Charged Bead Moving Along Axis of Charged Ring



Consider a negatively charged bead (mass m, charge -q) constrained to move without friction along the axis of a positively charged ring.

- Place bead on *x*-axis near center of ring: $|x| \ll a$: $E_x \simeq \frac{kQx}{a^3}$
- Restoring force: $F = -qE_x = -k_sx$ with $k_s = \frac{kQq}{a^3}$
- Acceleration: $a = \frac{F}{m} = -\frac{k_s}{m} x$
- Equation of motion: $\frac{d^2x}{dt^2} = -\frac{k_s}{m}x$
- Harmonic oscillation: $x(t) = A\cos(\omega t + \phi)$
- Angular frequency: $\omega = \sqrt{\frac{k_{\rm S}}{m}} = \sqrt{\frac{k Q q}{m a^3}}$

