(b) Which fuse lasts the longest time?

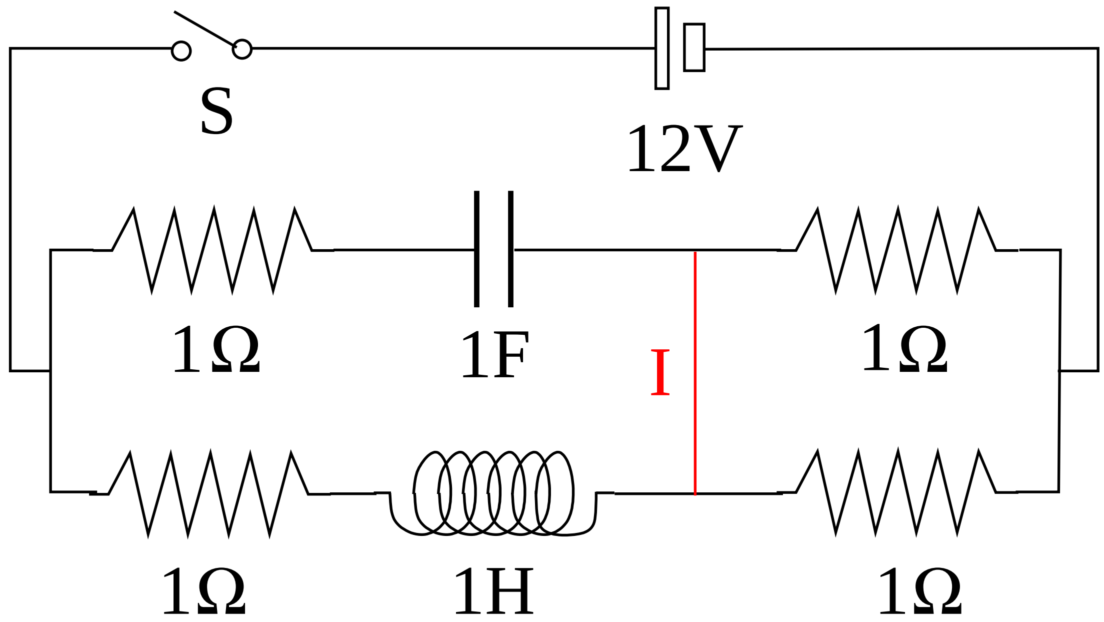


RL Circuit: Application (4)

Find the magnitude (in amps) and the direction $(\uparrow, \downarrow)$ of the current I

(a) right after the switch has been closed,

(b) a very long time later.

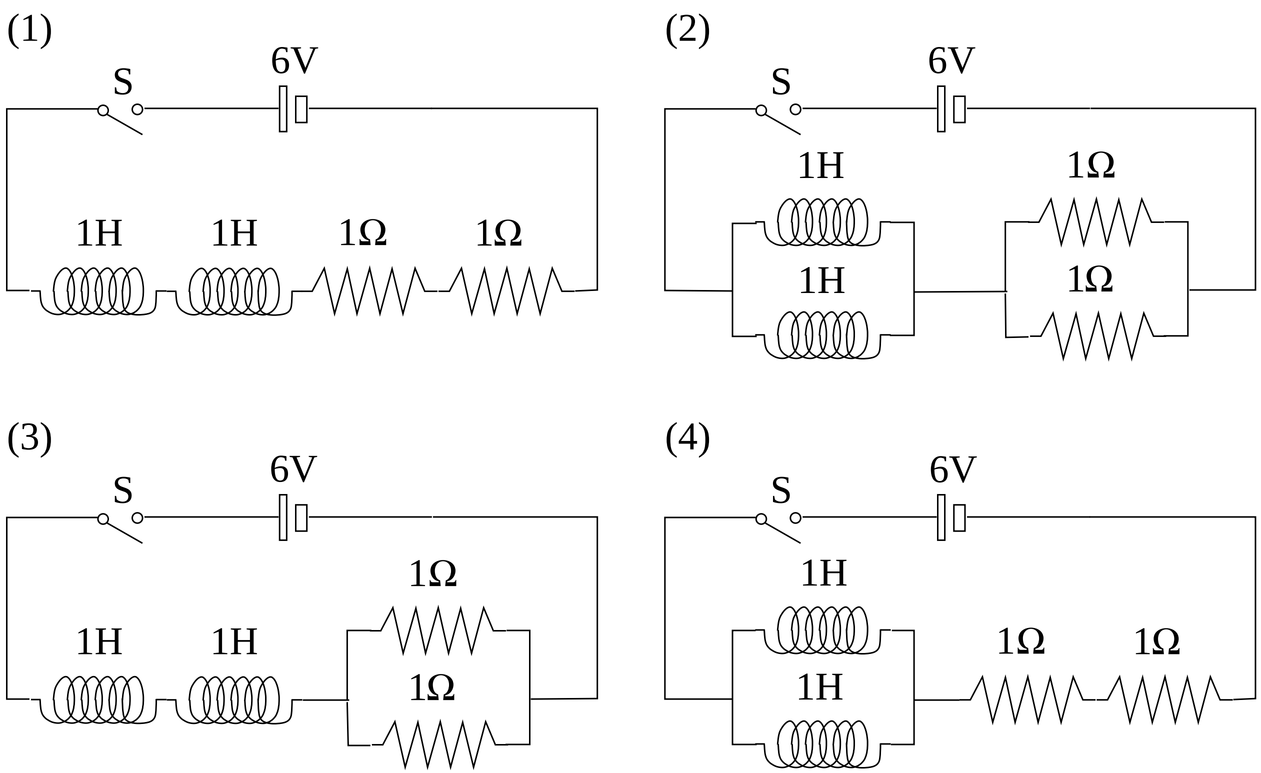


The switch in each RL circuit is closed at $t = 0$.
Rank the circuits according to three criteria:

(a) magnitude of current at $t =$1ms,

(b) magnitude of current at $t = \infty$,

(c) time it takes I to reach 63% of its ultimate value.

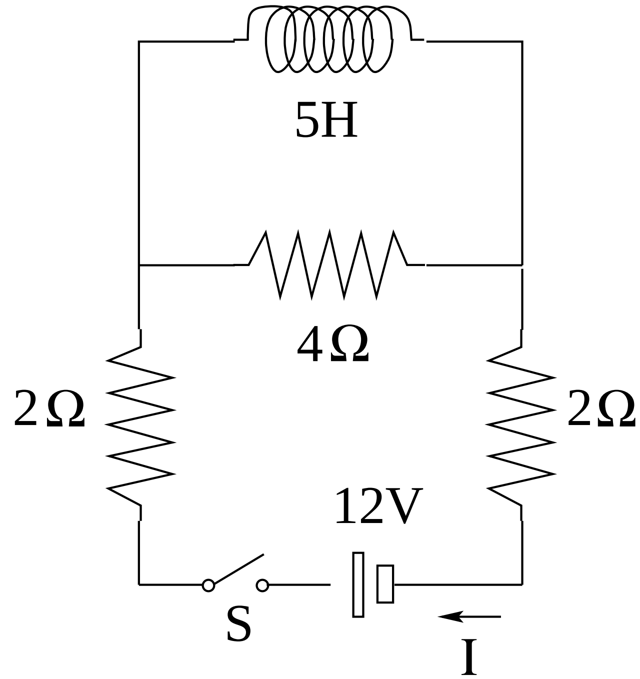


Intermediate Exam III: Problem #2 (Spring ’05)

In the circuit shown we close the switch S at time $t = 0$. Find the current I through the battery and the voltage V_L across the inductor

(a) immediately after the switch has been closed,

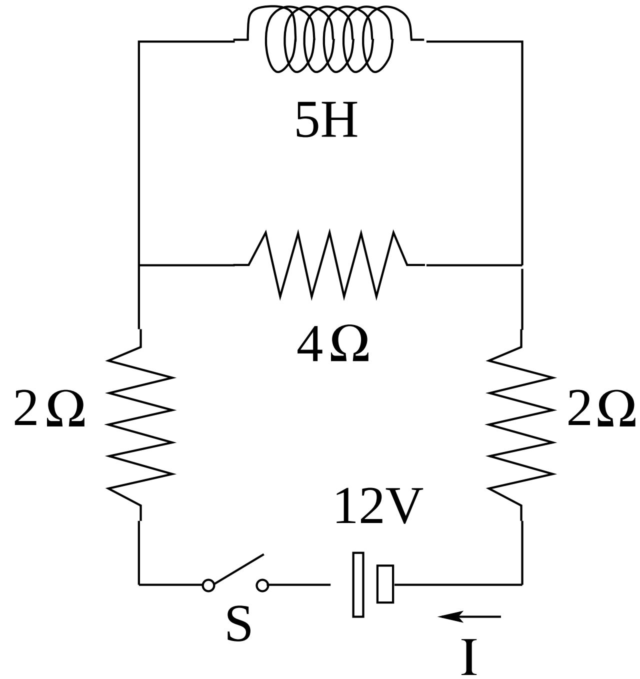
(b) a very long time later.



In the circuit shown we close the switch S at time $t = 0$. Find the current I through the battery and the voltage V_L across the inductor

(a) immediately after the switch has been closed,

(b) a very long time later.



Solution:

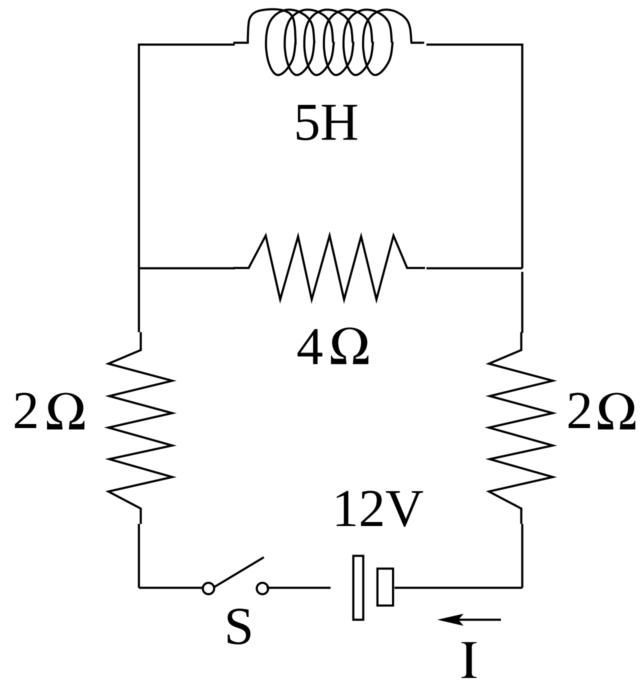
(a) $I = \dfrac{12\text{V}}{2\Omega + 4\Omega + 2\Omega} = 1.5\text{A}, \qquad V_L = (4\Omega)(1.5\text{A}) = 6\text{V}.$

Intermediate Exam III: Problem #2 (Spring '05)

In the circuit shown we close the switch S at time $t = 0$. Find the current I through the battery and the voltage V_L across the inductor

(a) immediately after the switch has been closed,

(b) a very long time later.



Solution:

(a) $I = \dfrac{12\text{V}}{2\Omega + 4\Omega + 2\Omega} = 1.5\text{A}, \qquad V_L = (4\Omega)(1.5\text{A}) = 6\text{V}.$

(b) $I = \dfrac{12\text{V}}{2\Omega + 2\Omega} = 3\text{A}, \qquad V_L = 0.$