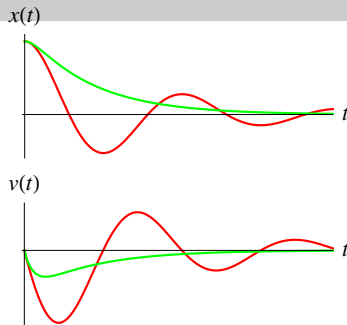


Mechanical Oscillator with Damping



- law of motion: $F = ma$, $a = \frac{d^2x}{dt^2}$
- law of force: $F = -kx - bv$, $v = \frac{dx}{dt}$
- equation of motion: $\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$



Solution for initial conditions $x(0) = A$, $v(0) = 0$:

(a) **underdamped motion:** $b^2 < 4km$

$$x(t) = Ae^{-bt/2m} \left[\cos(\omega' t) + \frac{b}{2m\omega'} \sin(\omega' t) \right] \quad \text{with} \quad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

(b) **overdamped motion:** $b^2 > 4km$

$$x(t) = Ae^{-bt/2m} \left[\cosh(\Omega' t) + \frac{b}{2m\Omega'} \sinh(\Omega' t) \right] \quad \text{with} \quad \Omega' = \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}$$