Electric Field of Charged Rod (4)

Symmetry dictates that the resulting electric field is directed radially (alternative derivation).

- Charge per unit length: $\lambda = Q/L$
- Charge on slice dx: $dq = \lambda dx$
- $dE = \dfrac{kdq}{r^2} = \dfrac{k\lambda dx}{x^2+y^2}$
- $dE_y = dE\cos\theta = \dfrac{dEy}{\sqrt{x^2+y^2}} = \dfrac{k\lambda y dx}{(x^2+y^2)^{3/2}}$
- $E_y = \displaystyle\int_{-L/2}^{+L/2} \frac{k\lambda y dx}{(x^2+y^2)^{3/2}} = \left[\frac{k\lambda y x}{y^2\sqrt{x^2+y^2}}\right]_{-L/2}^{+L/2}$
- $E_y = \dfrac{k\lambda L}{y\sqrt{(L/2)^2+y^2}} = \dfrac{kQ}{y\sqrt{(L/2)^2+y^2}}$
- Large distance $(y \gg L)$: $E_y \simeq \dfrac{kQ}{y^2}$
- Small distances $(y \ll L)$: $E_y \simeq \dfrac{2k\lambda}{y}$