Potential Energy of Charged Particle in Coulomb Field

- Electrostatic force: $\vec{F} = \dfrac{kqQ}{r^2}\hat{r}$ (conservative)
- Displacement: $d\vec{s} = d\vec{r} + d\vec{s}_{\perp}, \quad d\vec{r} = dr\hat{r}$
- Work: $W_{if} = \displaystyle\int_i^f \vec{F}\cdot d\vec{s} = kqQ\int_i^f \frac{\hat{r}\cdot d\vec{s}}{r^2} = kqQ\int_{r_i}^{r_f}\frac{dr}{r^2} = kqQ\left[-\frac{1}{r}\right]_{r_i}^{r_f} = -kqQ\left[\frac{1}{r_f}-\frac{1}{r_i}\right]$
- Potential energy: $U = -\displaystyle\int_{\vec{r}_0}^{\vec{r}}\vec{F}\cdot d\vec{s} = -\int_\infty^r F dr = -kqQ\int_\infty^r\frac{dr}{r^2} = k\frac{qQ}{r}$
- Electric potential: $V(r) = -\displaystyle\int_{\vec{r}_0}^{\vec{r}}\vec{E}\cdot d\vec{s} = -\int_\infty^r E dr = -kQ\int_\infty^r\frac{dr}{r^2} = \frac{kQ}{r}$