Work and Potential Energy in 3D Space

Consider a particle acted on by a force $\vec{F}$ as it moves along a specific path in 3D space.

- Force: $\vec{F}(\vec{r}) = F_x(x,y,z)\,\hat{i} + F_y(x,y,z)\,\hat{j} + F_z(x,y,z)\,\hat{k}$
- Displacement: $d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$
- Potential energy: $U(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}\cdot d\vec{s} = -\int_{x_0}^{x} F_x dx - \int_{y_0}^{y} F_y dy - \int_{z_0}^{z} F_z dz$
- Work: $W_{if} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}\cdot d\vec{s} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$

Note: The work done by a conservative force is path-independent.