Magnetic Moment of a Rotating Disk

Consider a nonconducting disk of radius R with a uniform surface charge density σ. The disk rotates with angular velocity $\vec{\omega}$.

Calculation of the magnetic moment $\vec{\mu}$:

- Total charge on disk: $Q = \sigma(\pi R^2)$.
- Divide the disk into concentric rings of width dr.
- Period of rotation: $T = \dfrac{2\pi}{\omega}$.
- Current within ring: $dI = \dfrac{dQ}{T} = \sigma(2\pi r dr)\dfrac{\omega}{2\pi} = \sigma\omega r dr$.
- Magnetic moment of ring: $d\mu = dI(\pi r^2) = \pi\sigma\omega r^3 dr$.
- Magnetic moment of disk: $\mu = \displaystyle\int_0^R \pi\sigma\omega r^3 dr = \frac{\pi}{4}\sigma R^4\omega$.
- Vector relation: $\vec{\mu} = \dfrac{\pi}{4}\sigma R^4\vec{\omega} = \dfrac{1}{4}QR^2\vec{\omega}$.